Phenomenological bosonization in multicomponent systems

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May 28, 2010

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Second Quantized Hamiltonian

$$H = \sum_j -\frac{\hbar^2}{2M_j}\int dx \psi_j^\dagger(x)\partial_x^2\psi_j(x') + \frac{1}{2}\int dx dx' \sum_{i,j} V_{ij}(x-x')\rho_i(x)\rho_j(x')$$

with:

$$[\psi_j^\dagger(x), \psi_j(x')]_\pm = \delta(x-x')$$
$$\rho_j(x) = \psi_j^\dagger(x)\psi_j(x)$$

(+=fermions, -=bosons)

V_{ij} are short ranged.

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Phenomenological bosonization

(Haldane, 1980)

$$\rho_j(x) = \rho_j^{(0)} - \frac{1}{\pi}\partial_x\phi_j + \sum_{m\geq 1}\frac{A_m^{(j)}}{\pi a}\cos(2m\phi_j(x) - 2\pi m\rho_j^{(0)}x)$$

$$\psi_j(x) = e^{i\theta_j(x)}\sum_p \frac{B_p^{(j)}}{\sqrt{2\pi a}}cos(p\phi_j(x) - \pi p\rho_j^{(0)}x)$$

$$\pi\Pi_j(x) = \partial_x\theta_j; [\phi_j(x), \Pi_k(x')] = i\delta_{jk}\delta(x - x')\,(\mathrm{CCR})$$

(p odd for fermions, even for bosons).

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Symmetries

- global gauge invariance: $\theta_j \to \theta_j + \alpha_j$
- translational invariance: $\phi_j \to \phi_j + \pi\rho_j^{(0)} a$
- Parity: $\phi_j(x) \to -\phi_j(-x)$ and $\theta_j(x) \to \theta_j(-x)$
- Time reversal: $\phi_j(x) \to \phi_j(x)$, $\theta_j(x) \to -\theta_j(x)$ and $i \to -i$

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Phenomenological Hamiltonian

Incommensurate case. Most general quadratic Hamiltonian:

$$H = \sum_{j,k} \int \frac{dx}{2\pi} \left[\pi^2 M_{jk} \Pi_j \Pi_k + N_{jk} \partial_x \phi_j \partial_x \phi_k\right],$$

compatible with symmetries.
Valid for low energy.

Microscopic to phenomenological (I)

Consider a system with finite size L.

Adding particles: $\rho_j \to \rho_j + q_j/L$.

$$\delta E_0 = \sum_j \frac{\partial E_0}{\partial N_j} q_j + \frac{1}{2} \sum_{j,k} \frac{\partial^2 E_0}{\partial N_j \partial N_k} q_j q_k$$

In phenomenological Hamiltonian: $\partial_x \phi_j \to \partial_x \phi_j - \pi q_j/L$.

$$\Rightarrow N_{jk} = \frac{L}{\pi} \frac{\partial^2 E_{GS}}{\partial N_j \partial N_k}.$$

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Microscopic to phenomenological (II)

Consider a system with finite size L.
Modify boundary conditions: $\psi_j(L) = e^{i\varphi_j}\psi_j(0)$.

$$\delta E_0 = \frac{1}{2}\sum_{j,k}\frac{\partial^2 E_0}{\partial\varphi_j\partial\varphi_k}q_j q_k$$

In phenomenological Hamiltonian: $\partial_x\theta_j \to \partial_x\theta_j + \varphi_j/L$.
Identification of ground state energy changes:

$$\Rightarrow M_{jk} = \pi L\frac{\partial^2 E_0}{\partial\varphi_j\partial\varphi_k},$$

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Galilean invariant case

Particle current: $J_j(x) = \partial_t \phi_j(x,t)/\pi$
E. O. M. $\Rightarrow J_j = \sum_k M_{jk} \partial_x \theta_k/\pi$
Galilean boost: $\theta_j(r) \rightarrow \theta_j(r) + M_j v r \Rightarrow \langle J_j \rangle = \frac{1}{\pi} \sum_k M_{jk} M_k v$
Under galilean boost: $\langle J_j \rangle = \rho_j^{(0)} v$

$$\Rightarrow \pi \rho_j^{(0)} = \sum_k M_{jk} M_j$$

Correlation functions in terms of Green's functions

$$\left\langle T_\tau e^{i\sum_j \lambda_j \phi_j(x,\tau)} e^{-i\sum_k \lambda_k \phi_k(0,0)} \right\rangle = e^{-\sum_{j,k} \lambda_j \lambda_k G_{jk}(x,\tau)}$$

$$\left\langle T_\tau e^{i\sum_j \lambda_j \theta_j(x,\tau)} e^{-i\sum_k \lambda_k \theta_k(0,0)} \right\rangle = e^{-\sum_{j,k} \lambda_j \lambda_k \bar{G}_{jk}(x,\tau)}$$

$$G_{jk}(x,\tau) = -\langle T_\tau(\phi_j(x,\tau) - \phi_j(0,0))\phi_k(0,0)\rangle$$

$$\bar{G}_{jk}(x,\tau) = -\langle T_\tau(\theta_j(x,\tau) - \theta_k(0,0))\theta_k(0,0)\rangle$$

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Expression of Green's functions

$$G_{jk}(x,\tau) = \frac{1}{\beta}\sum_{\omega_n}\int \frac{dq}{2\pi} G_{jk}(q,\omega_n)(e^{iqx-i\omega_n\tau}-1)e^{-|q|\alpha}$$

$$G_{jk}(q,\omega_n) = -\pi((\omega_n^2 + (MN)q^2)^{-1}M)_{jk}$$
$$\bar{G}_{jk}(q,\omega_n) = -\pi((\omega_n^2 + (NM)q^2)^{-1}N)_{jk}$$

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Equal time, zero temperature

$$\langle T_\tau e^{i\sum_a \lambda_a\phi_a(x,0)} e^{-i\sum_a \lambda_a\phi_a(0,0)}\rangle = \left(\frac{\alpha}{\sqrt{x^2+\alpha^2}}\right)^{\frac{1}{2}{}^t\lambda(MN)^{-1/2}M\lambda},$$

$$\langle T_\tau e^{i\sum_a \lambda_a\theta_a(x,0)} e^{-i\sum_a \lambda_a\theta_a(0,0)}\rangle = \left(\frac{\alpha}{\sqrt{x^2+\alpha^2}}\right)^{\frac{1}{2}{}^t\lambda(NM)^{-1/2}N\lambda},$$

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Positive temperature

$$\langle T_\tau e^{i\sum_a \lambda_a \phi_a(x,0)} e^{-i\sum_a \lambda_a \phi_a(0,0)} \rangle \sim e^{-\frac{\pi T}{2}{}^t\lambda N^{-1}\lambda|x|},$$
$$\langle T_\tau e^{i\sum_a \lambda_a \theta_a(x,0)} e^{-i\sum_a \lambda_a \theta_a(0,0)} \rangle = e^{-\frac{\pi T}{2}{}^t\lambda M^{-1}\lambda|x|},$$

Thermal lengths:

$$\xi_\phi(\lambda) = 2/(\pi T\, {}^t\lambda (N)^{-1}\lambda)$$
$$\xi_\theta(\lambda) = 2/(\pi T\, {}^t\lambda (M)^{-1}\lambda)$$

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Conclusions

1. Derivation of a phenomenological Hamiltonian
2. Relations coming from Galilean invariance
3. Expression of correlation functions

Reference: E. Orignac M. Tsuchiizu and Y. Suzumura Phys. Rev. A **81**, 053626 (2010).