Twin-width and parameterized complexity

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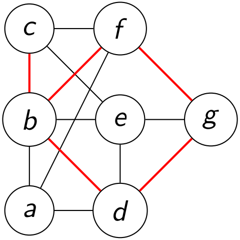
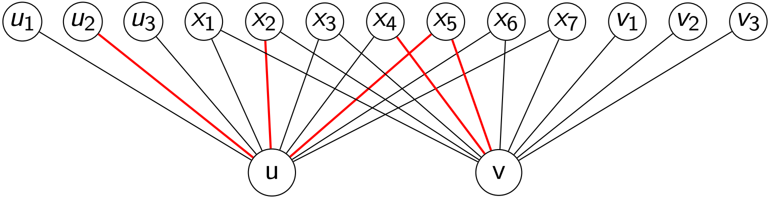
Trigraphs



Three outcomes between a pair of vertices:
edge, or non-edge, or red edge (error edge)

Contractions in trigraphs



Identification of two non-necessarily adjacent vertices

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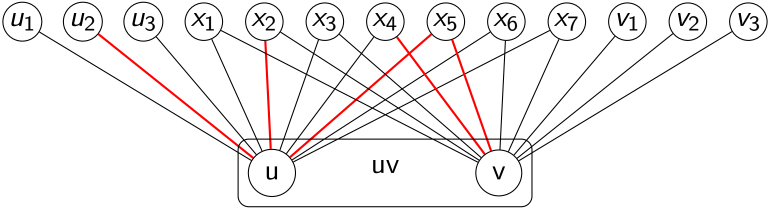

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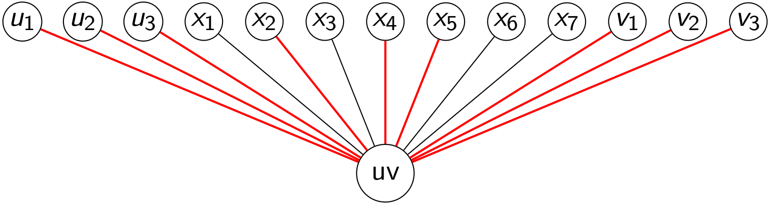
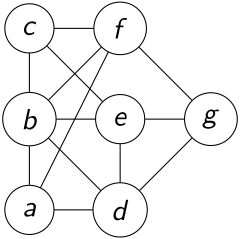


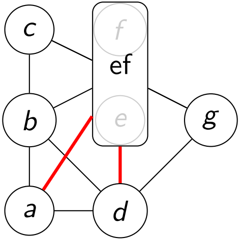
edges to $N(u) \triangle N(v)$ turn red, for $N(u) \cap N(v)$ red is absorbing

Contraction sequence



A contraction sequence of G:
Sequence of trigraphs $G = G_n, G_{n-1}, \ldots, G_2, G_1$ such that G_i is obtained by performing one contraction in G_{i+1}.

Contraction sequence



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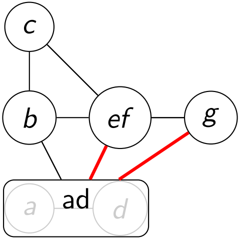

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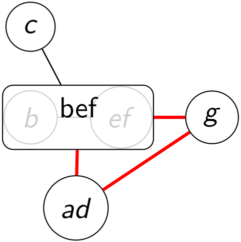

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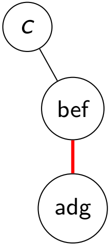

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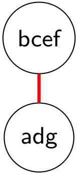

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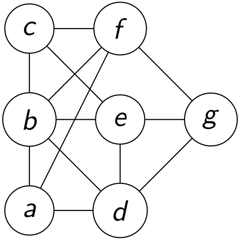
Twin-width

tww(G): Least integer d such that G admits a contraction sequence where all trigraphs have *maximum red degree* at most d.



Maximum red degree $= 0$

overall maximum red degree = 0

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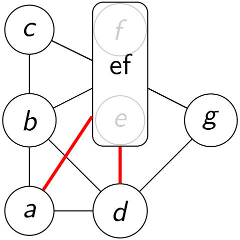

Maximum red degree $= 2$

overall maximum red degree = 2

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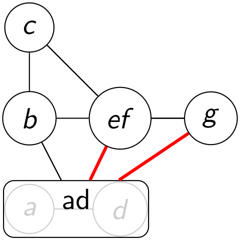

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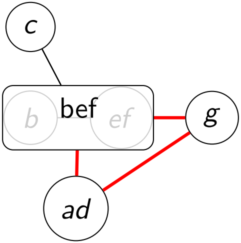

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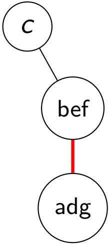

Maximum red degree $= 1$

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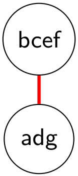

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Theorem ('20, '21, & '22)

The following families have bounded twin-width.

- *Bounded boolean-width graphs,*
- K_t*-minor free graphs, map graphs (with embedding),*
- *every hereditary proper subclass of permutation graphs,*
- *unit interval graphs, and posets of bounded antichain size,*
- d*-dimensional grids,* K_t*-free unit* d*-dimensional ball graphs,*
- *segment graphs without* $K_{t,t}$ *subgraph,*
- *visibility graphs of simple polygons with bounded* α*,*
- $\Omega(\log n)$*-subdivisions of all the* n*-vertex graphs,*
- *cubic expanders defined by iterative random 2-lifts from* K_4*,*
- *products of bounded twin-width graphs, one with bounded* Δ*,*
- *subgraphs of every* $K_{t,t}$*-free class above,*
- *first-order transductions of all the above.*

First-order model checking on graphs

GRAPH FO MODEL CHECKING **Parameter:** $|\varphi|$
Input: A graph G and a first-order sentence $\varphi \in FO(\{E\})$
Question: $G \models \varphi$?

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Example:

$$\varphi = \exists x_1 \exists x_2 \cdots \exists x_k \forall x \forall y \ (E(x,y) \Rightarrow \bigvee_{1 \leqslant i \leqslant k} x = x_i \vee y = x_i)$$

$G \models \varphi? \Leftrightarrow k$-VERTEX COVER

FO interpretations and transductions

FO interpretation: redefine the edges by a first-order formula
$\varphi(x, y) = \neg E(x, y)$ (complement)
$\varphi(x, y) = E(x, y) \vee \exists z E(x, z) \wedge E(z, y)$ (square)

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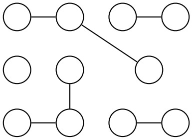
FO transduction: color by $O(1)$ unary relations, interpret, delete



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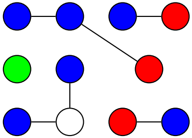

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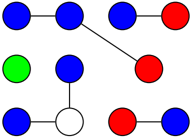

$$\varphi(x, y) = E(x, y) \vee (G(x) \wedge B(y) \wedge \neg\exists z R(z) \wedge E(y, z))$$
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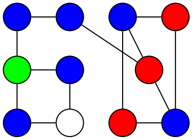

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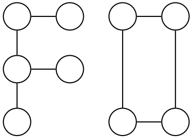

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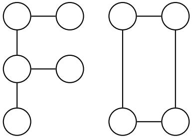

Theorem (B., Kim, Thomassé, Watrigant '20)
Any FO transduction of a bounded twin-width class has bounded twin-width.

Dependence and monadic dependence

A class $\mathscr{C}$ is
dependent, if the hereditary closure of every simple interpretation of $\mathscr{C}$ misses some graph
monadically dependent, if every transduction of $\mathscr{C}$ misses some graph [Baldwin, Shelah '85]

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Theorem (Downey, Fellows, Taylor '96)

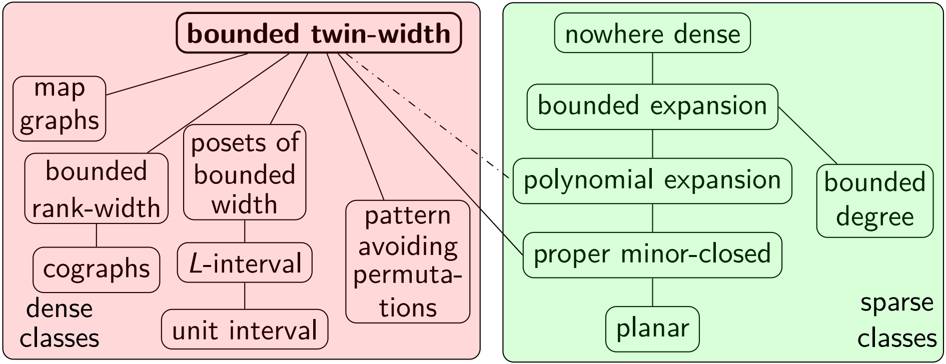
FO model checking is $AW[*]$-complete on general graphs,
thus unlikely FPT on independent classes.

Tractable: FO model checking is FPT on the class

Conjecture (Workshop in Warwick '16, Gajarský et al. '18)

Every monadically dependent class is tractable, with equivalence among hereditary classes.

Tractable classes



Theorem (B., Kim, Thomassé, Watrigant '20)

FO Model Checking *solvable in* $f(|\varphi|, d)n$ *on graphs with a* d*-sequence.*

Faster parameterized algorithms?

Theorem

*Given a d-sequence of the n-vertex input graph, k-*Independent Set *and k-*Dominating Set *can be solved in time $2^{O_d(k)} n$, whereas* Subgraph Isomorphism *and* Induced Subgraph Isomorphism *can be solved in time $2^{O_d(k \log k)} n$.*

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ETH lower bound: no $2^{o(n/\log n)}$ for Max Independent Set, hence no $2^{o(k/\log k)}n^{O(1)}$ for k-Independent Set

Question

Can we close this gap?

Imposing more on the red graphs

reduced $(\Delta + \mathrm{tw})$, reduced $(\Delta + \mathrm{pw})$, reduced bandwidth

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Question

Given a sequence witnessing bounded reduced bandwidth, can
MAX INDEPENDENT SET *be solved in time* $2^{O(\sqrt{n})}$*?*
k-INDEPENDENT SET *be solved in time* $2^{O(\sqrt{k})} n^{O(1)}$*?*

Threshold unbounded/bounded reduced bandwidth of s-subdivisions at $s = \Theta(n)$

Polynomial kernels?

Mostly negative results: k-Independent Set, k-Dominating Set, already for twin-width at most 4

k-Independent Set: easy OR-composition, complete sum of planar instances

k-Dominating Set [BKRTW '21]: more complicated reduction, the 4-sequence goes through a red wall

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Question

Does k-Dominating Set admit a polynomial kernel when reduced bandwidth is bounded?

How hard is computing twin-width?

Theorem (Bergé, B., Déprés '22)

It is NP-complete to decide if the twin-width is at most 4.

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Question

Is there an FPT $f(OPT)$-approximation of twin-width?

Question

Is twin-width at most k a "simpler" class? (for $k \leqslant 3$)

Small values of twin-width

- twin-width $0 =$ cographs;
- twin-width at most 1: polytime recognizable [BKRTW '21], perfect and bounded cliquewidth
- Is twin-width at most 2 polytime recognizable? simpler?
- Is twin-width at most 3 polytime recognizable? simpler?

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- Is twin-width at most 3 polytime recognizable? simpler?

Twin-width at most 1:
safe contraction (resulting in an induced subtrigraph), or
contraction of the only red edge

Twin-width at most 2

contains unit interval graphs
Max Independent Set is polytime solvable given a 2-sequence

Theorem (B., Geniet, Kim, Thomassé, Watrigant '21)

Max Independent Set *can be solved in time* $O^*(|\mathcal{R}|)$ *given a contraction sequence whose connected subsets in the red graphs form the set* $\mathcal{R}$.

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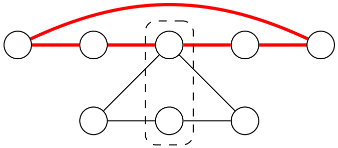

Safe contractions on a red cycle

Twin-width at most 3

The following *should* have twin-width 4:

- planar grids, and
- ($\geqslant 2 \log n$)-subdivisions of n-vertex graphs.

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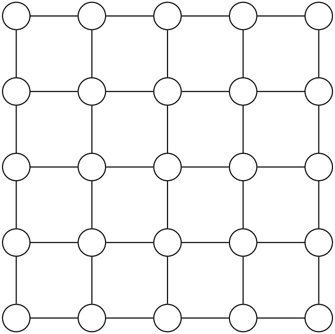
Question

Is every class of twin-width at most 3 and no $K_{t,t}$ subgraph of bounded cliquewidth?

Grids have twin-width at most 4



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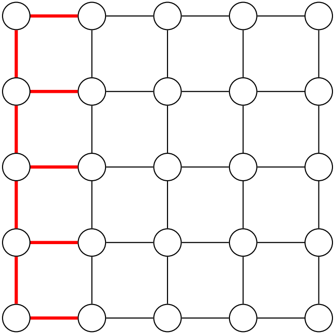

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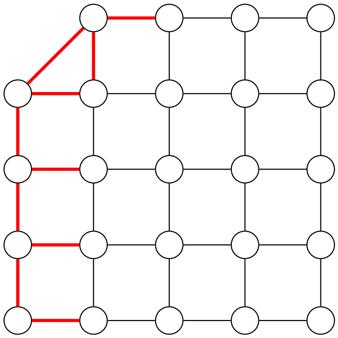

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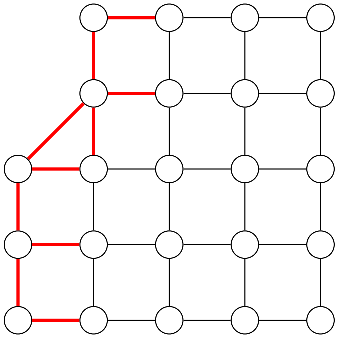

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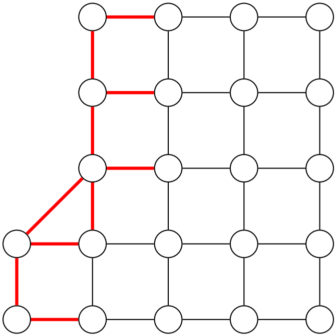

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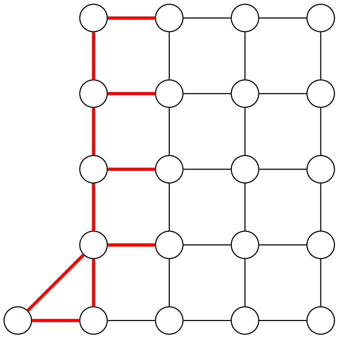

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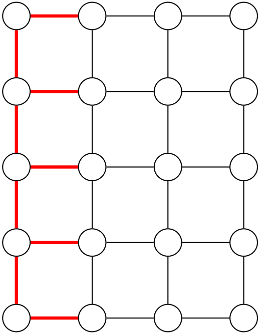

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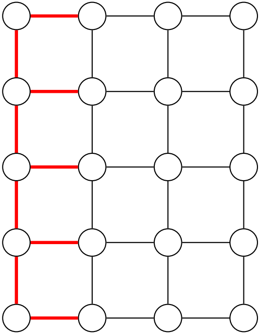
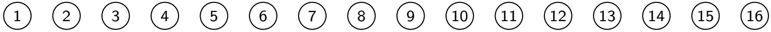


Exactly 4? The 6×8 grid has twin-width 3 [Schidler, Szeider '21]

($\geqslant 2\log n$)-subdivisions have twin-width at most 4



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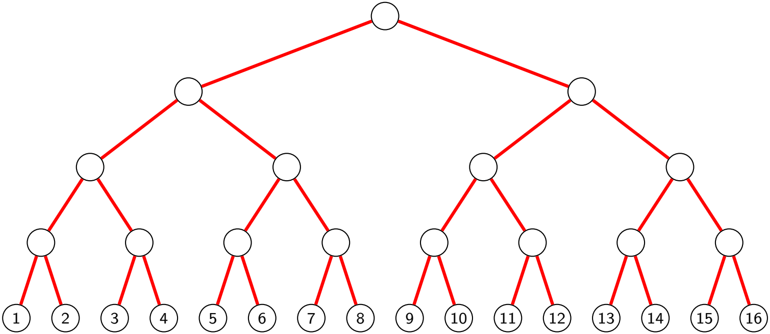
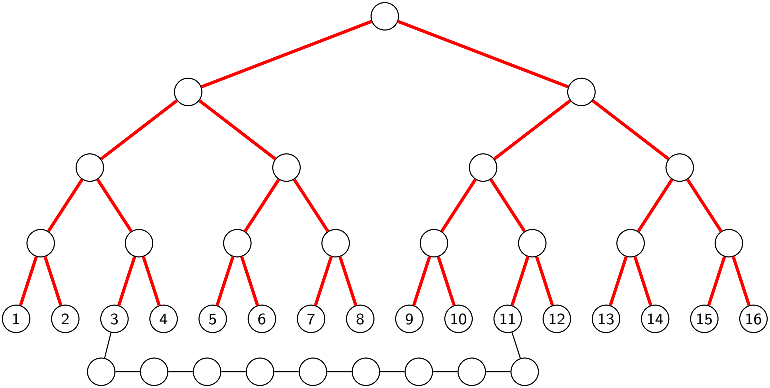


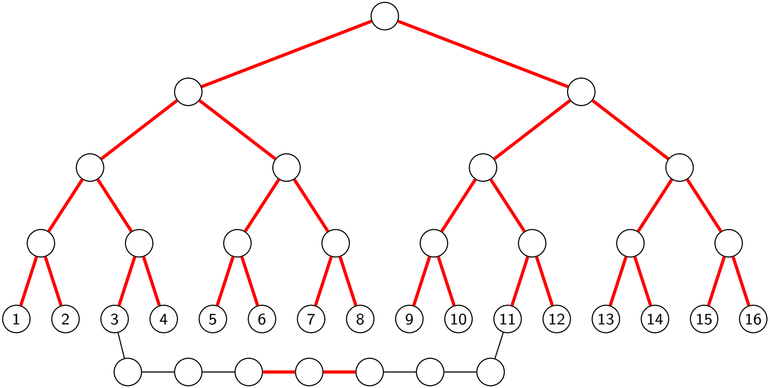
Add a red full binary tree whose leaves are the vertex set

($\geqslant 2 \log n$)-subdivisions have twin-width at most 4



Take any subdivided edge

$(\geqslant 2 \log n)$-subdivisions have twin-width at most 4



Shorten it to the length of the path in the red tree

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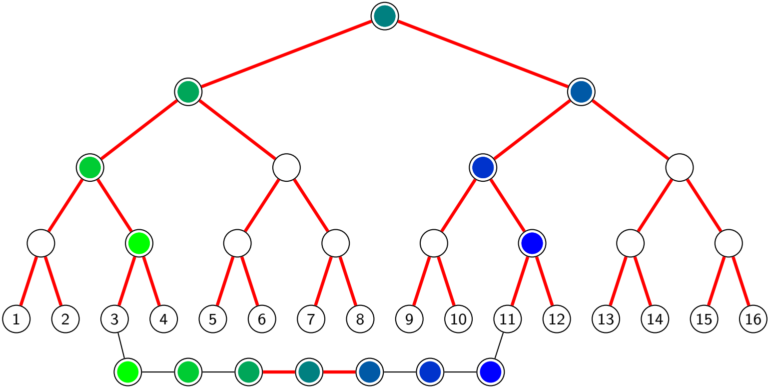

Zip the subdivided edge in the tree

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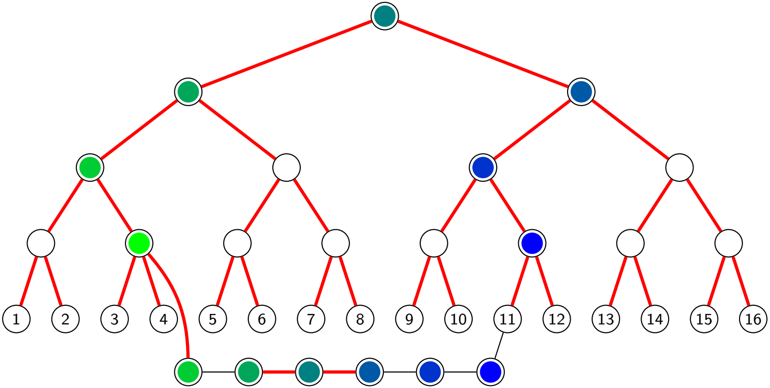

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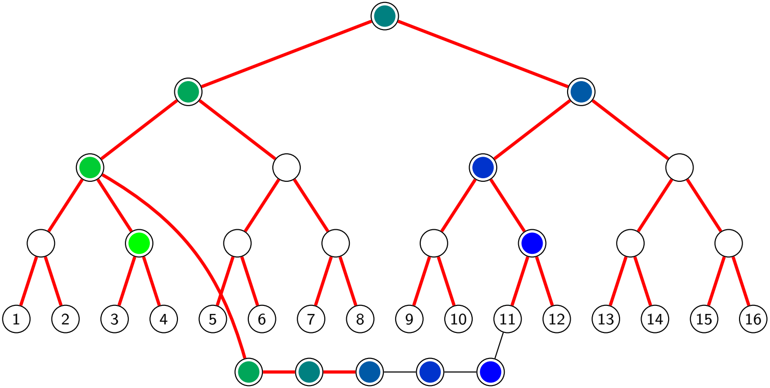

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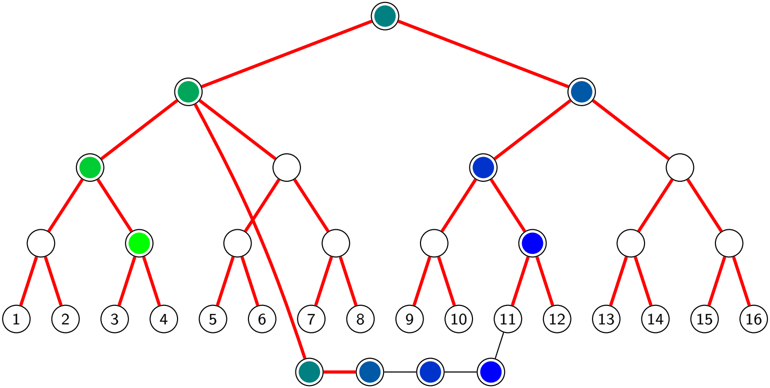

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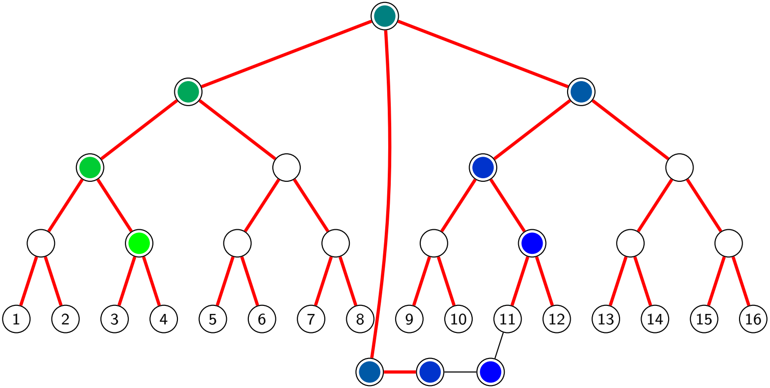

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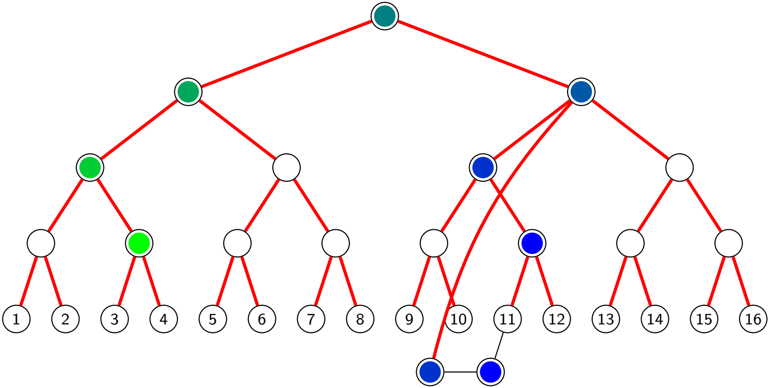

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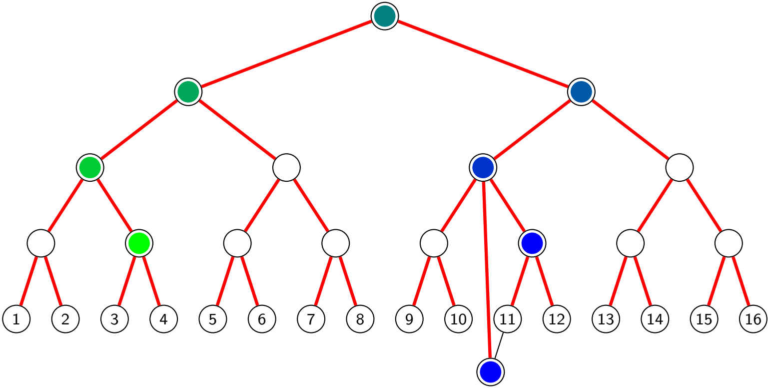

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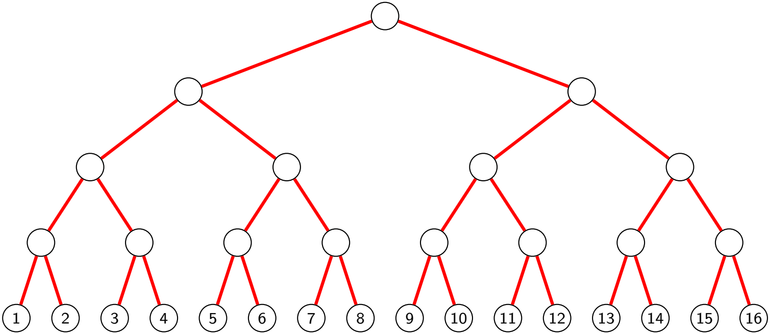
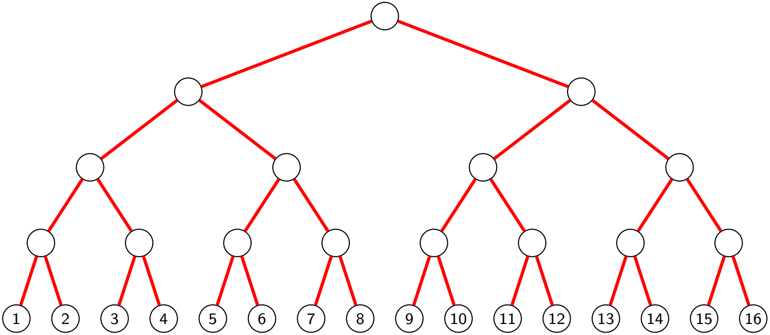


Move to the next subdivided edge

$(\geqslant 2 \log n)$-subdivisions have twin-width at most 4



Twin-width exactly 4?

Grid number, mixed number

$$\left[\begin{array}{cc|cc|cc|cc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ \hline
0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ \hline
0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 1
\end{array}\right]$$

4-grid minor

$$\left[\begin{array}{cc|ccc|ccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ \hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
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1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 1
\end{array}\right]$$

3-mixed minor

$\mathrm{gn}(M) =$ largest k such that M has a k-grid minor
$\mathrm{mxn}(M) =$ largest k such that M has a k-mixed minor

Grid number, mixed number

$$\left[\begin{array}{cc|cc|cc|cc}
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3-mixed minor

$\mathrm{gn}(G) = \min\ \mathrm{gn}(M)$ among every adjacency matrix M of G

$\mathrm{mxn}(G) = \min\ \mathrm{mxn}(M)$ among every adjacency matrix M of G

Twin-width and mixed/grid number

Theorem (B., Kim, Watrigant, Thomassé '20)

For every graph G, $\frac{mxn(G)-1}{2} \leqslant tww(G) \leqslant 2^{2^{O(mxn(G))}}$.

Corollary

For every graph G, $tww(G) \leqslant 2^{O(gn(G))}$.

Twin-width and mixed/grid number

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Theorem (B., Déprés '22)

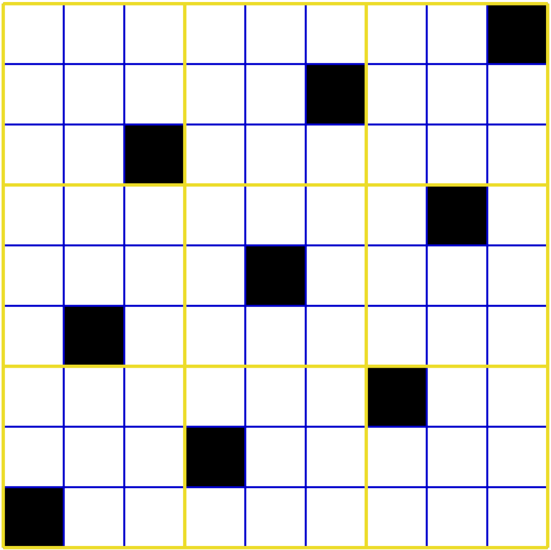
$\forall c < 1$, $\exists$ *a class $\mathcal{C}$ of unbounded twin-width such that* $\forall G \in \mathcal{C}$,

$$tww(G) > 2^{c \cdot (gn(G)-2)}.$$

Question

Is the double-exponential dependence in mixed number necessary?

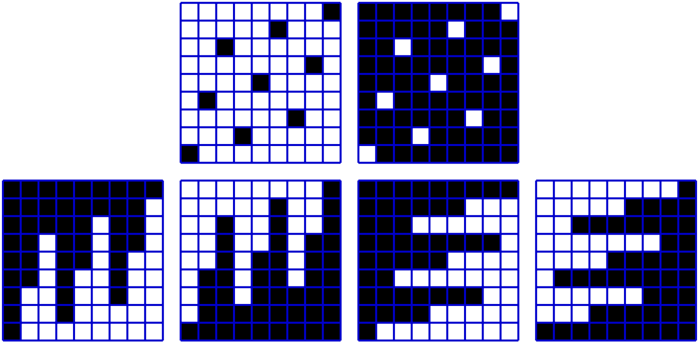
k-grid permutation



Here with $k = 3$

The 6 universal patterns of unbounded twin-width

$\exists f$ s.t. all the adjacency matrices of a graph of twin-width $\geqslant f(k)$ contains a k-grid permutation submatrix or one of its 5 encodings



Twin-width win-win

Goal: compute FO-definable parameter p in FPT time in $\mathcal{C}$.

Show that $\exists f$ non-decreasing, such that $\forall G \in \mathcal{C}$ an $f(p(G))$-sequence of G can be computed in FPT time

- Width $> f(k)$: report $p(G) > k$
- Width $\leqslant f(k)$: use FO model checking algorithm

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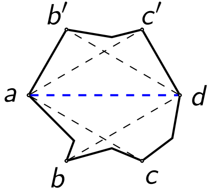
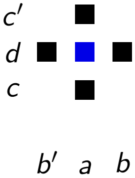
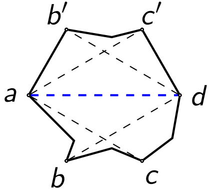
$\to$ k-Independent Set in visibility graphs of simple polygons

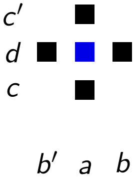
Ordering along the boundary of the polygon

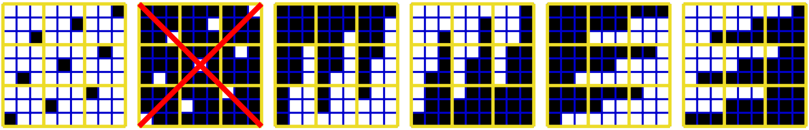




Ordering along the boundary of the polygon

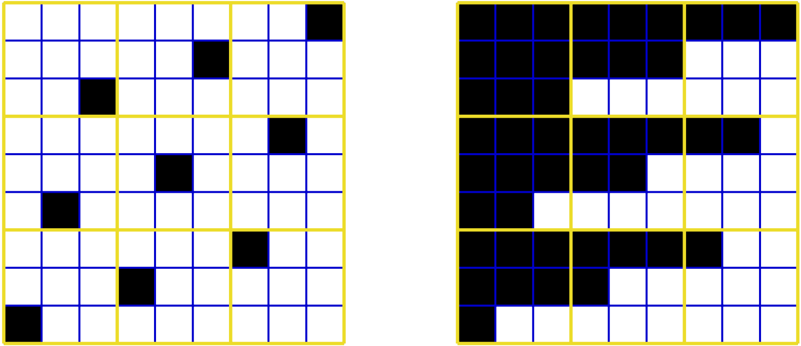






Extractions

Here we only need a decreasing pattern



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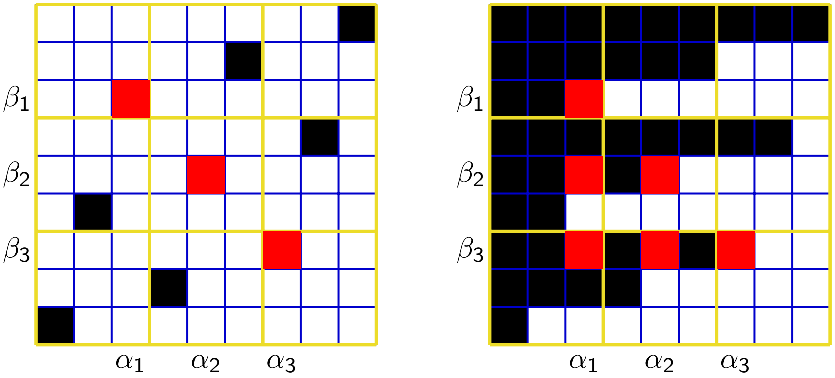

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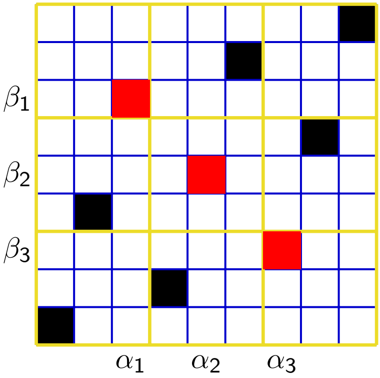
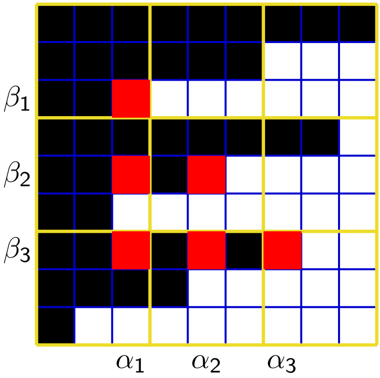
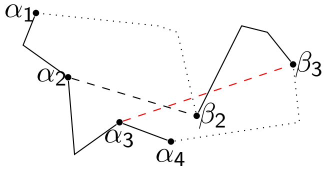




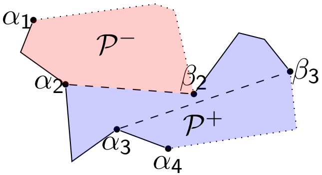
By Ramsey's theorem, we can assume that the α_is and the β_is both induce a clique.

Geometric arguments



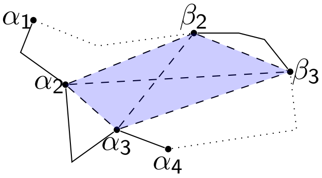
Quadrangle $\alpha_2\alpha_3\beta_3\beta_2$ is not self-crossing

Geometric arguments



Quadrangle $\alpha_2\alpha_3\beta_3\beta_2$ has to be convex

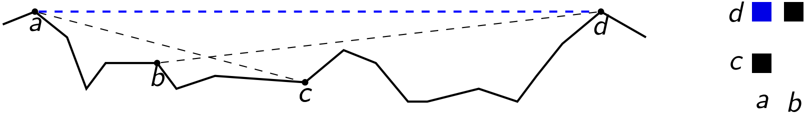
Geometric arguments

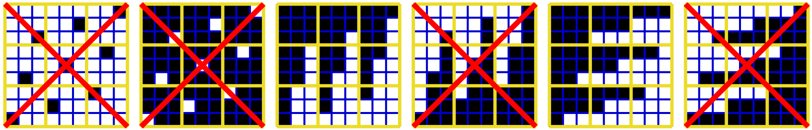


Then $\alpha_2, \alpha_3, \beta_3, \beta_2$ induce K_4, a contradiction

Visibility graphs of 1.5D terrains

Order along x-coordinates





Visibility graphs of 1.5D terrains

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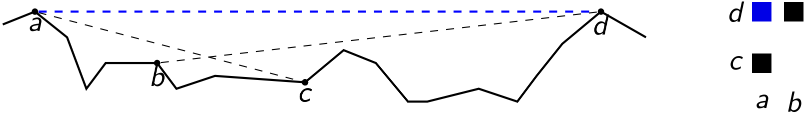

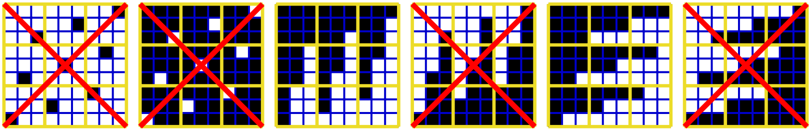

Question

Is 1.5D TERRAIN GUARDING *or* k-DOMINATING SET *in visibility graphs of 1.5D terrains FPT?*

Delineation

$\mathcal{D}$ is **delineated** if for every hereditary $\mathcal{C} \subseteq \mathcal{D}$,
$\mathcal{C}$ has bounded twin-width $\Leftrightarrow \mathcal{C}$ is monadically dependent

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$\mathcal{D}$ is **effectively delineated** if further twin-width is FPT approximable in $\mathcal{D}$

Effectively delineated classes

Ordered graphs, permutation graphs, interval graphs, etc.

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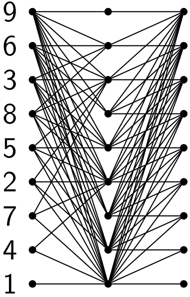
Ordered graphs, permutation graphs, interval graphs, etc.

Find a natural ordering of the vertex set

- no universal pattern → bounded twin-width
- universal pattern → "transversal pair," witness of monadic independence



transversal pair

Non-delineated classes

Bounded degree, split graphs, segment graphs, visibility graphs of simple polygons, etc.

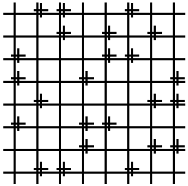
Non-delineated classes

Bounded degree, split graphs, segment graphs, visibility graphs of simple polygons, etc.

To show that $\mathcal{D}$ is not delineated:

Exhibit two transdutions T, T' and $\mathcal{C} \subseteq \mathcal{D}$ such that $T(\mathcal{C})$ contains all subcubic graphs and $T'(\{\text{subcubic graphs}\})$ contains $\mathcal{C}$

- T implies that $\mathcal{C}$ has unbounded twin-width
- T' implies that $\mathcal{C}$ is monadically dependent



Questions on delineation

Question

Are tournaments delineated?

Question

Are visibility graphs of terrains delineated?

Question

Are unit segments delineated?

Question

Is non delineation equivalent to transduction equivalent to subcubic graphs?

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Thank you for your attention!