Stage : Multilevel derivative free optimization

at ALMA MATER STUDIORUM - Università di Bologna
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The efficient solution of optimization problems arising in real applications increasingly calls for the development of efficient and easy-to-use implementations of derivative-free algorithms [1]. In applications such as engineering design, medical image registration and design of algorithms (amongst many others), optimization problems are often defined by functions computed by costly simulation. Functions have therefore to be treated as expensive black-boxes (EBB), and due to the high computational cost involved, it is important to use optimization algorithms that produce reasonably good solutions within a limited number of function evaluations.

In this stage we will deal with the main challenge in the solution of simulation-based optimization problems via derivative-free optimization (DFO) : the DFO scalability. In this context, the internship student will study multilevel optimization strategies.

The basic idea of multilevel strategies is to exploit the fact that many problems can be represented at different scales [2], for instance an infinite dimensional problem can be discretized choosing different grid parameters, problems in imaging can be represented at different levels of resolution reducing the number of pixels.

The main idea of the multilevel method is to consider a hierarchy of problems of reduced dimensions, rather than directly solving a large problem with many variables. The computational cost can be reduced by not performing expensive computations (such as the step computation) at higher levels (the ones with the largest number of parameters) and instead transferring cheap information computed at lower levels to the higher ones. These strategies are well-known in optimization [3,4] and it has been shown that they are really effective in reducing the cost of the optimization process and to speed up the convergence. Such strategies are however highly dependent on the information contained in the derivatives. In the context of the internship, we will extend these ideas to the derivative-free context.

Required skills

The stage will focus on both numerical implementations and theoretical aspects. For this reason programming skills in python or Matlab are required.

References

[1] Conn, A.R., Scheinberg, K. and Vicente, L.N., *Introduction to Derivative-Free Optimization* , MPS-SIAM Series on Optimization, 2009.

[2] Briggs, W. L. and Henson, V.E. and McCormick, S. F., *A multigrid tutorial*, SIAM, 2000.

[3] Migdalas, A., Pardalos, P. M. and Värbrand, P., *Multilevel optimization: algorithms and applications*, Springer Science & Business Media, 2013.

[4] Calandra, H. and Gratton, S., Riccietti, E. and Vasseur, X., *On a multilevel Levenberg–Marquardt method for the training of artificial neural networks and its application to the solution of partial differential equations*, Optimization Methods and Software, 2020