Stage : Solution of ill-posed inverse problems in partial differential equations by neural networks

at Università di Firenze
Advisors: Stefania Bellavia (stefania.bellavia@unifi.it), Elisa Riccietti (elisa.riccietti@ens-lyon.fr)

Partial differential equations (PDEs) are widely used in modeling physics, biology and engineering problems. They can be solved numerically using a wide variety of methods such as finite differences, finite elements or spectral methods. However, when dealing with high-dimensional problems, the computational cost of the solution becomes a key challenge. To overcome this, recent works have studied the use of neural networks to approximate the solution of PDEs. In fact, in modern machine learning practice, the prior knowledge of a modeled problem is often not used. The authors of [3,4] propose a new approach that exploits the underlying physical law that governs the dynamics of a system to act as a regularization agent that reduces the size of the domain of admissible solutions. This work led to the emergence of Physics Informed Neural Networks (PINNs), neural networks that have knowledge of the underlying law of physics. With this new paradigm, they managed to achieve similar results to other state-of-the-art methods for solving PDEs. However their framework is limited to the direct solution of PDEs and inverse problems that are well posed.

Ill-posed inverse problems are problems whose solutions are unstable under data perturbation, meaning that even small perturbations in the data can lead to large perturbations in the solution. An important example of ill-posed inverse problems is given by parameter recovery in PDEs. For instance consider the following PDE:

$$\Delta u(x) + c(x)u(x) = f(x).$$

Given the function f and some noisy measurements of the solution of the PDE u, we want to recover the parameter $c(x)$.

The aim of this stage is to solve this kind of ill-posed problems using deep learning techniques and PINNs. The arising training problem has been shown to be ill-posed. The objective of the stage will be to develop regularizing training strategies, that is optimization methods able to correctly train the neural network and recover the unknown parameter c in a stable way, despite the perturbations in the measurements [1,2].

Required skills

The stage will focus on both numerical implementations and theoretical aspects. For this reason programming skills in python are required.

References

[1] Stefania Bellavia and Elisa Riccietti. *On an elliptical trust-region procedure for ill-posed nonlinear least-squares problems*, Journal of Optimization Theory and Applications, 2018.

[2] Heinz Engl, Martin Hanke, and Andreas Neubauer. *Regularization of inverse problems*. Kluwer Academic, 2000.

[3] Maziar Raissi, Paris Perdikaris, and George Em Karniadakis. *Physics informed deep learning (part I): Data-driven solutions of nonlinear partial differential equations*, https://arxiv.org/abs/1711.10561, 2017.

[4] Maziar Raissi, Paris Perdikaris, and George Em Karniadakis. *Physics informed deep learning (part II): Data-driven discovery of nonlinear partial differential equations*, https://arxiv.org/abs/1711.10566, 2017.