(see also : http://rses.anu.edu.au/seismology/projects/tireg)

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Using Voronoi diagrams to assess the resolvability of a given seismic parameter

In seismic tomography, our ability to resolve a given parameter at a given location depends strongly on the distribution of rays which is always irregular. We propose a strategy to find a 2D 'optimized' parameterization of the model in which each geographical point belongs to the smallest cell for which a quality criterion, related to the resolution of a given seismic parameter, is satisfied. The resulting 'optimized' parameterization is almost always irregular and the size and shape of each cell reflects the way the considered parameter can be resolved. The 'optimized' 2D parameterization therefore provides information about the way a given seismic parameter can be resolved from the ray coverage.

1.Parameterization using natural neighbours

In 2-D, the Voronoi diagram of an irregular set of node divides the plane into a set of regions, one for each node, such that any point in a particular region is closer to that region's node than to any other node (Fig. 4).

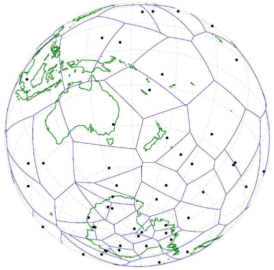

Fig. 4 : a) An example of a Voronoi diagram on the Earth's surface. The boundaries of each cell are great circles equi-distant from the defining nodes, which are plotted as filled circles. Each Voronoi cell contains that part of the Earth's surface closest to the defining node. In this case the nodes are randomly positioned.

2. Building an 'optimised' Voronoi diagram

2.1 The strategy of deleting nodes

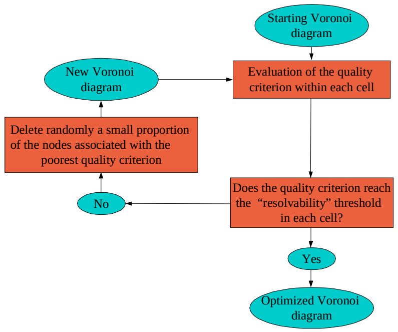

Fig. 5 : Flow-chart of the iterative procedure used to generate optimized Voronoi diagrams with respect to any quality criterion. At each iteration, the cells neighbouring those deleted increase in size and capture more rays, which make them more likely to satisfy the quality criterion.

If, from the Voronoi diagram of Fig. 4, we delete a subset of the initial set of nodes, it is always possible to built from the new set of nodes a new Voronoi diagram (see Fig. 5). In this new diagram, any point that was previously associated to a given node remains associated to the same node if this node has not been deleted. The points associated to a deleted node will be incorporated to the neighbouring cells. In other words, **after deleting a set of nodes, the remaining cells can only 'grow' or 'stay the same'.**

2.2 Starting Voronoi diagram

In the period range of analysis used in global surface wave tomography (40 s -300 s) the shortest wavelengths to be used are about 160 km and limitations due to the ray theory make it difficult to resolve structure smaller than a few hundred of kilometers. We therefore impose a square *2* x *2* degree cell as a minimum size for the starting Voronoi cells. Then we follow the flow-chart of Fig. 5 to build the 'optimized' Voronoi diagram.

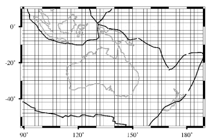

Fig. 5 : Starting Voronoi diagram corresponding to a regular grid of 2° x 2°. From this starting Voronoi diagram, a new Voronoi diagram is buid in such a way that a particular quality criterion is met in each cell.

3. Application : resolving the azimuthal anisotropy of long period SV waves

Current waveform inversion techniques (e.g. Cara and Lévêque, 1987; Nolet 1990) provide a path-average shear velocity model compatible with a multi-modes surface wave seismogram. From a set of path-average models related to paths with different azimuths it is possible to retrieve the azimuthal variation of long period shear waves. Our ability to resolve this azimuthal variation depends on the azimuthal distribution of rays. Here we refine the cellular structure of a starting Voronoi diagram by developing a quality criterion which ensures resolution of anisotropic structure in each of the final cells. The resulting 'optimized' Voronoi diagram provides a measure of our ability to resolve the azimuthal anisotropy of SV waves from the ray coverage.

3.1 A simple quality criterion for the azimuthal anisotropy of surface waves

A long period SV wave propagating horizontally in a slightly anisotropic medium at depth *z* experiences an azimuthal variation of the form (see e.g. Montagner and Nataf, 1986; Lévêque et al., 1998) :

$$SV(z) = SV_0(z) + SV_1(z)\cos(2\theta) + SV_2(z)\sin(2\theta)$$

where θ is the azimuth. A similar relation but with a 4θ variation can be obtained for long period SH waves. In most studies, authors concentrate on the 2θ azimuthal variation of long period Rayleigh or SV waves which is the easiest to retrieve and to interpret.

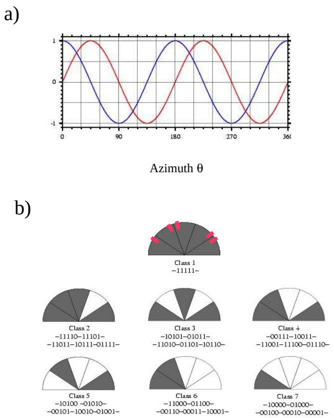

Fig. 8 : a) The cos (2θ) and sin(2θ) azimuthal variation can be retrieved only if the azimuthal range of 180° is sampled by at least 3 paths with different azimuths. b) The quality criterion for the cos (2θ), sin(2θ) azimuthal variaton of surface waves. By imposing that each cell of our final Voronoi diagram belong to class 1 (at least one path in each 36° box) we make sure that in each cell, the azimuthal variation of Rayleigh waves is well resolved. Red bars simulate the worst azimuthal sampling that we can encounter in class 1, where three different azimuths are sampled.

3.2 Retrieving the 2θ azimuthal variation from regional tomography

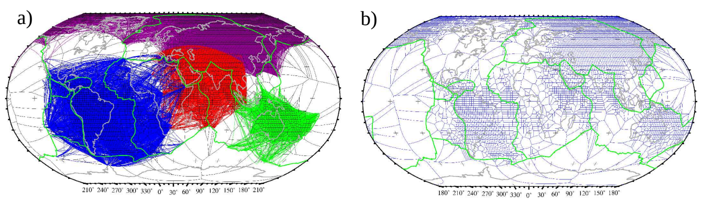

Fig. 9 : a) Ray coverage superimposed to the 'optimized' Voronoi diagram. b) Optimized Voronoi diagram.

We apply our procedure to the heterogeneous coverage resulting from an assemblage of regional studies. In most of the well sampled regions of our study the initial *2* x *2* degree cells remain in the optimized Voronoi diagram, meaning that the 2θ azimuthal variation can geometrically be resolved in each cell. A cell elongated in the east-west direction suggests that changes in anisotropic directions are easier to resolve in the north-south direction than in the east-west direction.

3.3 Retrieving the 2θ azimuthal variation from global tomography (synthetic experiment)

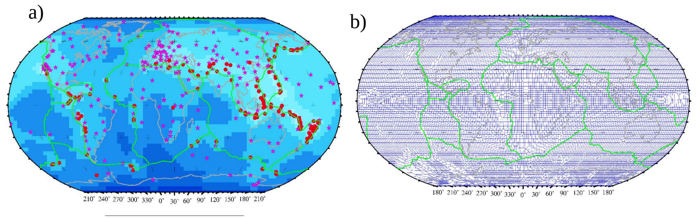

Fig. 10 : a) Ray density averaged over 4 x 4 degrees cells, distribution of events (red circles) and stations (purple stars). b) Optimized Voronoi diagram.

With a coverage comparable to what can be achieved in modern global tomography (here 37320 paths with lengths greater than 1200 km) it is possible to retrieve the 2θ azimuthal variation almost everywhere in the Earth. Note however that the actual horizontal resolution achieved in surface wave tomography results from the compromise between what can be geometrically resolved and what can be resolved from the physics of surface waves...

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