Continuous regionalization of massive surface waves datasets

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Summary : We propose a new scheme to speed up the inclusion of the a priori information in the estimation of the inverted model using the least square criterion. The approach, applied in the case where the unknowns are functions of a continuous variable and when a Gaussian a priori covariance function controls the horizontal degree of smoothing in the inverted model, render possible the inversion of several tens of thousands of seismograms in few hours on a standard workstation. It is highly parallelizable and allows to invert massive datasets using a sophisticated a priori information which minimizes some artifacts observed with spherical harmonic parameterizations. In addition we propose a procedure to obtain a qualitative estimation of how well a given parameter can be resolved from the ray coverage. The new approach is demonstrated using a real dataset of 36513 Rayleigh waves fundamental and higher modes seismograms.

1. Surface wave tomography by continuous regionalization :

1.1 Forward problem :

We assume propagation along the great circle. For a source receiver path *i:*

$$1/C_i(T) = 1/L_i \int_i 1/C(T,\theta,\phi)\,ds$$

T is the period, *C* is the phase velocity, θ and ϕ are the coordinates of the geographical points along great circle *i*. From a set of path average measurements $C_i(T)$ the regionalization consists in retrieving the local velocities $C(T,\theta,\phi)$.

1.2 Inversion :

The inverse problem is underdetermined and the solution is stabilized through the use of an a priori covariance function on the model :

$$C_{m0}(r,r') = \sigma(r)\sigma(r')\exp\left(\frac{-\Delta^2_{r,r'}}{2L^2_{corr}}\right)$$

The inversion is done using the formalism of Tarantola and Valette (1982b) where the unknowns are functions of a continuous variable and the relationship between the data and the parameters is assumed to be linear. The solution of the inverse problem is :

$$m(r) = m_0(r) + \frac{1}{L_i}\sum_i \left[\int_{path\,i} ds_k\,\sigma(r_k)\sigma(r)\exp\left(\frac{-\Delta^2_{r_k,r}}{2L^2_{corr}}\right)\sum_j S^{-1}_{ij}\left(d_{0j} - \int_{path\,j} G_j(r'')m_0(r'')dr''\right)\right]$$

with :

$$S_{ij} = C_{d0_{ij}} + \frac{1}{L_i}\frac{1}{L_j}\int_i \int_j \sigma(r_i)\sigma(r_j)\exp\left(\frac{-\Delta^2_{r_i,r_j}}{2L^2_{corr}}\right)ds_i\,ds_j$$

1.3 Computational limitations :

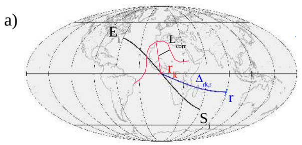
The estimation of *m(r)* presents several practical difficulties that become severe limitations for application to large datasets. The first one resides in the estimation of the integral (see Fig.1a) :

$$A_i(r) = \frac{1}{L_i}\int_{path\,i} ds_k\,\sigma(r_k)\sigma(r)\exp\left(\frac{-\Delta^2_{r_k,r}}{2L^2_{corr}}\right)$$

The second practical difficulty resides in the computation of S_{ij}, which requires the estimation of the double integral(see Fig. 1b) :

$$B_{ij} = \frac{1}{L_i}\frac{1}{L_j}\int_{path\,i}\int_{path\,j}\sigma(r_k)\sigma(r_l)\exp\left(\frac{-\Delta^2_{r_k,r_l}}{2L^2_{corr}}\right)ds_k\,ds_l$$

a)



b)

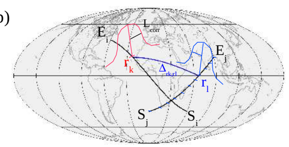


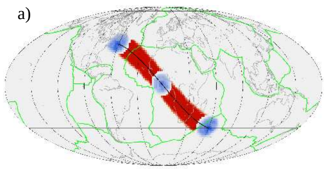
Fig. 1 : a) The estimation of $A_i(r)$ everywhere on the Earth requires us to compute $C_{m0}(r_k,r)$ between each point r_k of a given path *i* and each geographical point *r* of the model before integrating this contribution along path *i*. b) For each pair of ray paths *(i,j)* the calculation of B_{ij} involves a double integral of $C_{m0}(r_k,r_l)$ along paths *i* and *j*, where r_k is the *k*th point along path *i* and r_l the *l*th point along path *j*. In both cases, when the distance $\Delta_{r,r'}$ between two points *r* and *r'* is large compared to L_{corr} the exponential term tends toward zero and its computation can be skipped. However, a computation of the distance $\Delta_{r,r'}$ and a comparison with L_{corr} are required a large number of times, making the inversion impracticable when the number of paths exceeds a few thousand (see Fig. 3a).

2. Optimization :

We optimize the computation of $A_i(r)$ and B_{ij} by exploring for each point r_k of the great circle *i*, only the 'influence zone' of the point for which the contribution of the exponential term to the integrals is significant. The exploration is stopped when the limit of this 'influence zone' is reached, avoiding a large number of useless $\Delta_{rk,r}$ computations and confrontations with L_{corr} .

Computation of A :

a)



Computation of B :

b)

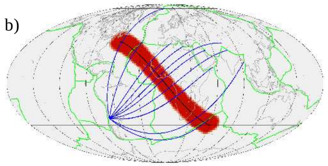


Fig. 2 : For each point of the great circle our code locates the current cell, and explores the model in a region located within 2.64 L_{corr}(this corresponds to an amplitude of 3% of the maximum of the gaussian filter). The influence zone of the path (in red) is the juxtaposition of the influence zones of each point of the path (in blue for 3 points on Fig. 2a). Only the model contributions (Fig. 2a) or the path contribution (Fig. 2b) located within the influence zone of the ray are considered in the estimation of $A_i(r)$ and B_{ij}. No computation is made outside the influence zone.

3. Performances of the new code :

a)

Number of Paths	Computation time (original scheme)	Computation time (optimized scheme)
60	50 s	1 s
111	116 s	2 s
223	288 s	5 s
395	762 s	8 s
805	2499 s	21 s
1532	7862 s	53 s
3090	33889 s	186 s
5672		763 s
11303		3422 s
16936		7956 s
21184		14282 s

b)

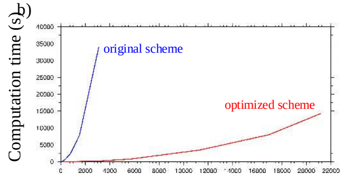
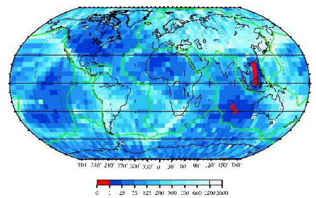


Fig. 3 : *a)* Tests performed on a PC equipped with a single 2.4 Ghz Pentium 4 processor, with 2 Gb of Ram. The values in table *a)* correspond to the curve in *b)*. All the tests have been performed by including azimuthal anisotropy in the inversion and adopting a discretization of the final model in cells of *2 x 2* degrees. Note that we also use a conjugate gradient solver instead of inverting the data by data matrix S which probably contributes to improve the performance of our code when the number of paths is large.

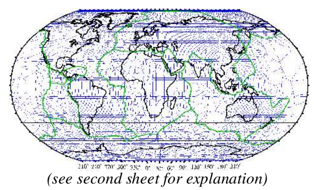
We are currently working on a parallel version of the new scheme (the parallelization is performed over rays) that should further increase the number of seismograms that can be considered routinely. The new scheme will be made available on the web soon...

Application : Global tomography (36513 path average Sv models obtained from multi-modes Rayleigh waveforms)

Ray density



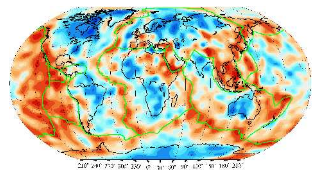
Optimized Voronoi diagram



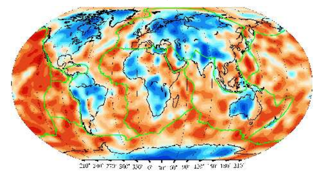
(see second sheet for explanation)

SV-wave heterogeneities at different depths :

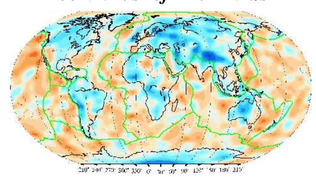
100 km Vs ref = 4.41 km/s



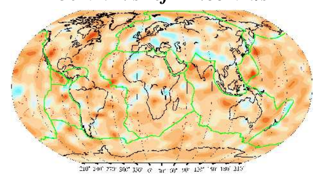
150 km Vs ref = 4.42 km/s



200 km Vs ref = 4.44 km/s

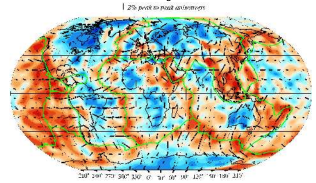


250 km Vs ref = 4.66 km/s



SV-wave azimuthal anisotropy (shown with black bars) :

100 km Vs ref = 4.41 km/s



150 km Vs ref = 4.42 km/s