Self Assembly

(talk for the AERES evaluation)

Eric Rémila

based on **Florent Becker**'s Ph. D. thesis.

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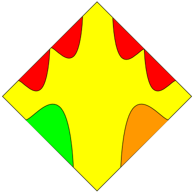
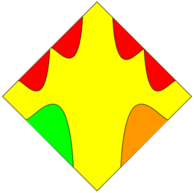
Principle

- A set of *Wang tiles*



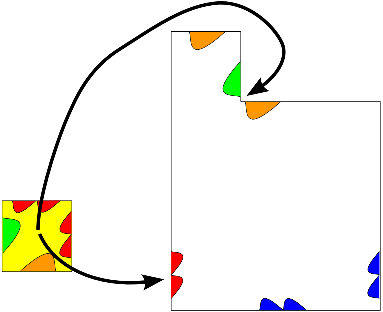
Principle

- A set of *Wang tiles*
- with *glues* of different strengths



Principle

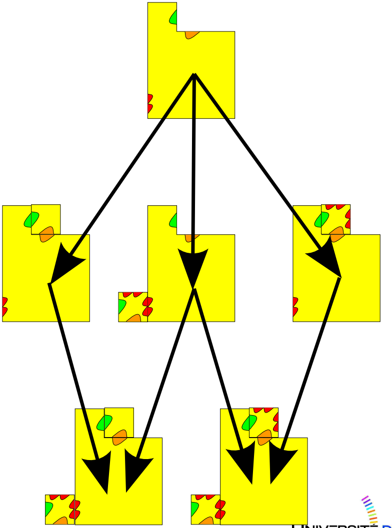
- A set of *Wang tiles*
- with *glues* of different strengths
- The sum of link strengths must be larger than the *temperature* for a possible aggregation of a new tile



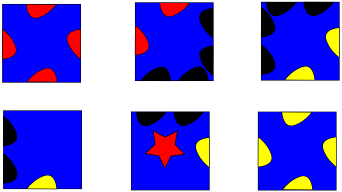
Example with $T = 2$.

The notion of dynamics

- We want to describe the assembly process, taking account parallelism and non-determinism
- Partial order of *productions*
- Language generated by the tile set: final productions
- originality: we want to generate stable languages up to homotheties, instead of a unique shape

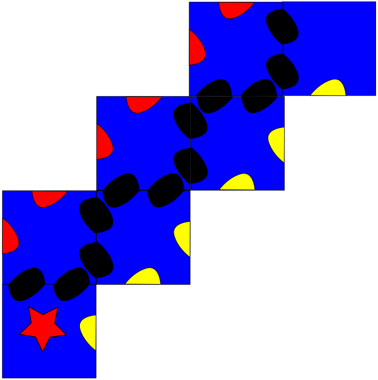


A simple example



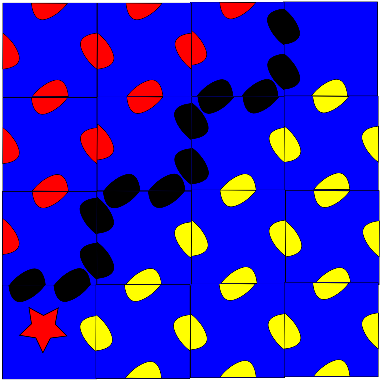
What is the language generated by this tile set, at temperature 2 ?

A simple example



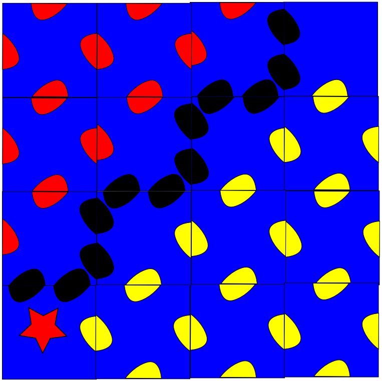
with strength 2 glues, creation of a diagonal line .

A simple example



Completion.

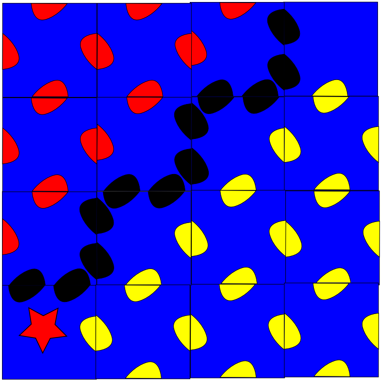
A simple example



Conclusion: the given tileset

- allows to construct all squares (with size ≥ 2),.

A simple example



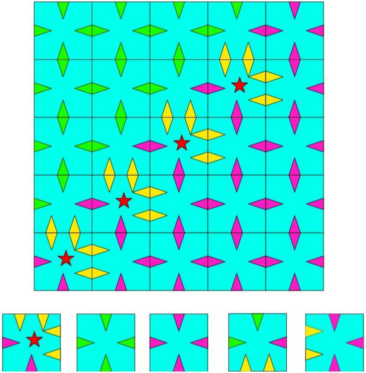
Conclusion: the given tileset

- allows to construct all squares (with size ≥ 2),
- only allows to construct squares (bicolor effect).

The context
(a page of advertising)

- Crystal growth.
- DNA self-Assembly.
- Biological computing.
- Nanotechnology.

Tile set optimality result



The smallest tile set which generates the language of squares contains 5 tiles.

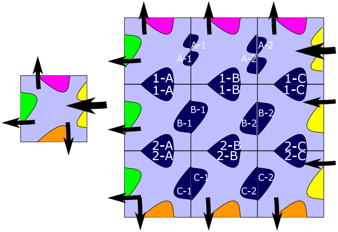
Question: Assume that we have a tile set S which generates a shape language L. Can we deduce a shape language S' which generates the shape language $3L$?

- In the general case, this is not possible.

Scaling results

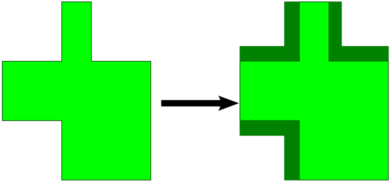
Question: Assume that we have a tile set S which generates a shape language L. Can we deduce a shape language S' which generates the shape language $3L$?

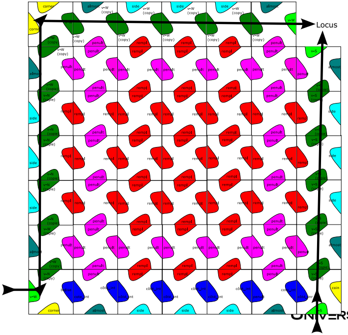
- In the general case, this is not possible.
- If the dynamics induced by S satisfies an *order condition* (which is true for all samples in the literature), then the dynamics can be controlled and, therefore, this can be done.



Approximative scaling results

If the dynamics induced by S satisfies the *RC condition* (Rothemund, Winfree) and no tile contains two strength 2 glues, (which is true for most of samples in the literature), then this can be approximatively done.
Moreover, $S' = S \cup U$, where U only depends on the set of glues of S (universality).





A new ingredient: the time

For each tile t, we fix a concentration k_t.
The associated continuous time Markov chain is defined by:

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- States: productions,
- Transitions: tile additions,
- Transition time: the possible addition of the tile t is done according to an exponential law with parameter k_t (i. e. the average time for the transition is $1/k_t$).
- Construction time for a production P: the average time for reaching P.
  This is a canonical modelization of successive aggregations, starting from the root, in a soup with low concentrations.

The parallel model (discrete time)

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- At each step, we add simultaneously all the possible tiles of P,
- Parallel time : number of parallel steps to get P.
- **Theorem:** Under the order condition, we have:

$$\textit{parallel time} = \textit{continuous time}$$

up to a constant which only depends on concentrations,
This allows to study the parallel time (this is easier).

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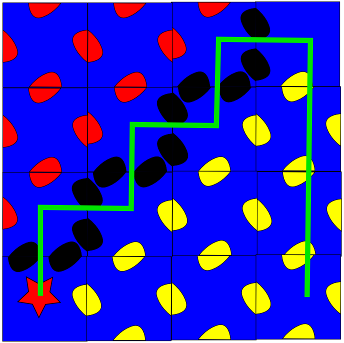
The time in our sample



- The (parallel) construction time of the $n \times n$ square is $3n$.

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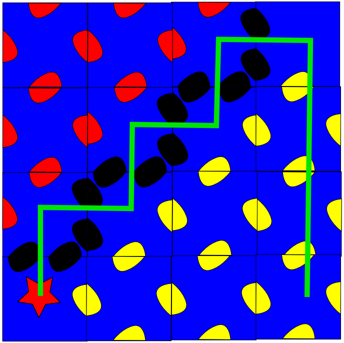
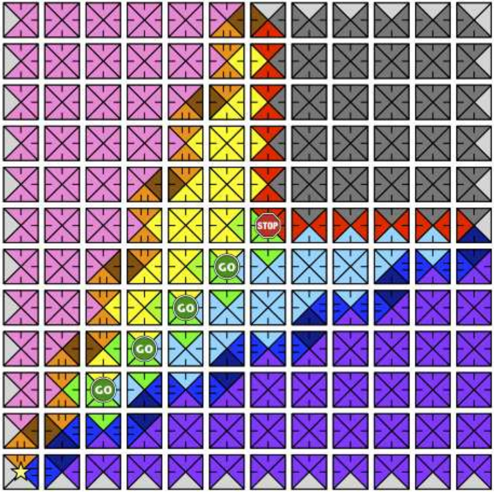
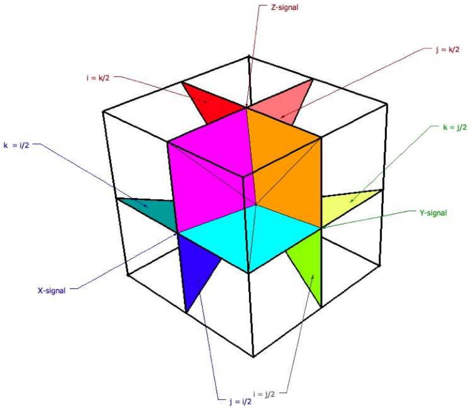


- The (parallel) construction time of the $n \times n$ square is $3n$.
- Can we do it faster ? Can we find a tile set which constructs squares in the optimal time $2n$?

Time optimal construction of squares

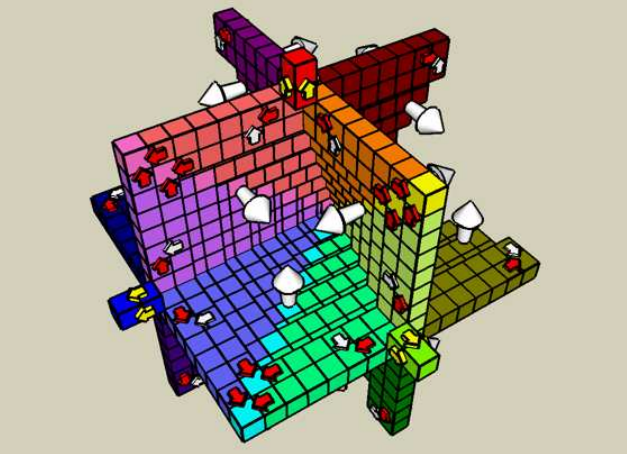
YES, we can !





Theorem: There exists a tile set which constructs cubes (with sides ≥ 2) in temperature 3.

Extension en dimension 3 (with temperature 3)



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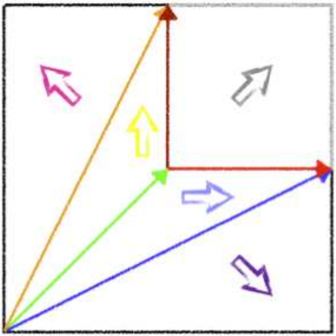
Programming language

(Pictures are better than a thousand tiles)

Given a language of shapes, how to design a tile set which generates this language?
We introduce a *self-assembly programming language* (with signals and collisions) which plays the role of a high level language.



Open questions

- construction of other languages of geometric chains (polygons, circles, ...)
- construction of tilings of the whole plane (quasi-periodic or more complex)
- more in higher dimensions
- working on other underlying lattices (euclidean, or even hyperbolic)

"This is the END"