Performance Evaluation - Multiple Choice Quiz

*Instructions*: for each question, fill the empty box or encircle all true answers (there is at least one true answer).

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>ANSWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Let Ω be a set and ( P ) a probability measure on ( F ) ( \sigma )-algebra over ( \Omega ) containing all singletons, is ( P ) fully characterized by all the values ( \mathbb{P}({\omega}), \omega \in \Omega )?</td>
<td>YES NO</td>
</tr>
<tr>
<td>2 Same question if ( \Omega ) is countable</td>
<td>YES NO</td>
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<tr>
<td>3 Let ( A_n, n \geq 1 ), be a set of events such that ( \mathbb{P}(A_n) = 1 ). Is it always true that ( \mathbb{P}(\bigcap_{n&gt;1} A_n) = 1 )?</td>
<td>YES NO</td>
</tr>
<tr>
<td>4 Is this statement true: “if two events are disjoint, then they are independent”?</td>
<td>YES NO</td>
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<tr>
<td>5 Let ( A, B, C ) three random variables such that ( A ) and ( B ) are independent, ( B ) and ( C ) too, ( C ) and ( A ) too. Is it true that the variables ( {A, B, C} ) are independent?</td>
<td>YES NO</td>
</tr>
<tr>
<td>6 Consider a set of independent events, then for any subset of events, they remain independent?</td>
<td>YES NO</td>
</tr>
<tr>
<td>7 Consider a set of independent events, if we replace some events by their complementary, then the events of the new set remain independent?</td>
<td>YES NO</td>
</tr>
<tr>
<td>8 Let ( X ) be a real random variable with values in ( \mathbb{N} ) and mean ( \mathbb{E}(X) ). Is it always true that ( \mathbb{E}(X) = \sum_{x \in \mathbb{N}} \mathbb{P}(X &gt; x) )?</td>
<td>YES NO</td>
</tr>
<tr>
<td>9 The time to get connected to the ENS Wifi is 5 seconds on average. Is it true that the probability that this time exceeds 20 seconds, is lower that 25%?</td>
<td>YES NO</td>
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<tr>
<td>10 You play Heads &amp; Tail with a biased coin which gives Heads with probability ( p ). You draw the coin until Heads appears, each trial being independent of all others. How many trials will you need on average?</td>
<td>( \frac{1}{1-p} ) ( \frac{1}{p} ) ( \frac{1-p}{p} ) ( +\infty )</td>
</tr>
<tr>
<td>11 You watch a system having ( n ) possible states. Each observation is independent and returns one state randomly with the uniform distribution. What is the order of magnitude of the average number of observations required to see all possible states?</td>
<td>( n ) ( n \log n ) ( n^2 ) ( +\infty )</td>
</tr>
<tr>
<td>12 You measure the performances of a system with a simulator. Each simulation is independent of others and returns a performance measure which follows the same (unkown) law whose mean is ( m ). Let ( X_n, n \geq 1 ), be the sequence of measures and ( M_n = (X_1 + \cdots + X_n)/n ) their empirical mean, is it always true that this empirical mean tends to ( m ) almost surely?</td>
<td>YES NO</td>
</tr>
<tr>
<td>13 A noisy communication channel is disturbed by a sum of ( n ) independent random variables where each one adds to the signal value either ( +1/n, ) or ( -1/n, ) with respective probabilities ( 1/2. ) When ( n \rightarrow +\infty, ) the total noise converges to the following random law?</td>
<td>Normal Poisson does not converge!</td>
</tr>
<tr>
<td>14 A test from industry detects flawed processors with probability 99%, but may state a good processor is flawed in some 1% of the cases. Statistics indicate that 1% of processors are really flawed. You are using the test on a processor and it indicates a flaw. What is the probability that the processor is really flawed?</td>
<td>20% 50% 90% 99%</td>
</tr>
</tbody>
</table>
### Question 15
Two ticket offices open simultaneously with one waiting client in each queue. Office A processes each client request with exactly 30 seconds. Office B processes each client request with a time following an exponential law of parameter $0,1\text{sec}^{-1}$. To minimize the average waiting time, which office would you recommend?

**Table 1: Waiting Time Comparison**
<table>
<thead>
<tr>
<th>Office</th>
<th>Average Waiting Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30 seconds</td>
</tr>
<tr>
<td>B</td>
<td>$\lambda^{-1} = \frac{1}{0,1} = 10$ seconds</td>
</tr>
</tbody>
</table>

**Recommendation:** Office A

### Question 16
A monkey is typing numbers randomly on a pocket calculator, each new number follows a uniform probability and is independent of the previous numbers. What is the probability that the monkey ends by typing all decimals of $\pi$?

**Answer:** $\frac{1}{\pi}$

### Question 17
The monkey is back. What is the probability that some day he will type consecutively all decimals of your phone number?

**Answer:** \(\frac{1}{10^n}\)

### Question 18
Let $M = (m_{ij})$ be an $n \times n$ matrix, provide necessary and sufficient conditions over coefficients $m_{ij}$ so that $M$ is the transition matrix of some Markov chain over $n$ states.

**Answer:**
- Necessary condition: $\sum_{j=1}^{n} m_{ij} = 1$ for all $i$
- Sufficient condition: $m_{ij} \geq 0$ for all $i, j$

### Question 19
Consider a Markov chain $M$ over states $\{1, 2, 3, 4\}$ whose transition matrix $P$ is given on Figure 1. Is this chain irreducible?

**Answer:** Yes

### Question 20
Is this chain $M$ aperiodic?

**Answer:** Yes

### Question 21
Among the next vectors $[\pi_1, \pi_2, \pi_3, \pi_4]$ over states $\{1, 2, 3, 4\}$, which ones are invariant distributions for the chain $M$?

**Answer:**
- $[\frac{1}{2}, \frac{1}{2}, 0, 0]$
- $[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}]$
- $[0, 0, 0, 0]$

### Question 22
Let $\pi = [\pi_1, \pi_2, \pi_3, \pi_4]$ be a vector such that $\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$, does the vector sequence $\pi P^n, n \to +\infty$, always converge?

**Answer:** Yes

### Question 23
ALOHA is?

**Options:**
- a mobile phone standard
- a media access control protocol
- a Hawaiian greeting

**Answer:** a media access control protocol

### Question 24
You are looking for the book “One thousand exercises in probability” in several libraries. Each library has a 40% chance to own the book and, in that case, there is a 50% probability that it has already been borrowed. You are ready to visit 4 libraries. Does it ensures that you will find this book available with more that 50% probability?

**Answer:** Yes

### Question 25
On an ethernet channel which has just been freed, $k \geq 2$ computers are ready to send a message, each one in a time slot which is chosen randomly and independently among the next $n$ slots, with an uniform distribution. When the collision probability, i.e. the probability that two computers choose the same slot, is larger than 1/2?

**Answer:**
- $k = \lceil \frac{n}{2} \rceil$
- $k = \lceil 2\sqrt{n} \rceil$
- $k = \lceil \log_2 n \rceil$

### Question 26
At an oral exam, a candidate who did not rehearse at all, must choose between three answers. Two of them are wrong and the third one is true. Once the candidate has chosen an answer, the examiner shows one of the two other answers and indicates that the one he shows is wrong. Then he asks the candidate whether he wishes to change his answer. Should the candidate change?

**Answer:** Yes

### Question 27
Every second, a frame enters a router with probability 1/2 and the router serves a waiting (if any) frame with probability 1/2. All is independent and the router is initially empty. What is the probability that its queue never exceeds 100?

**Answer:**
- $0$
- $1/2^{100}$
- $1/2$
- $1$

### Question 28
What is the average time (in seconds) until the router above is empty again?

**Answer:**
- $1$
- $2$
- $2048$
- $\infty$

**Matrix $P$**

$$
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1/2 & 0 & 1/2 & 0 \\
0 & 0 & 0 & 1 \\
1/2 & 0 & 1/2 & 0
\end{bmatrix}
$$

**Figure 1 – Matrix $P$**
Free box to comment or justify your answers.