ENS Lyon Straining Camp. Day 01.

26 october 2015

Problem A. «Alex Origami Squares»

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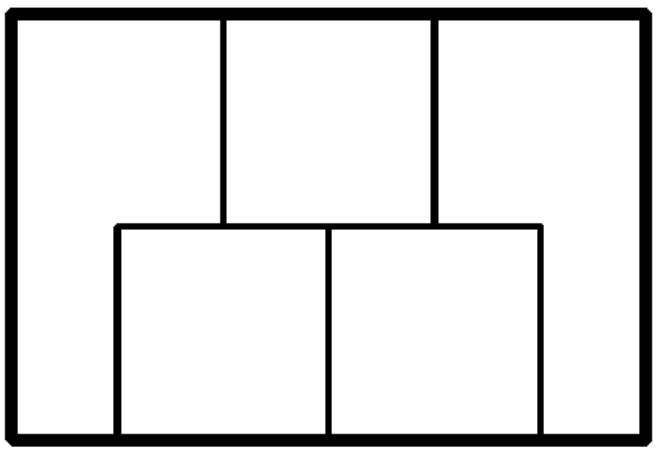
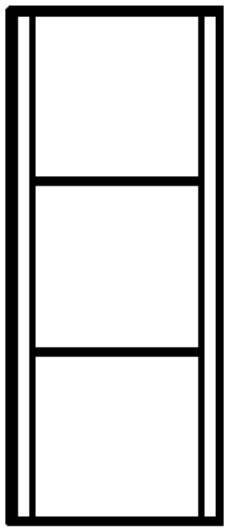
Problem

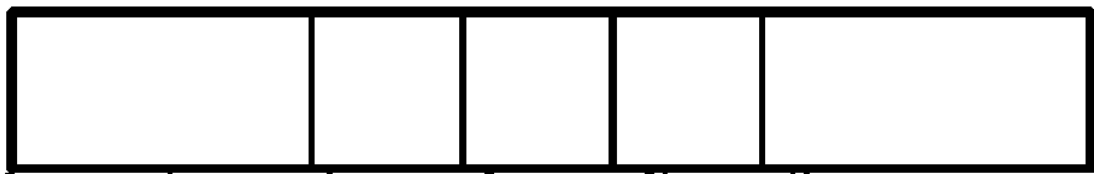
- You are given rectangle
- You need to cut three equal squares of maximum area out of him





Solution

```
max(
  min / 2.0, // Case 1
  min(
    max / 3.0, // Case 2
    min // Case 3
))
```



Problem B. «Black and White»

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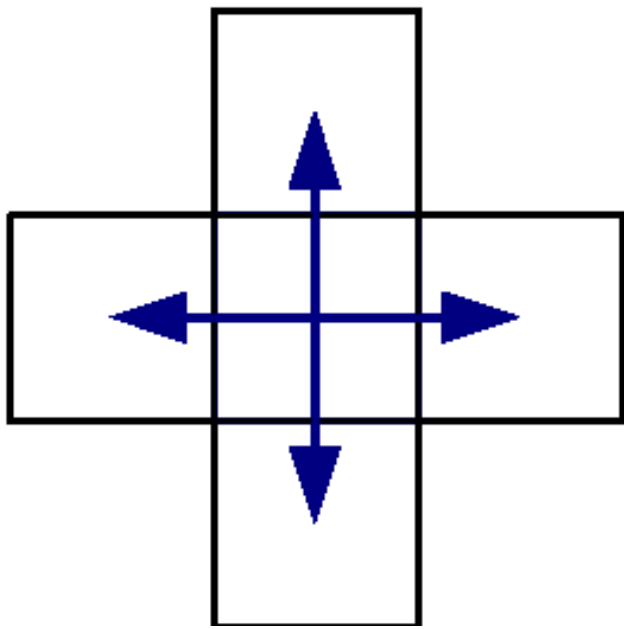
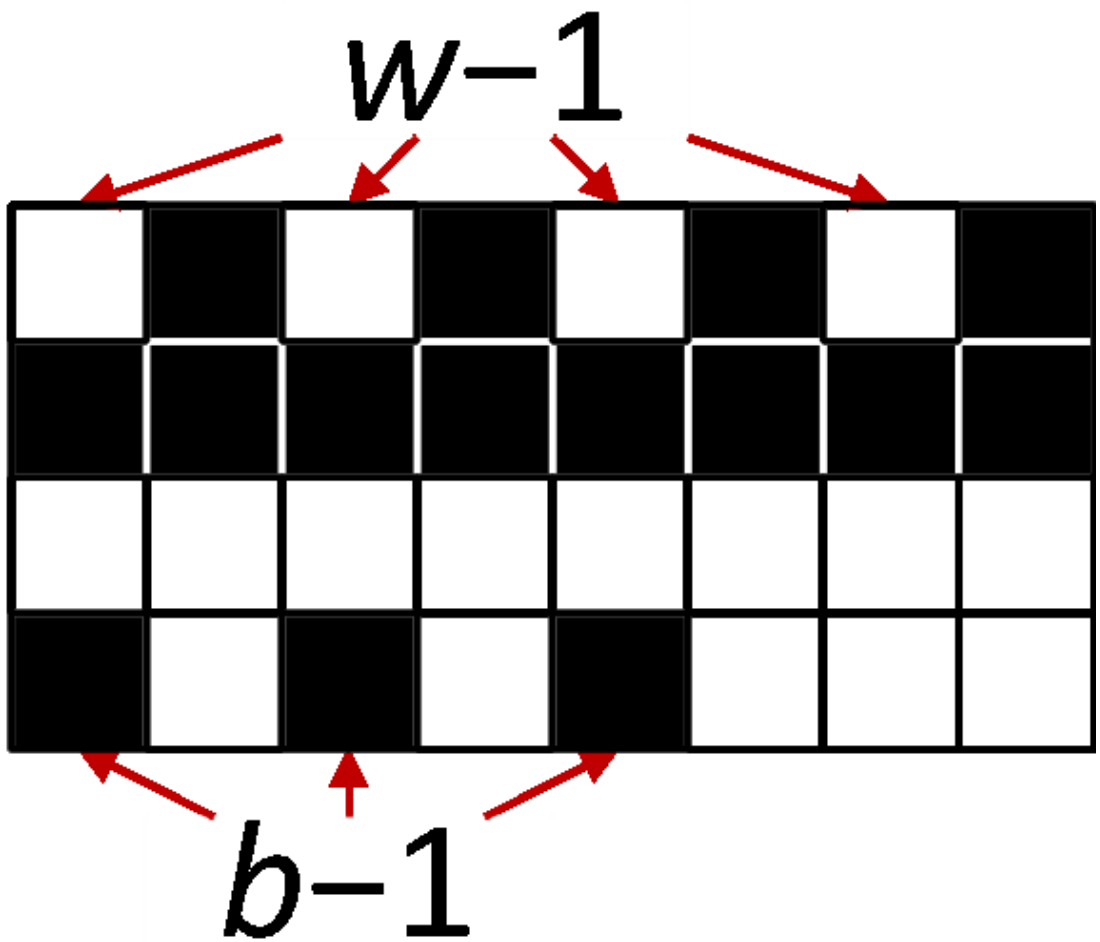
Problem

- Build the rectangle
  - b black areas
  - w white areas
- areas are 4-connected



Solution

Example of construction:



Problem C. «Concatenation»

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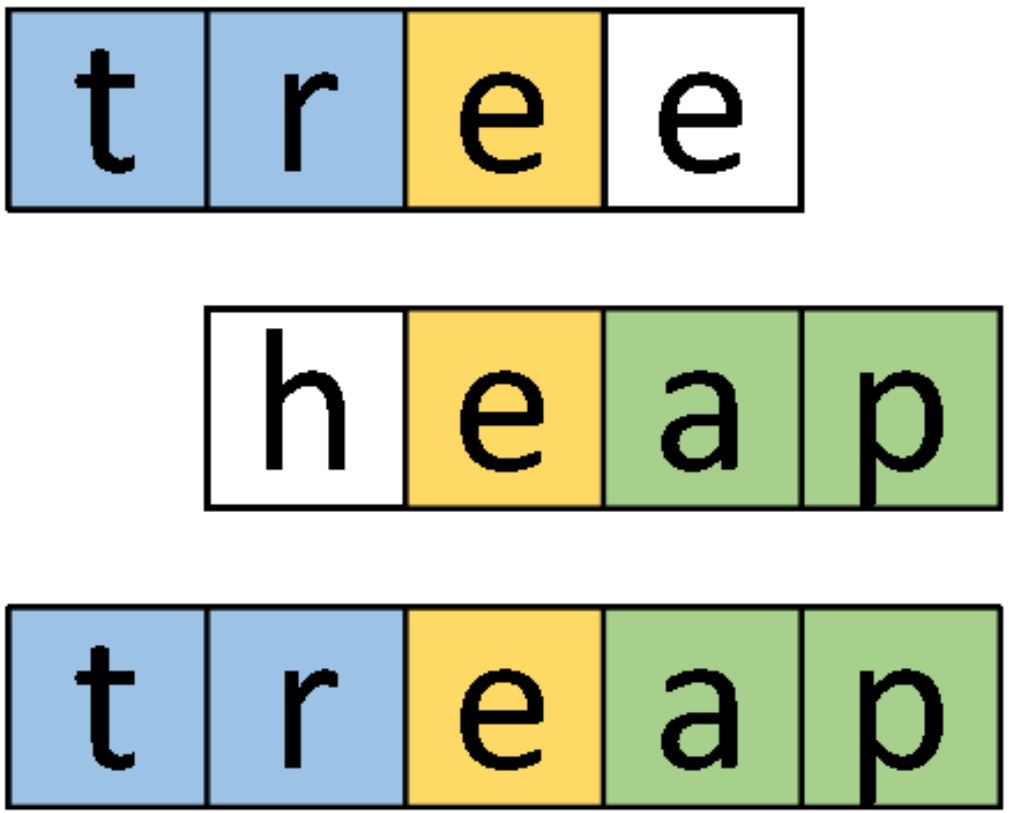
Problem

Calculate the number of different strings as a concatenation of

- non-empty substring of s_1
- non-empty substring of s_2

Solution

When there are duplicates? When there is equal letter.



Solution

- Calculate:
  - $c_{1,i}$ — the number of letter i in s_1
  - $c_{2,i}$ — the number of letter i in s_2
- Answer: $|s_1| \cdot |s_2| - \sum_{i='a'}^{'z'} c_{1,i} \cdot c_{2,i}$
- `int` overflow

Problem D. «Distruibution in Metagonia»

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Problem

Represent x as the sum of terms in form $2^a 3^b$ which doesn't diveide each other.

Example:

$$10 = 4 + 6$$

Solution

- n is even
  - solve the problem for $\frac{n}{2}$
  - multiply the result by 2
- n is odd
  - solve the problem for $n - 3^b$, where $3^b \leq n < 3^{b+1}$
  - add 3^b to answer

Problem E. «Easy arithmetics»

Problem E. «Easy arithmetics»

Problem

Add additional pluses and minuses to the sum to maximize the result
Example:

$$10 + 20 - 30 \rightarrow 10 + 20 - 3 + 0$$

Solution

- $+d_1 d_2 \ldots d_n \to$ left as this
- $-d_1 d_2 \ldots d_n \to -d_1 + d_2 \ldots d_n$
  - exclusion $d_2 = 0$. $-d_1 0 \ldots d_n \to -d_1 + 0 + d_3 \ldots d_n$

Problem F. «Fygon»

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Problem

Calculate total number of `lag` operations.

- return the polynom of n
- there are only loops `for` v `in range(`l`):`

```
for i in range(n):
    for j in range(i):
        for x in range(5):
            lag
lag
```

Solution

- The degree of the polynom is equal to the depth of nesting of loops (less than or equal to 6)
- The answer is calculated as:
  - calculate number of `lags` for $n = 1, 2, 3, 4, 5, 6, 7$
  - make interpolation
  - calculate the coefficients
- Need to write the rational number class

Problem G. «Graph»

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Problem

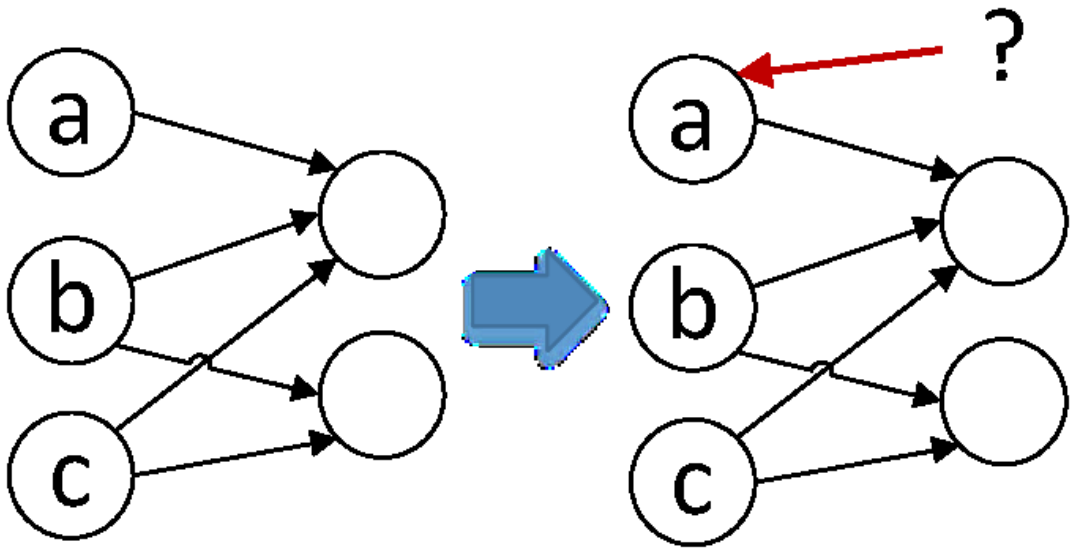
- You are given directed acyclic graph
- You need to add k edges to maximize the minimal topological sort

Solution

- Generate minimal topological sort
- The vertex is added if:
  - there are no input edges
  - it has minimal number

Solution

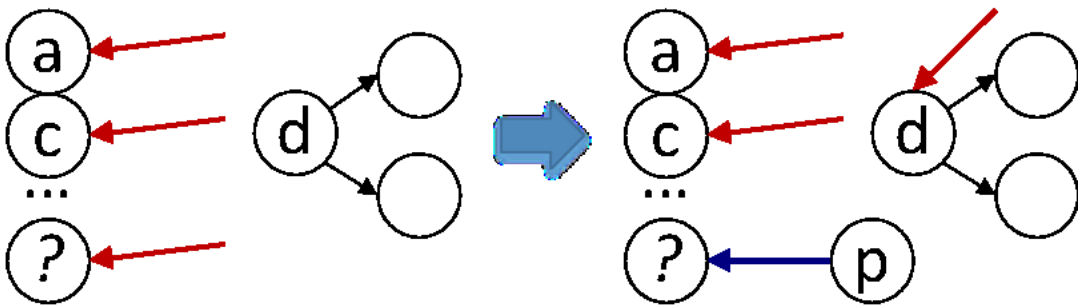
Multiple candidates: add edge to minimal, if there are edges



Solution

One candidate:

- all less than d, then print d and remove d
- maximal ? bigger than d, then add an edge from the last printed vertex to ? and add an edge to d



Solution

If there are no additional edges left:

- always put the minimal vertex without new edge
- put vertices with new edge in decreasing order

Solution

- Vertex:
  - the number of input edges
  - the list of outgoing edges
  - the deletion in $O(e)$
- Sorted sets:
  - candidates
  - the vertices with new edge
- Total work time: $O(E \log E)$

Problem H. «Hash Code Hacker»

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Problem

- You need to generate n different strings with the same hash value
- Hash is calculated as $s_0 \cdot 31^{n-1} + s_1 \cdot 31^{n-2} + \ldots + s_{n-1}$

Solution

- From statement: $hash(\texttt{edHs}) = hash(\texttt{feHs}) = h$
- Then:
  - $hash(\texttt{edHsedHs}) = 31^4 h + h$
  - $hash(\texttt{edHsfeHs}) = 31^4 h + h$
  - $hash(\texttt{feHsedHs}) = 31^4 h + h$
  - $hash(\texttt{feHsfeHs}) = 31^4 h + h$

Problem I. «Insider's information»

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Problem

- There are secret ranking in the form of m triples (a_i, b_i, c_i):
  - $a_i \leq b_i \leq c_i$
  - or $c_i \leq b_i \leq a_i$
- You need to generate rating, such that at least $\frac{m}{2}$ conditions are satisfied

Solution

- Because the truples are consistent:
  - then exists value x from the triples, which doesn't equal to any b_j
  - choose such value x and remove triples with them for a while
- Solve the problem for the rest triples
  - bring back removed triples
  - if we put x as the leftmost element, then some part of triples L will be satisfied
  - if we put x as the rightmost element, then some part of triples R will be satisifed
  - at least one $|L|$ or $|R|$ is not less than the half

Problem J. Journey to the “The World’s start”

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Problem

- Travel from station 1 to station n
  - in a time less than or equal to T
  - spend time to exit and reenter the stations
- There are travel passes for r stations
- You need to buy the cheapest, which satisfies the condition

Solution

- Binary search for r:
- How to calculate the fastest time?
  - dynamic programming problem
  - $t_i = d_i + \min_{i-r \le j < i} t_j$
  - use segment tree or set
- The total work time: $O(n \log^2 n)$

Problem K. «Kingdom trip»

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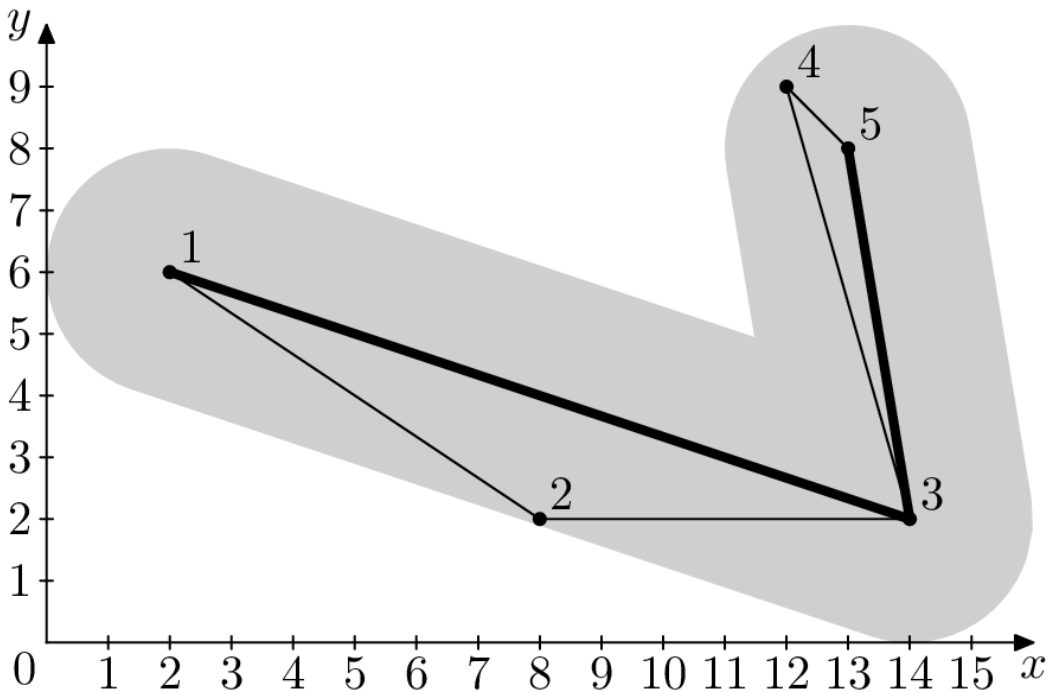
Problem

- You need to make the path more straight
- The only condition is that the distance from vertices to new path $\leq d$

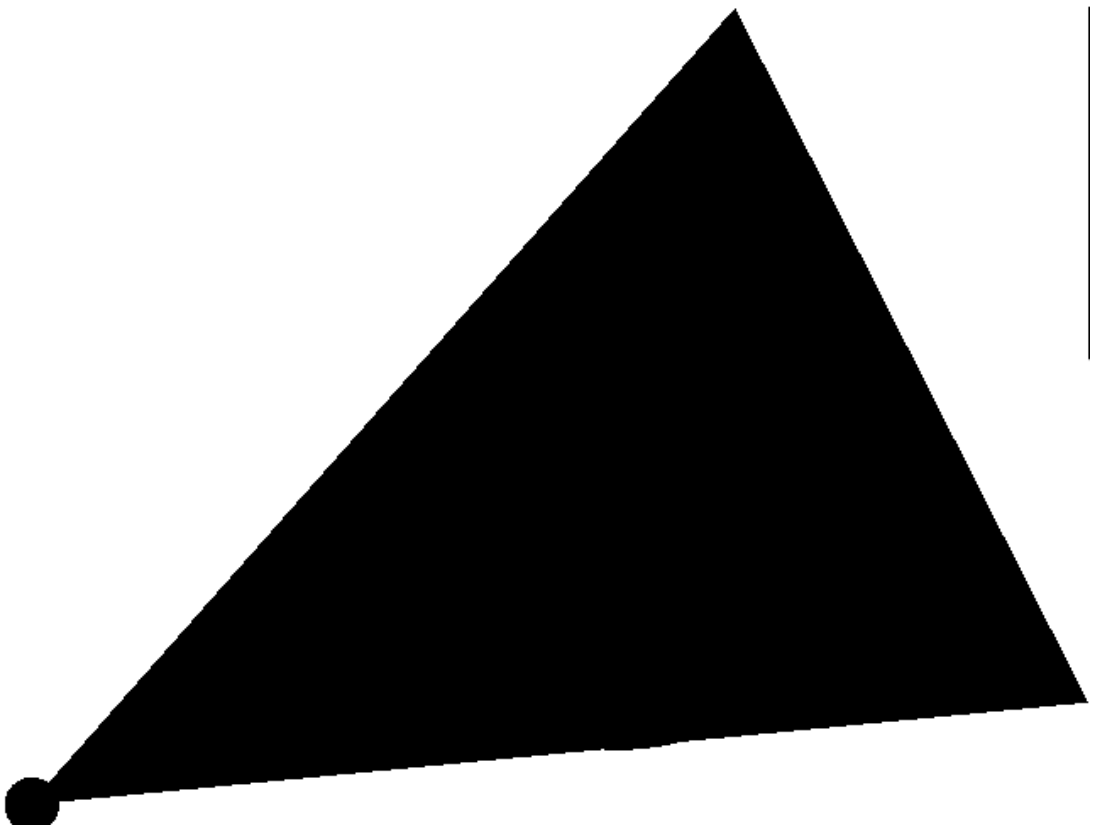


Simplification

- The distance to segment is hard
- The distance to two lines is better
- But easier to think, that the vertices are circles with radius d

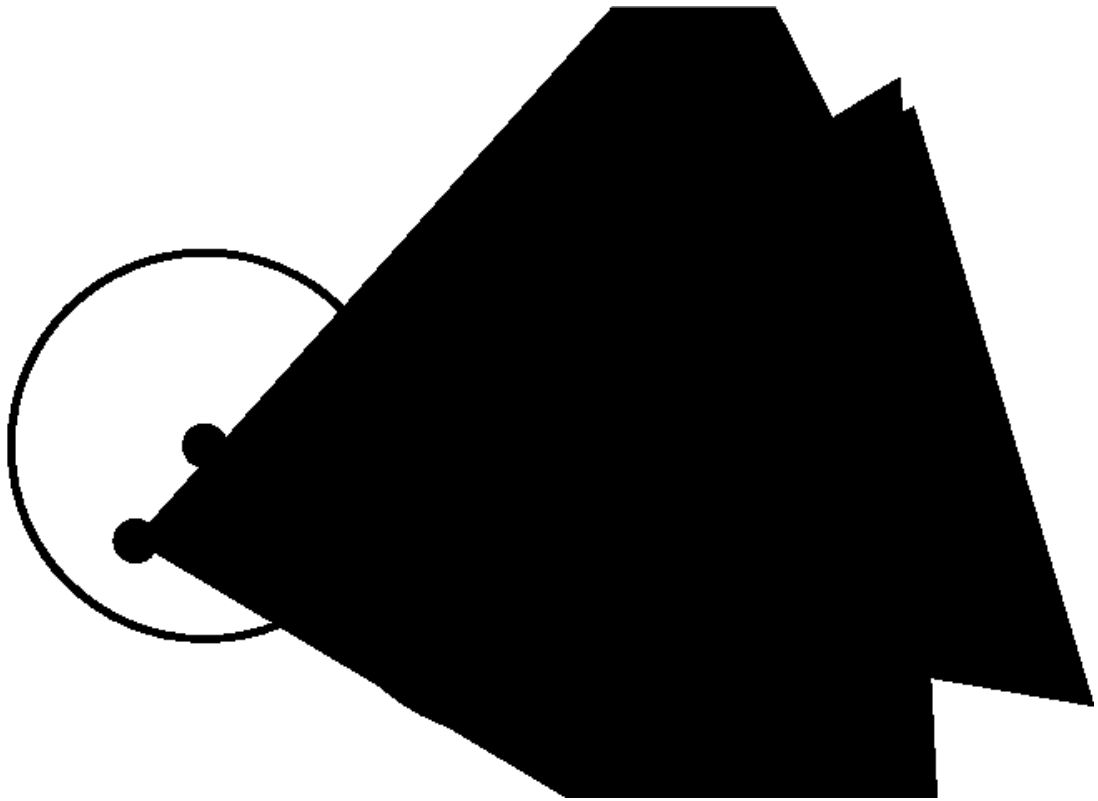
Solution

One point



Solution

Many points:



Solution

- Iterate through the start vertex
- Iterate through the end vertex
  - intersect angles
  - hit the angle
- Dynamic programming: the path on "good" pairs

Problem L. «Lucky chances»

Problem L. «Lucky chances»

Problem

- You are given the matrix with numbers
- You need to find:
  - for each element
  - for each direction
  - chosen element should be bigger than others

Questions?