Scheduling under Unavailability and Uncertainty Constraints

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Outline

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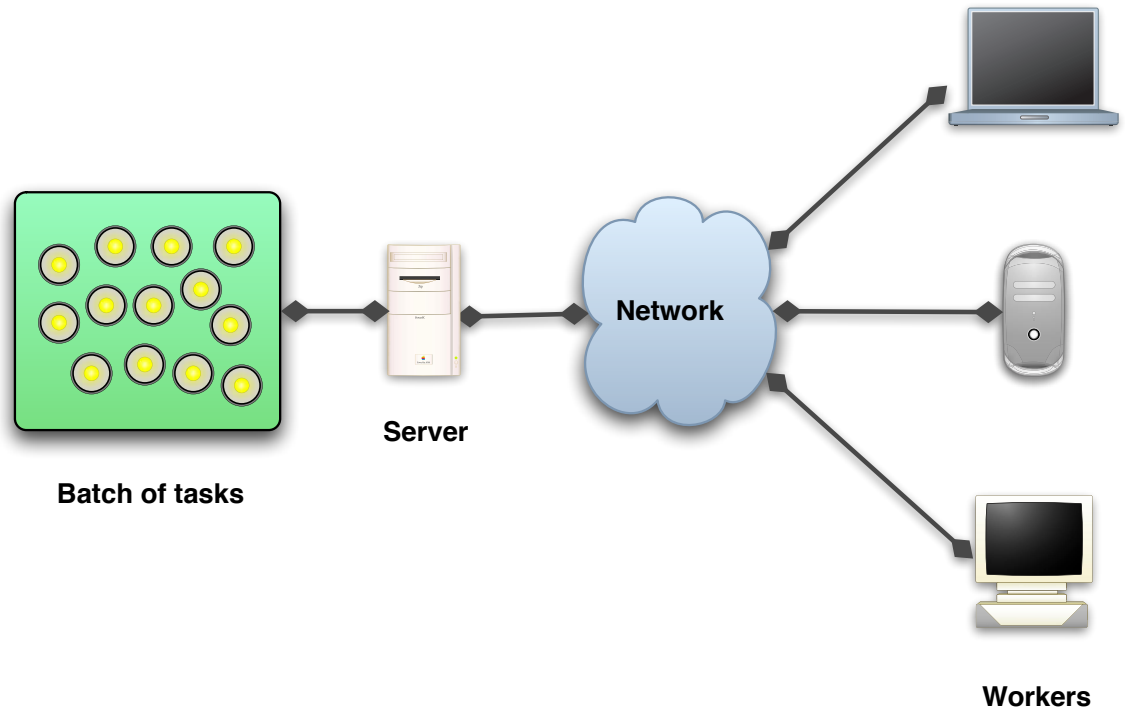
Desktop Grids

Distributed Computing

- The server assigns **tasks** to **workers**.
- Each worker returns a **result** for the corresponding job.
- Example: BOINC-based projects.



Examples

SETI@home

Goal: analyze radio signals, searching for signs of extra terrestrial intelligence.
Anybody can participate by running a free program that downloads and analyzes radio telescope data.
Participants gain credits for performing work.

Folding@home

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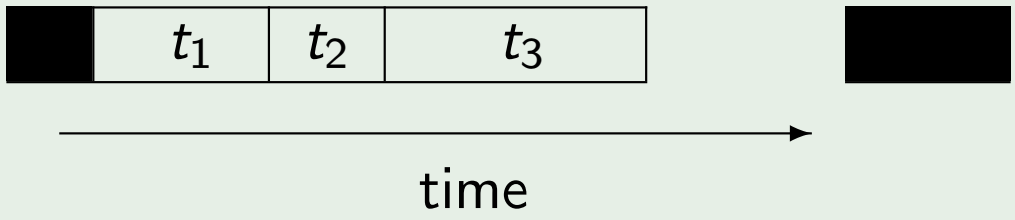

Interruption

Interrupted tasks need to be re-executed.

Tackling Uncertainty

A proactive mechanism and a reactive re-execution strategy (each interrupted task is executed as soon as possible without delaying next scheduled tasks).

Unavailability and Uncertainty constraints

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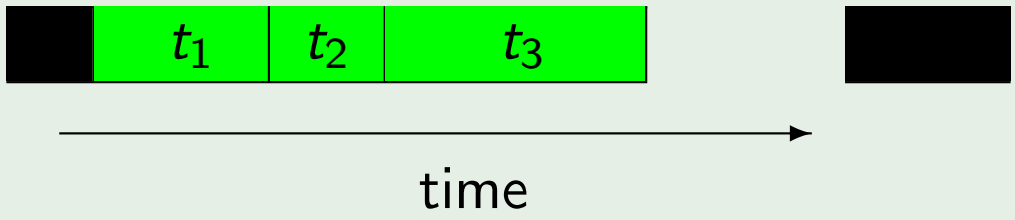

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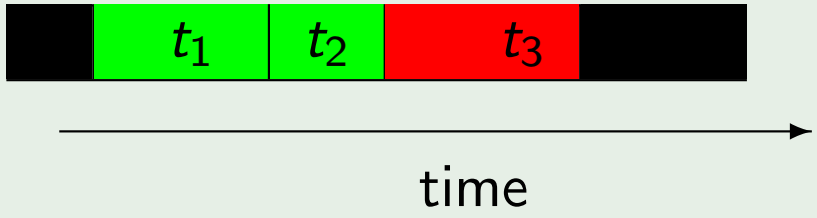

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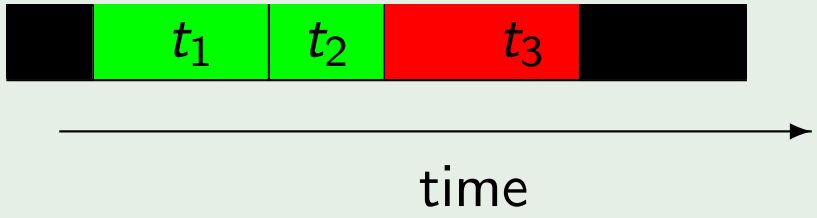

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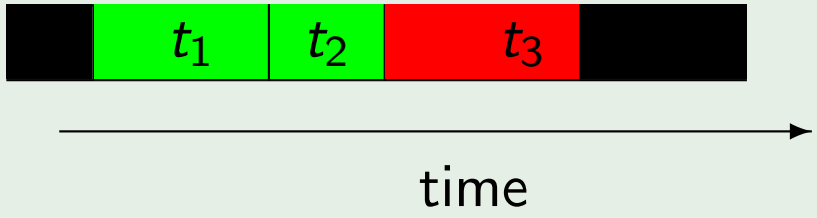

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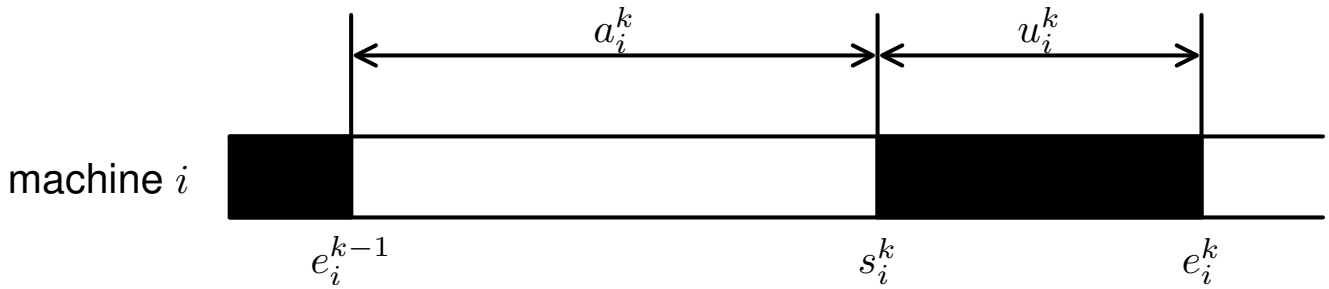

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- Earliness: $\delta_i^k \leq a_i^k$
- Task durations: p_i with $2 \max p_i \leq \min a_i^k$

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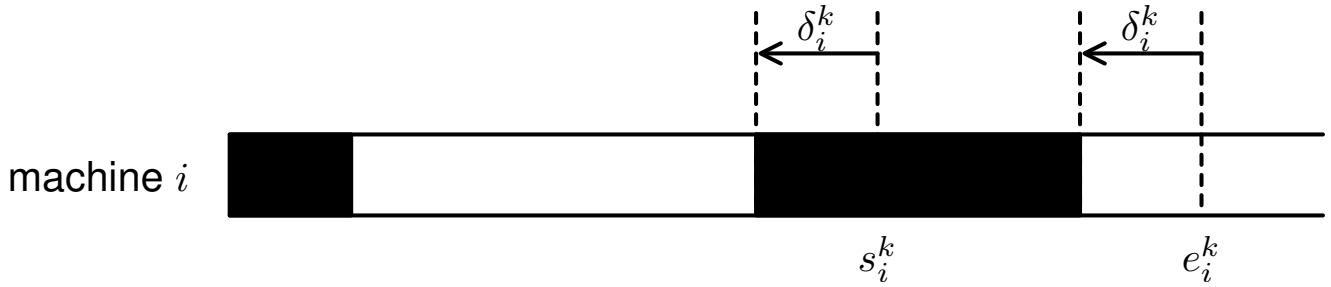

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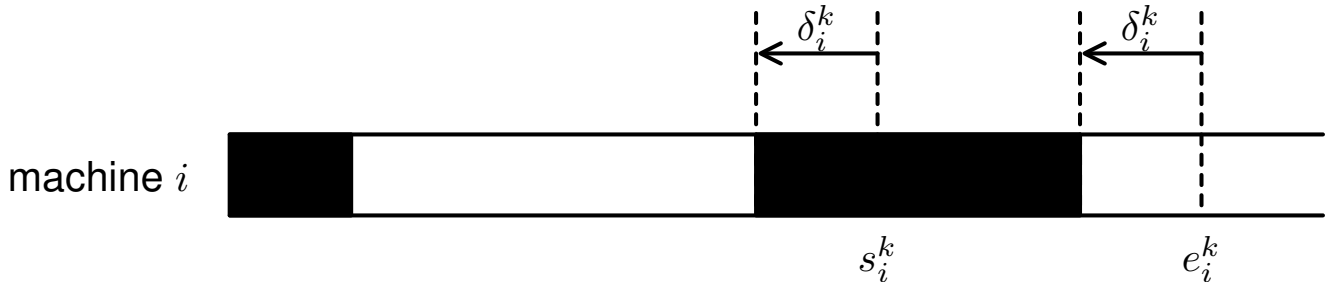

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Often used in desktop grids to overcome the high volatility.

Limits of Checkpointing

- High cost.
- Cannot always be used (application-specific).

Our approach

Avoid the cost of checkpointing in desktop grids with good predictability (enterprise computing).

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Stability

The *stability* is the ratio between the worst disturbed makespan and the baseline makspan.

Bi-Objective Scheduling

Find an allocation function with minimal makespan and minimal value of stability.

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The slack must greater or equal to the maximum length of the jobs assigned to the corresponding availability.

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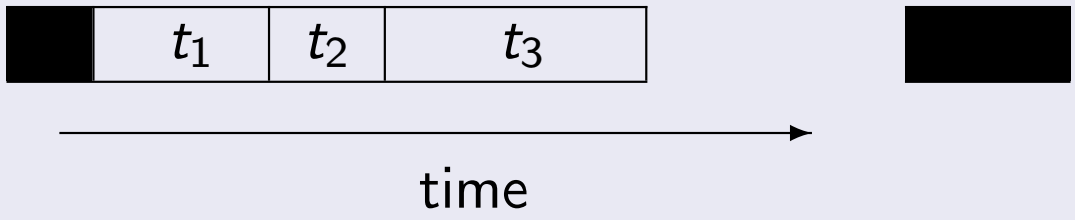

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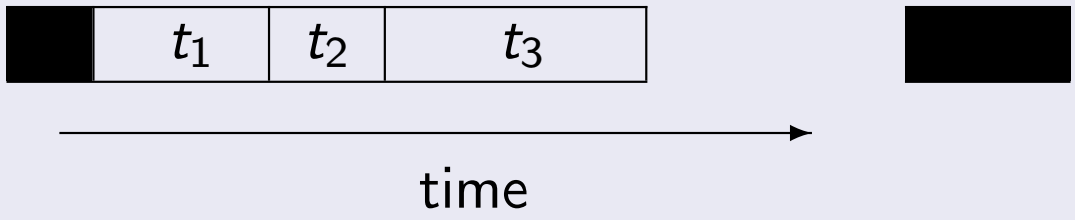

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Complexity

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Minimizing the makespan while respecting the slack rule.

Relation with a modified knapsack problem

On each availability, there must be as much elements as possible.
Let p_i be the profit of element i and $s_i = p_i$ its size. The capacity of the knapsack is C. Element i is selected if $x_i = 1$. Then, we want to maximize $\sum_i x_i p_i$ given the constraint $\sum_i x_i s_i + \max_i x_i s_i \leq C$.

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The slack is proportional to a parameter β (0 for performance, 1 for stability).

Stability Bound

The worst case stability using this approach is

$$r_S = \begin{cases} \frac{5}{2} - \beta + \gamma & \text{if } \beta \neq 1 \\ 1 & \text{otherwise} \end{cases}$$

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Schemes for Scheduling Tasks

General Principle

- Consider each availability in a given order.
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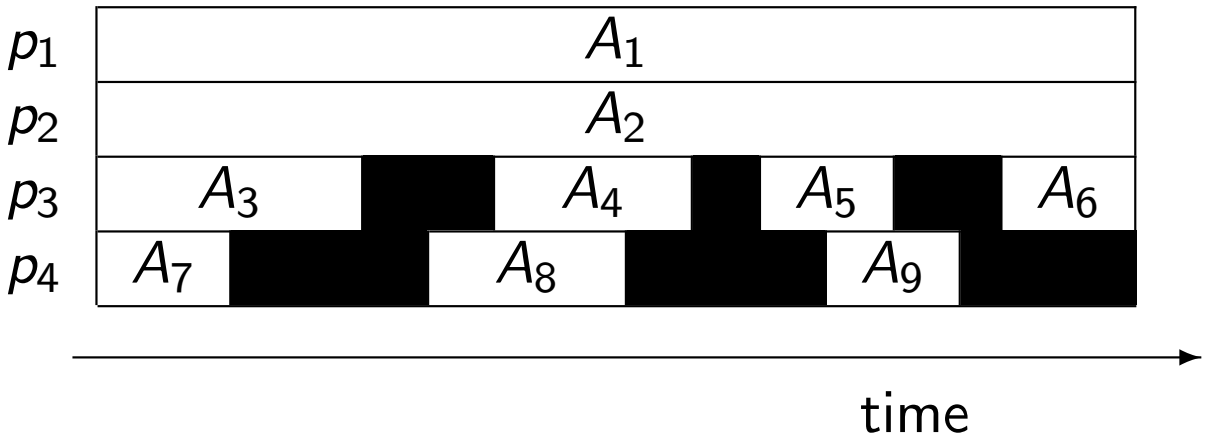

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Principal Steps

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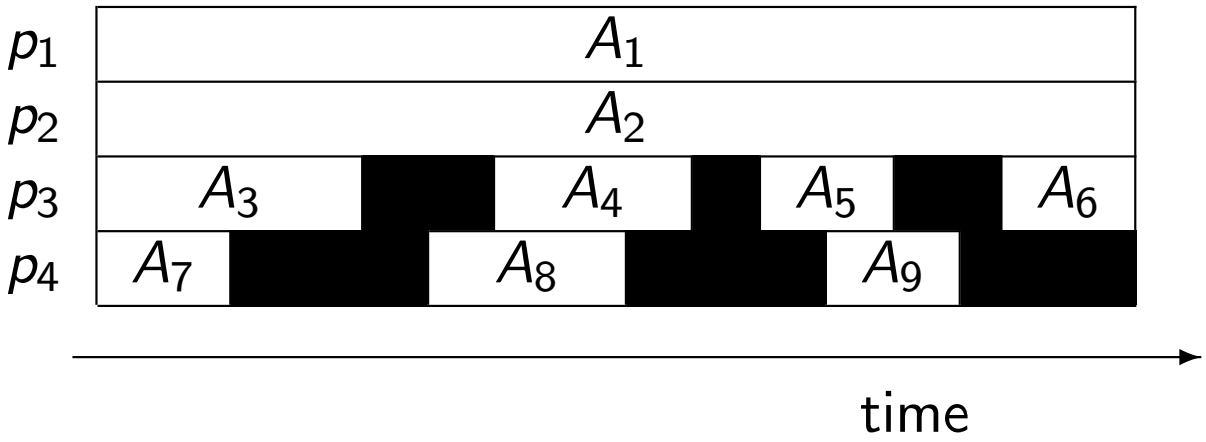

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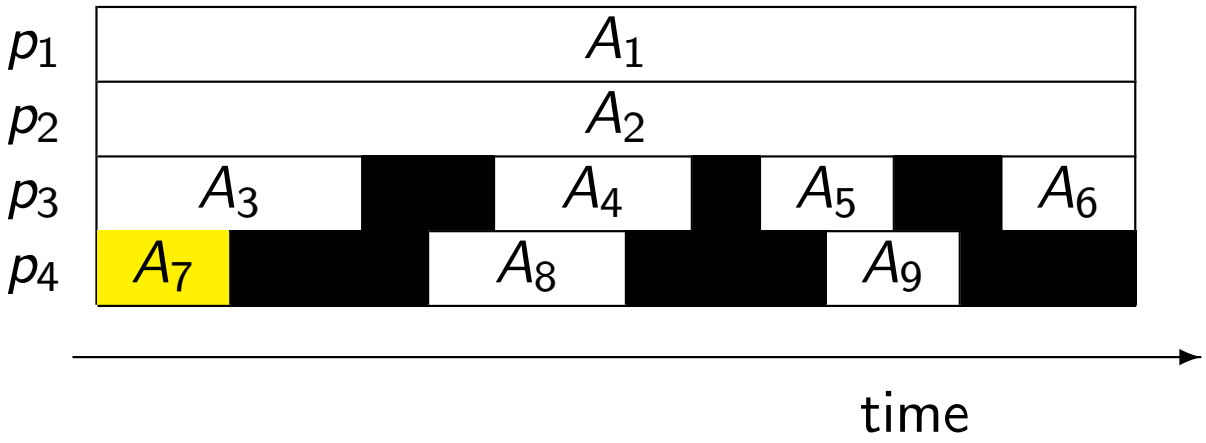

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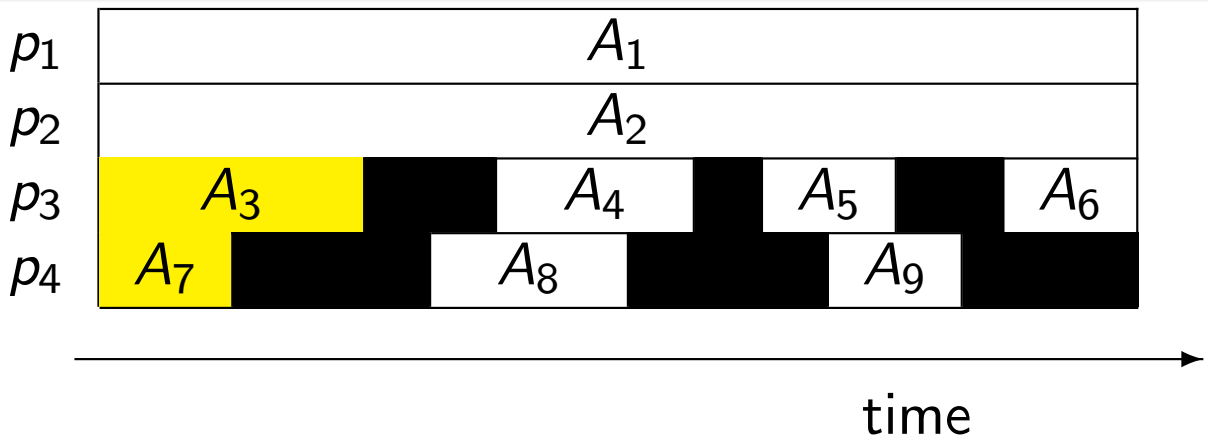

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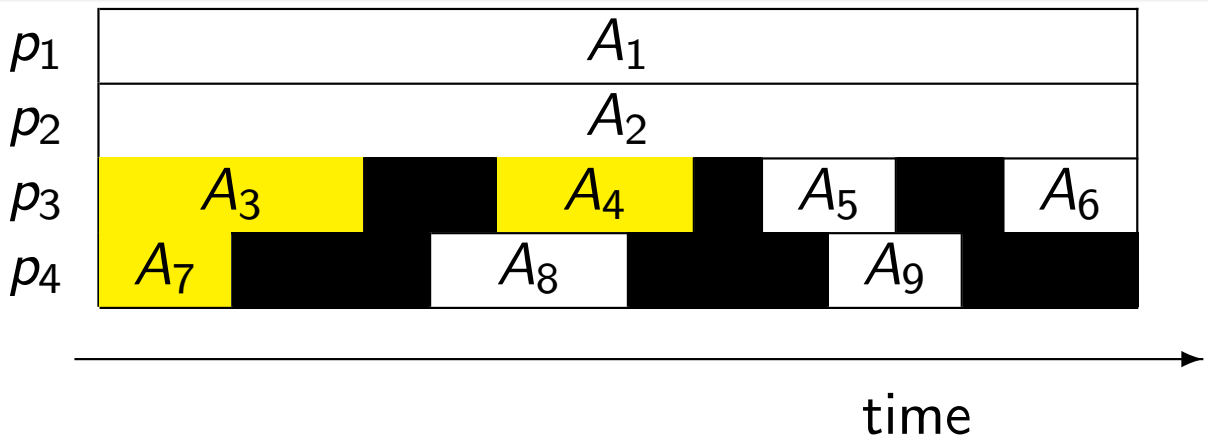

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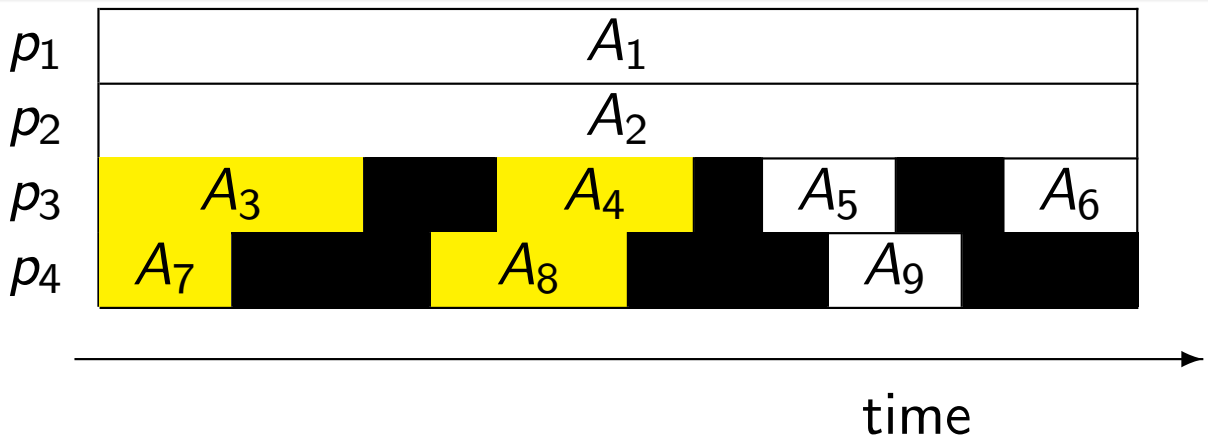

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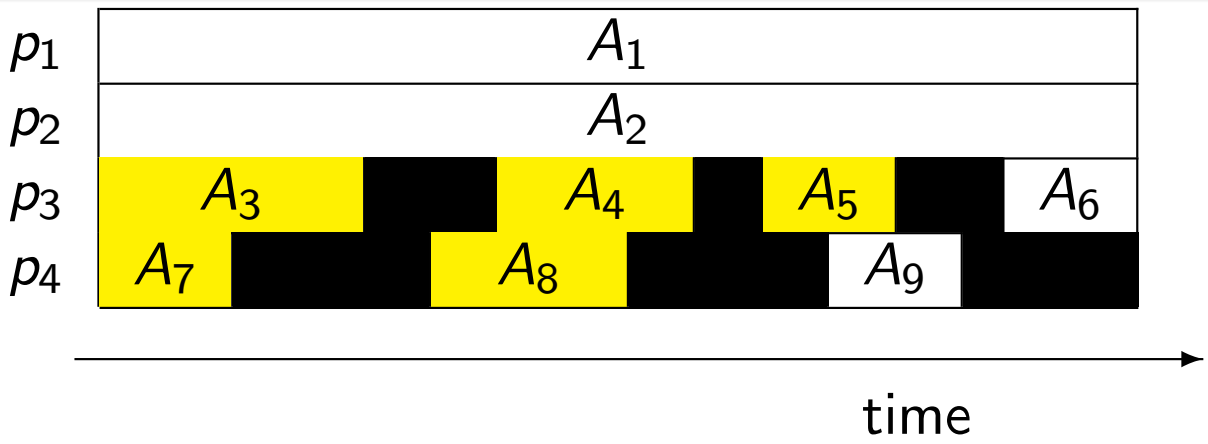

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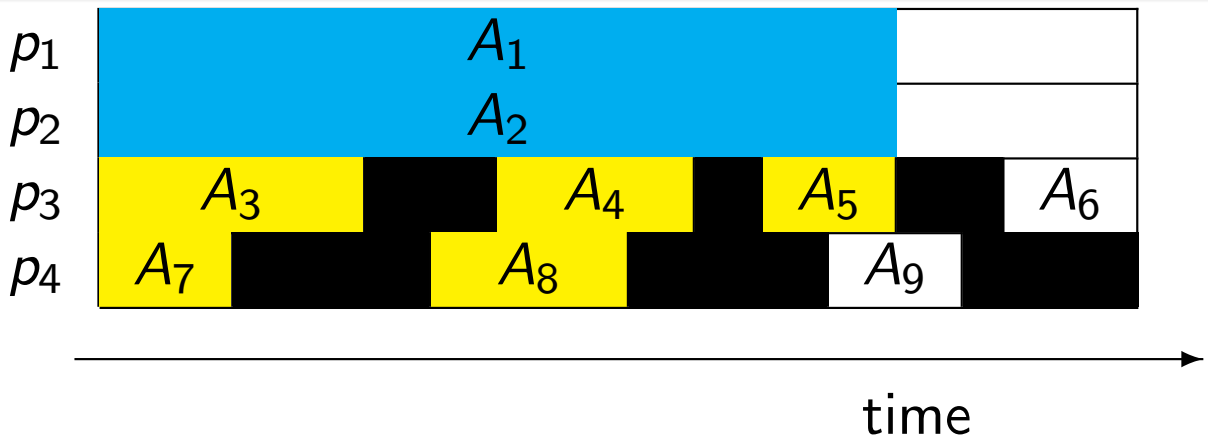

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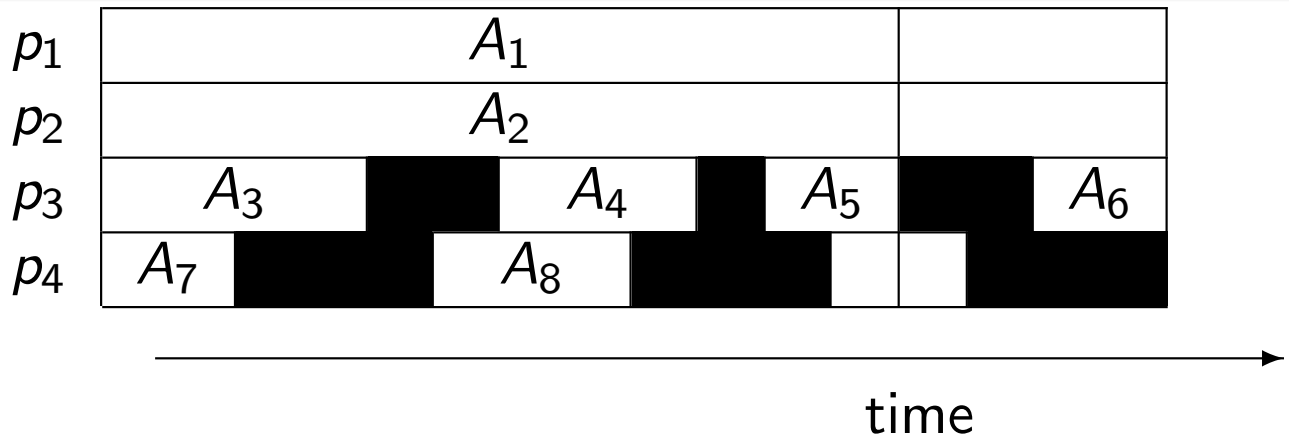

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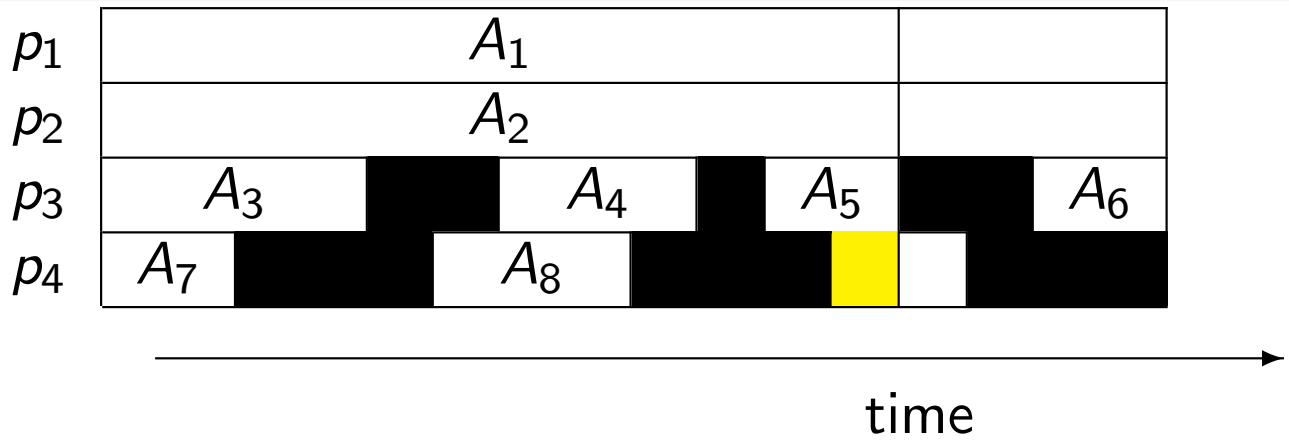

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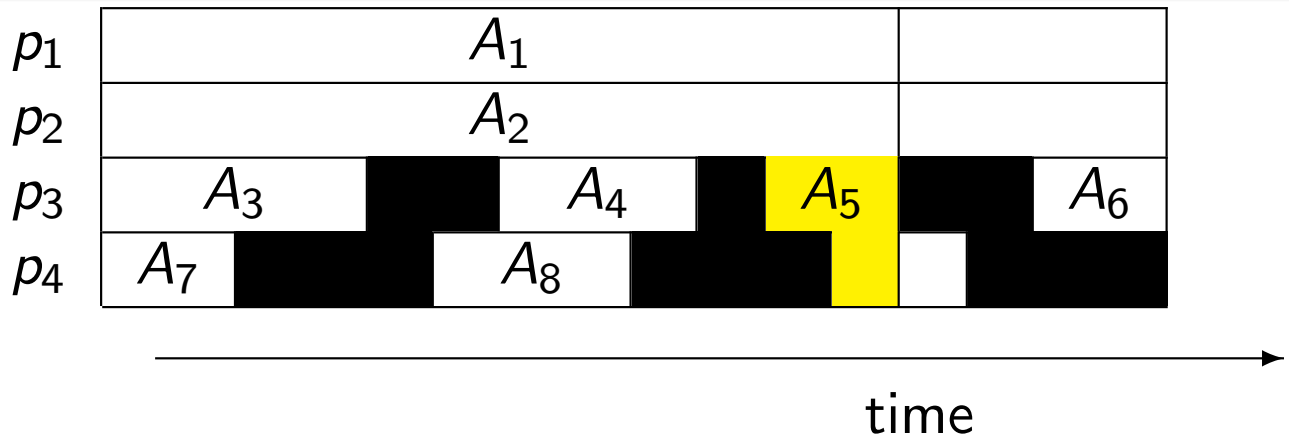

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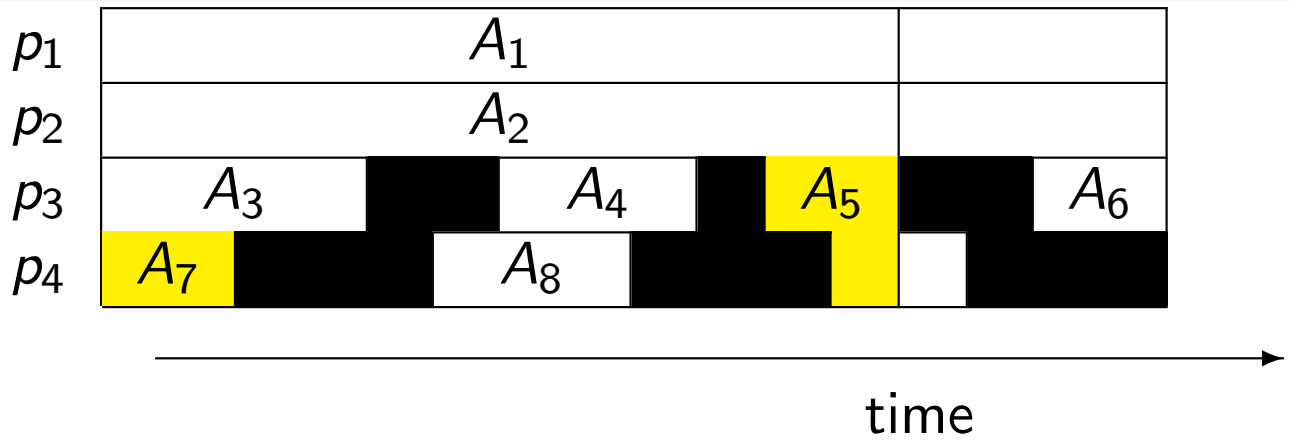

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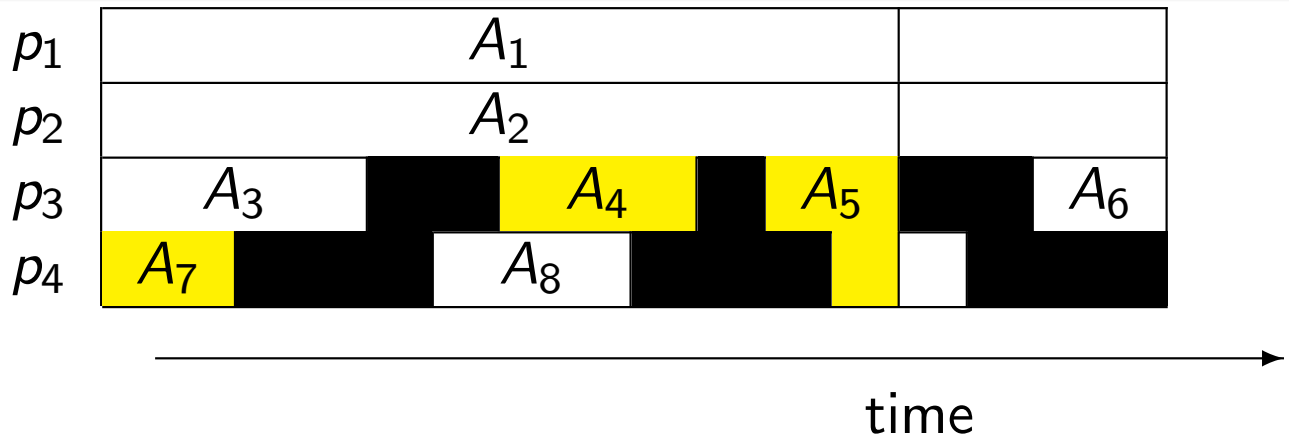

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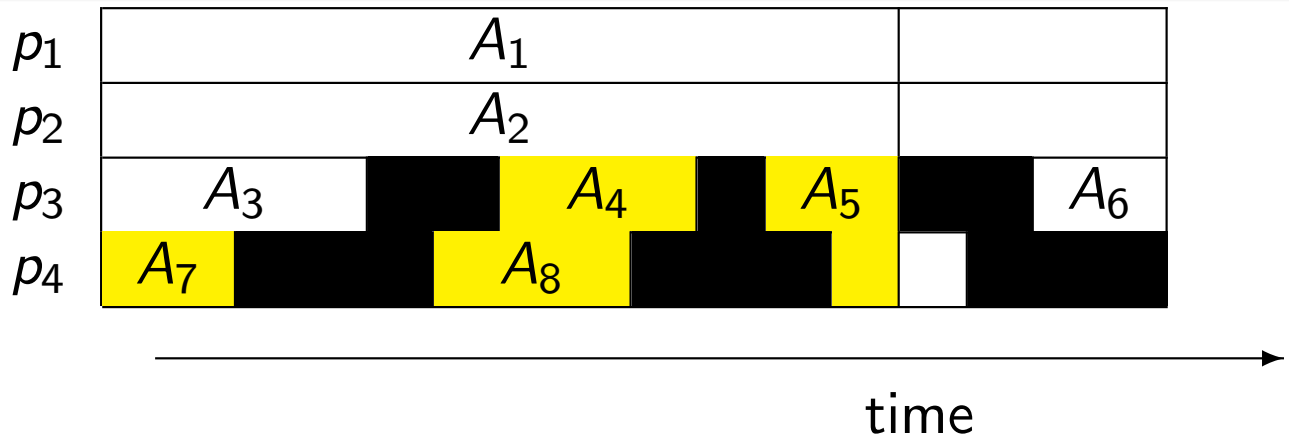

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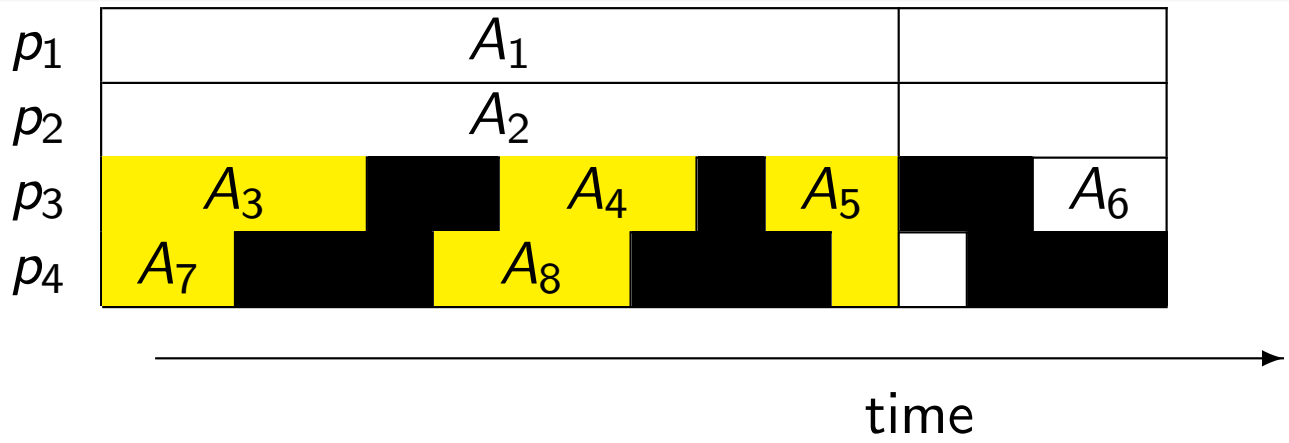

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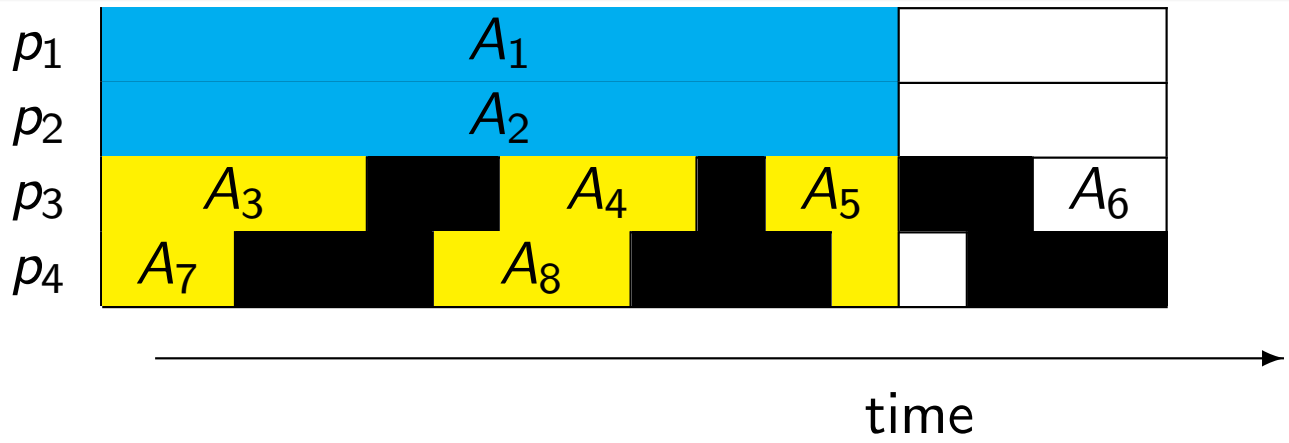

Filling Heuristics

Filling Availabilities

Each availability is considered sequentially:

First Fit Decreasing Schedule from the largest task to the shortest.

First Fit Increasing Schedule from the shortest task to the largest.

Filling Free Processors

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Dynamic Programming

Main steps

1. Sort elements by non-decreasing size.
2. Solve the related knapsack problem without the slack constraint.
3. Add the slack constraint and use the previous results.

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Summary

Scheme	Availabilities	Free processors
Greedy	FFD/FFI/DP	LPT/SPT
Dual approximation	all	all

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Settings

Traces

- Availability from SETI@home.
- Workload from Docking@home.
- GAPS: greedy strategy with FFD for the unavailabilities and LPT for the free processors.

Protocol

- 100 instances are generated from the trace (workload and availability).
- For each instances, a schedule is obtained.
- The makespan ratio is measured (baseline makespan over a lower bound).
- The schedule is disturbed 30 times (with each unavailability starting early such that every possible start dates are equiprobable) and the median stability is selected.

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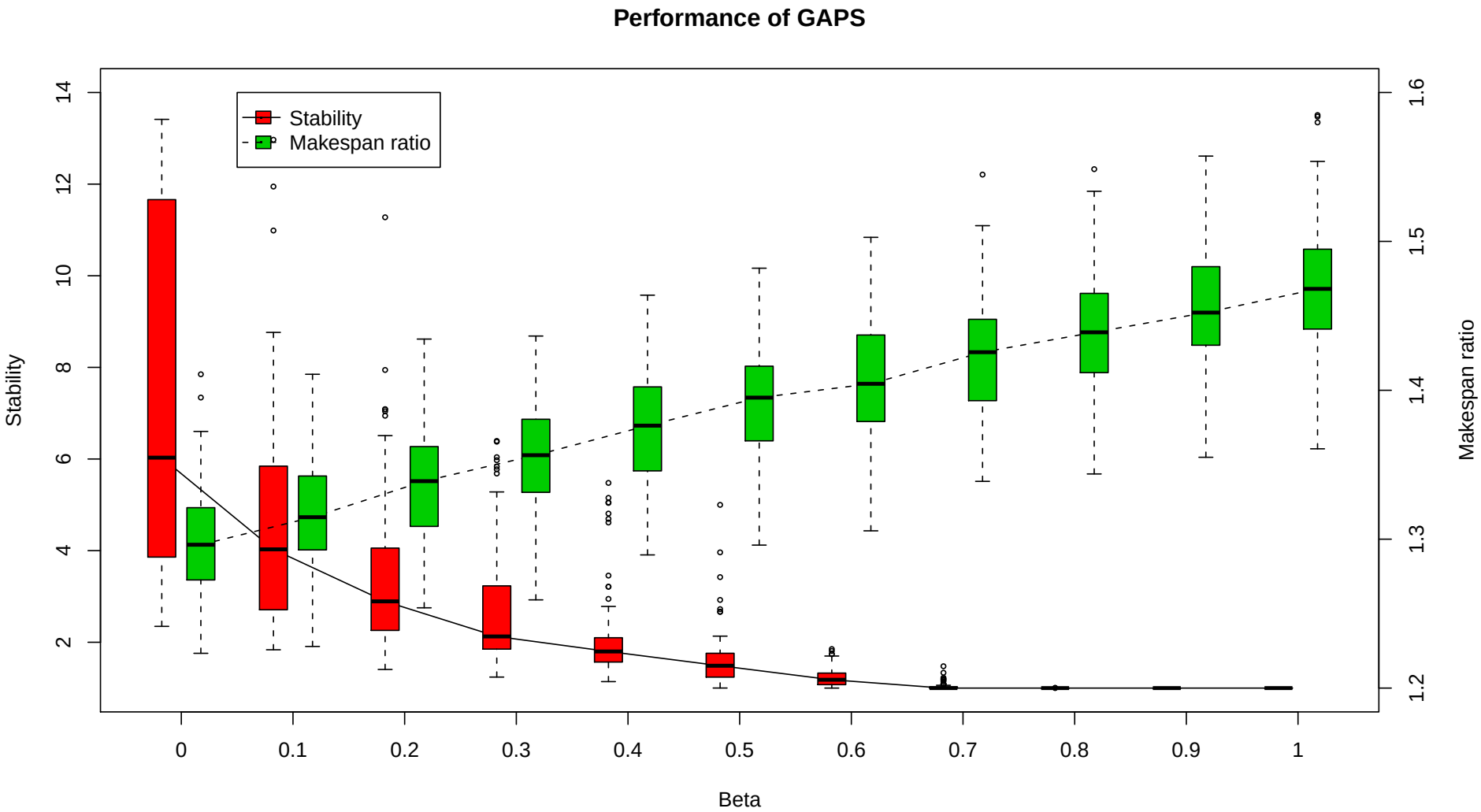

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- Propose a condition with bound on the stability.
- Several strategies (schemes and filling heuristics).
- Empirical validation of one approach.

Perspective

- Adapt a PTAS (from Hochbaum and Shmoys, 1988).
- Study hypotheses (worst case analysis, deterministic u_i^k).
- Compare with a checkpointing-based mechanism.
- Take into account fairness between multiple projects.

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- Adapt a PTAS (from Hochbaum and Shmoys, 1988).
- Study hypotheses (worst case analysis, deterministic u_i^k).
- Compare with a checkpointing-based mechanism.
- Take into account fairness between multiple projects.