

# Energy-aware mappings of series-parallel workflows onto chip multiprocessors

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Scheduling in Aussois

# Motivation and introduction

- Energy saving: crucial problem
- Chip MultiProcessor (CMP): present and future of the processor
- Streaming applications: ubiquitous in many domains, like image processing
- Most task graphs are Series-Parallel Graphs (SPGs)


# Outline of the talk

- 1 Framework
- 2 Complexity results
- 3 Heuristics
- 4 Simulations

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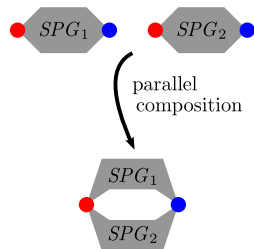
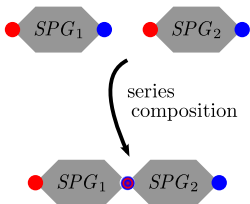
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# Series-parallel graph


- $G$  is a SPG if  $G$  is a composition of two SPGs
-  is a SPG
- Compositions

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- Compositions: series, parallel
- Label

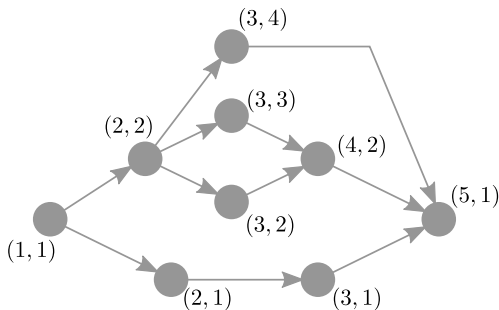
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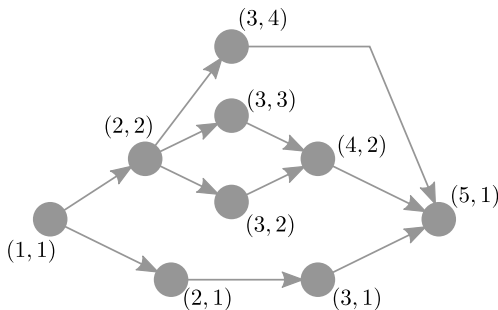
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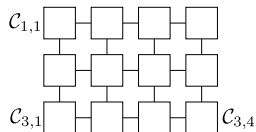
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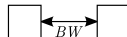
⇒ bounded elevation

# Target platform

- Cores  $\mathcal{C}_{u,v}$  on a  $p \times q$  grid

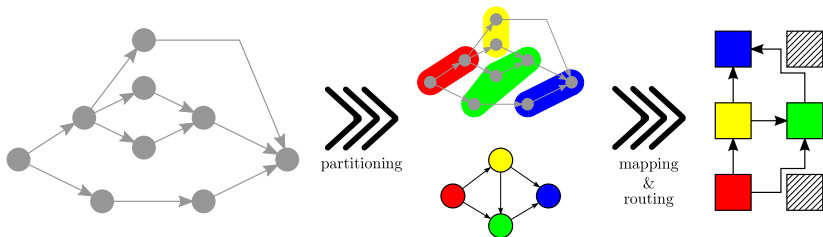


- Bidirectional links of bandwidth  $BW$



- $\mathcal{C}_{u,v}$  running at speed  $s_{u,v} \in \{s^{(1)}, \dots, s^{(M)}\}$

# Mapping strategy

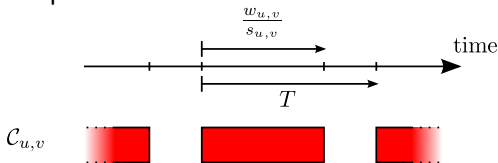


- Partitioning: if  $S_i$  and  $S_j$  mapped onto  $\mathcal{C}_{u,v}$  and  $S_i \rightarrow S_k \rightarrow S_j$ , then  $S_k$  mapped onto  $\mathcal{C}_{u,v}$
- Mapping:
  - One-to-one
  - Routing free, but single-path

$\Rightarrow w_{u,v}$  : work computed by  $\mathcal{C}_{u,v}$   
 $b_{(u,v) \rightarrow (u',v')}$  : communication from  $\mathcal{C}_{u,v}$  to  $\mathcal{C}_{u',v'}$

# Objective functions

- Given period  $T$
- Energy of computations



$$E^{(\text{comp})} = |\mathcal{A}| \times P_{\text{leak}}^{(\text{comp})} \times T + \sum_{C_{u,v} \in \mathcal{A}} \frac{w_{u,v}}{s_{u,v}} \times P_{s_{u,v}}^{(\text{comp})},$$

where  $\mathcal{A}$  is the set of active cores

- Energy of communications

$$E^{(\text{comm})} = P_{\text{leak}}^{(\text{comm})} \times T + \left( \sum_{u,v} \sum_{u',v'} b_{(u,v) \rightarrow (u',v')} \right) \times E^{(\text{bit})}$$

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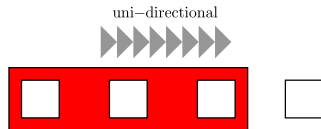
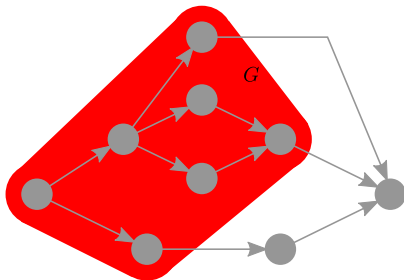
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# Uni-directional uni-line CMP

Polynomial with bounded elevation:  
dynamic programming algorithm

$$\mathcal{E}(G, k) = \min_{G' \subseteq G} \left( \mathcal{E}(G', k-1) + \mathcal{E}^{\text{cal}}(G \setminus G') \right),$$

- where
- $G'$  is admissible
  - outgoing communications of  $G'$  do not exceed  $BW$

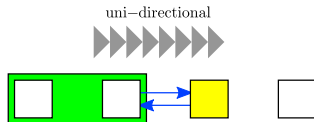
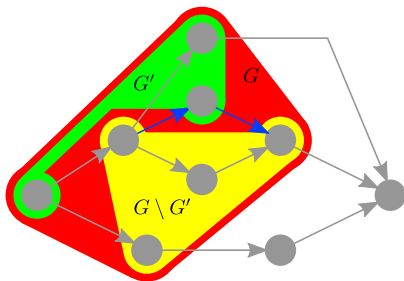


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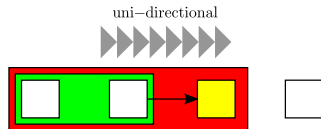
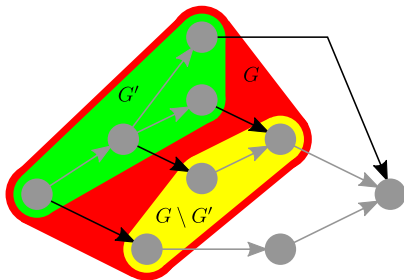


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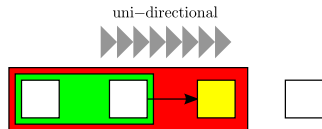
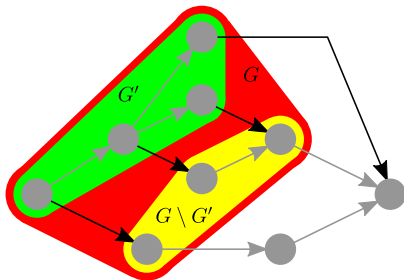


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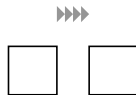
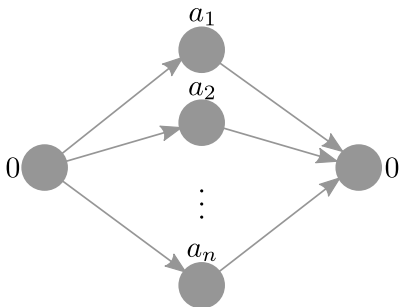
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**Polynomial**  
 $O(q \times n^{y_{\max}+1})$

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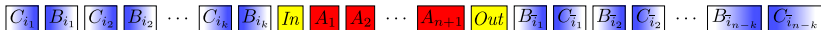
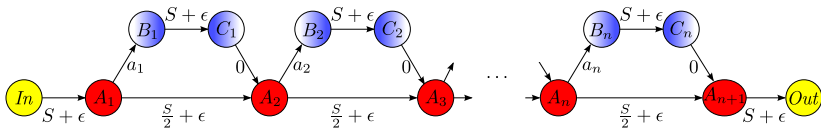
NP-complete with unbounded elevation:  
reduction from 2-PARTITION



Single speed  $\frac{\sum_{i=1}^n a_i}{2}$

# Bi-directional uni-line CMP

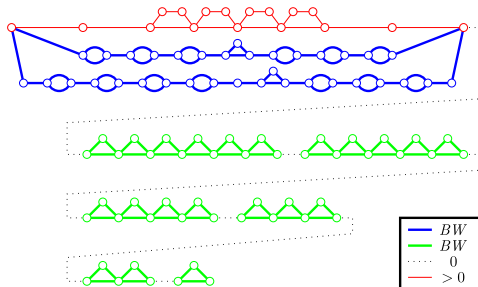
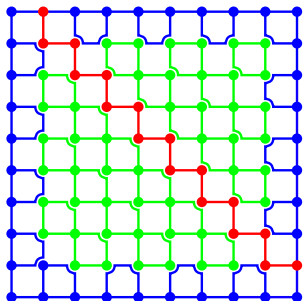
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$$BW = \frac{3S}{2} + \epsilon$$

# Bi-directional square CMP

NP-complete with bounded elevation:  
reutilization of previous proof



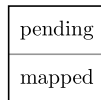
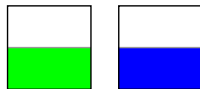
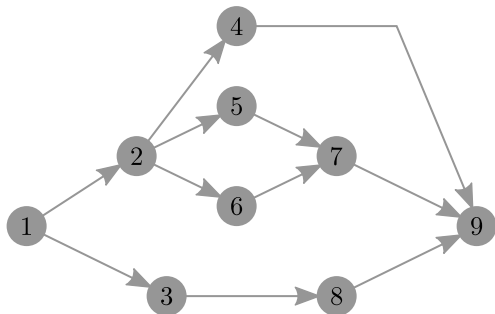
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# Heuristic summary

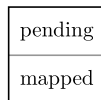
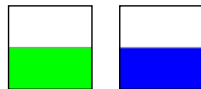
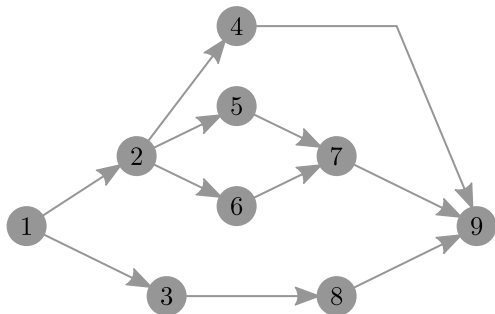
- Random heuristic: random speeds, random assignments, XY routing
- Greedy heuristic: starting from  $C_{1,1}$ , process as many nodes as possible, give following nodes to right and down cores; iterate on those cores
- 2D dynamic programming algorithm
- 1D heuristics (2D CMP configured as a snake):
  - Optimal solution on 1D uni-directional CMP
  - Previous 2D dynamic programming algorithm onto snake

# Greedy

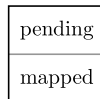
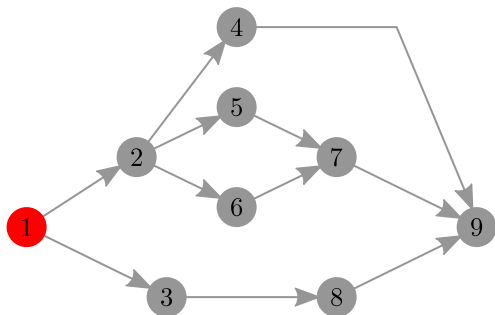




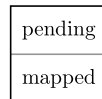
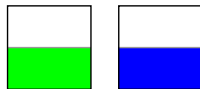
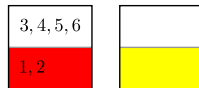
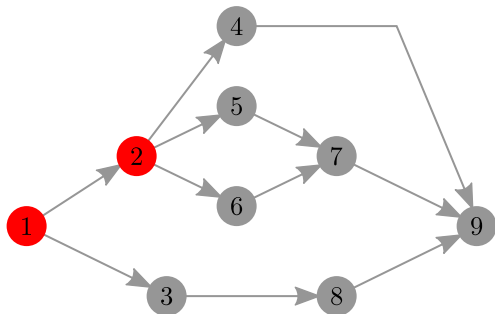
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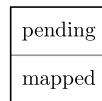
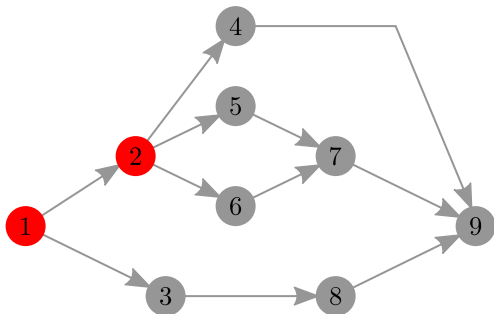
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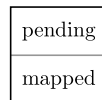
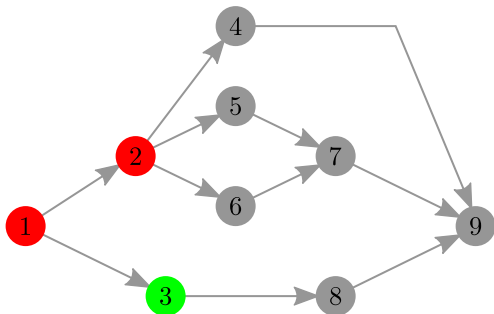
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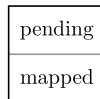
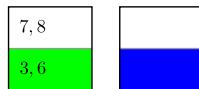
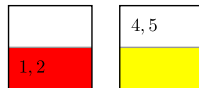
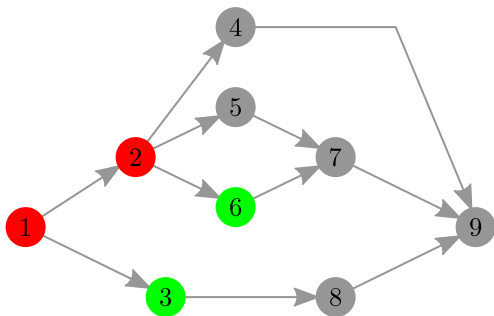
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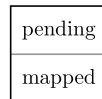
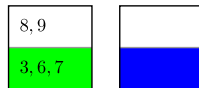
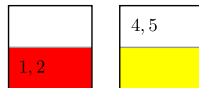
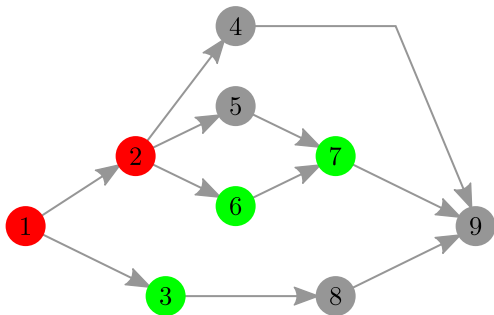
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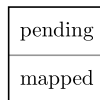
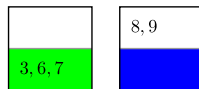
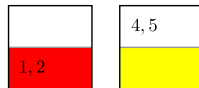
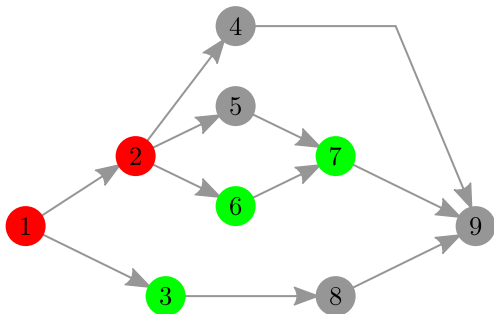
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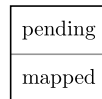
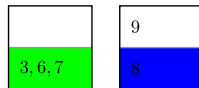
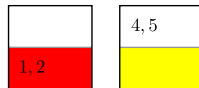
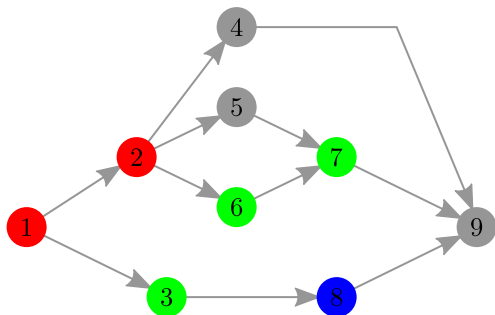


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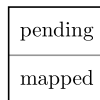
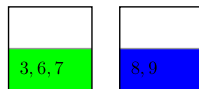
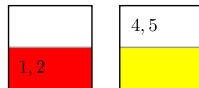
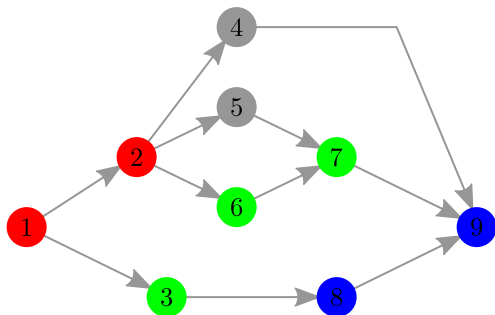




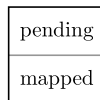
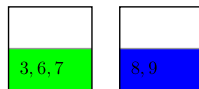
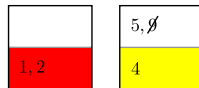
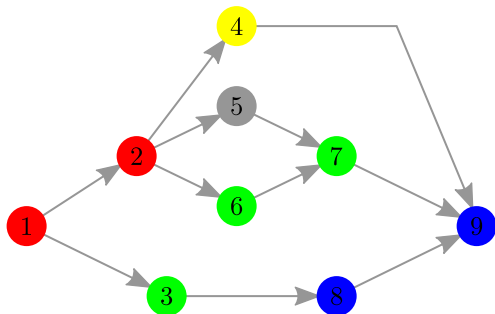
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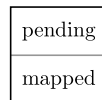
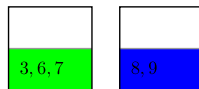
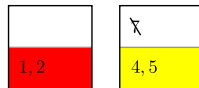
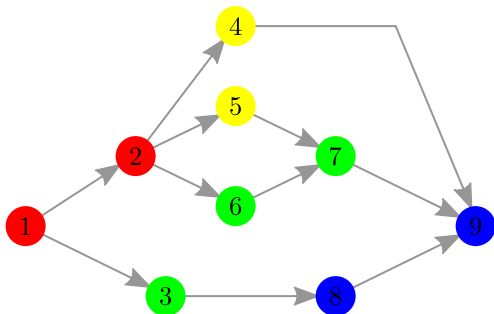
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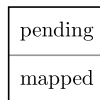
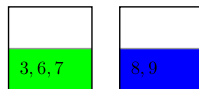
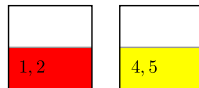
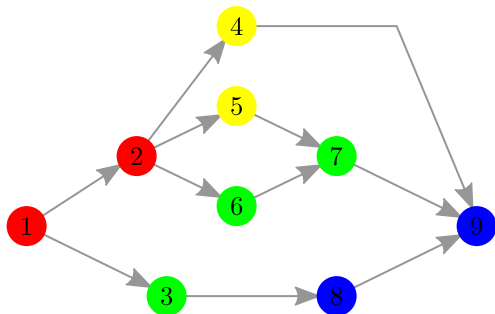
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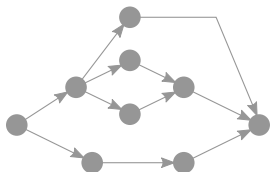
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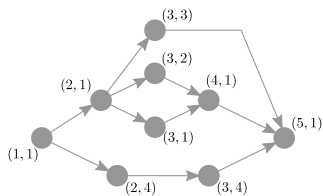
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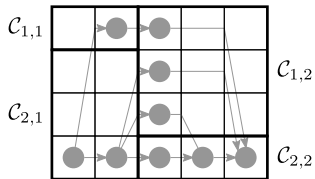
## DPA2D



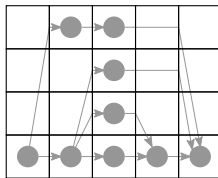
label



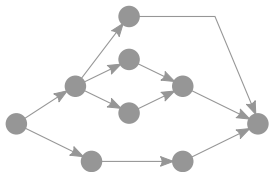
reorganize



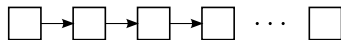
map



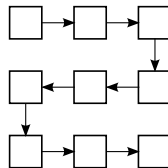
# DPA1D, DPA2D1D



map



reconfigure



# Outline of the talk

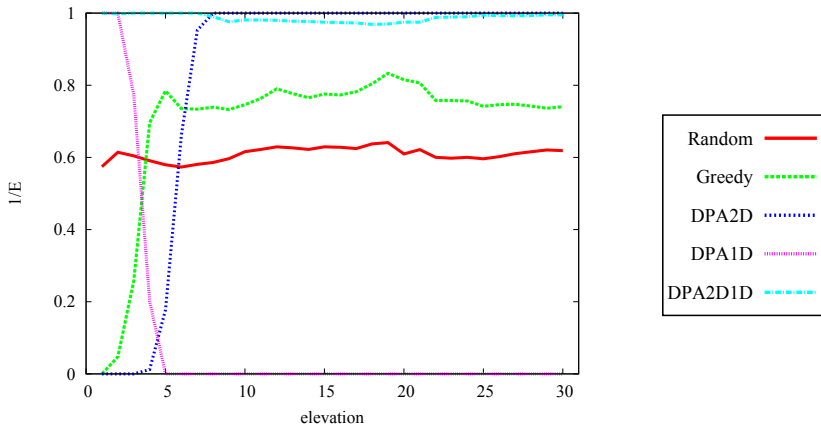
- 1 Framework
- 2 Complexity results
- 3 Heuristics
- 4 Simulations**



# Simulation settings

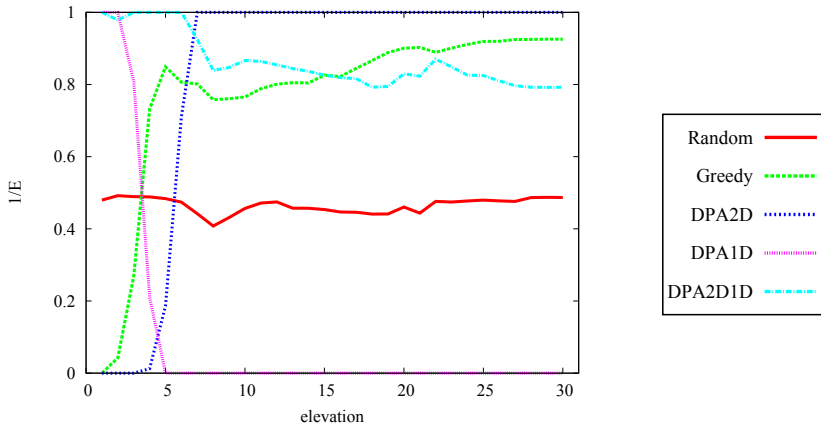
- Random SPGs
  - Averaged over 100 applications
  - 150 nodes
  - Elevation: from 1 to 30
- Real-life SPGs
  - 12 workflows
  - From 8 to 120 nodes
  - Elevation: from 1 to 17
- Varying communication-to-computation ratio

# Random SPGs; computation intensive



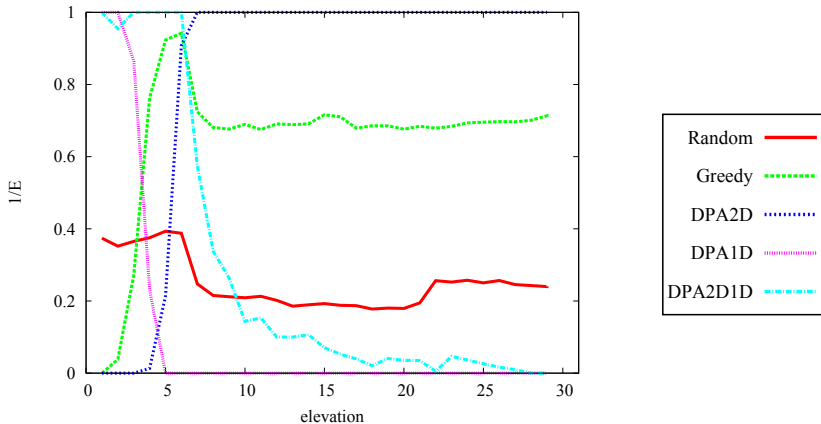
- **DPA1D** best then very rapidly useless
- **DPA2D1D** always close to best
- **DPA2D** bad then rapidly best
- **Greedy** intermediate

# Random SPGs; balanced



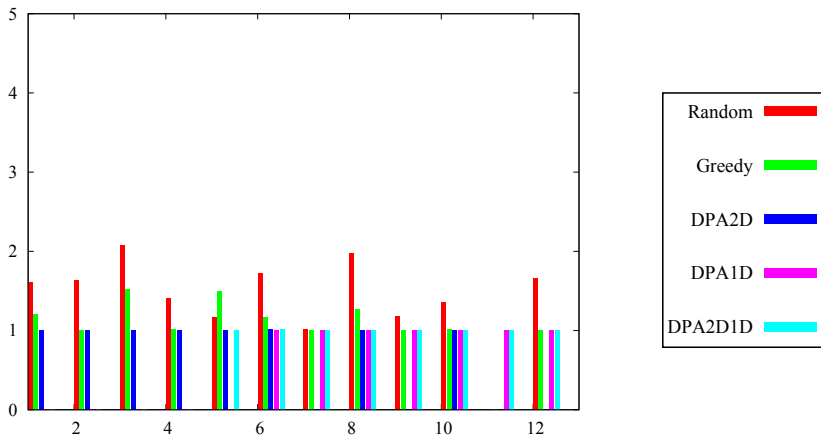
- DPA1D, DPA2D and Greedy better compared to Random
- DPA2D1D crumbles

# Random SPGs; communication intensive

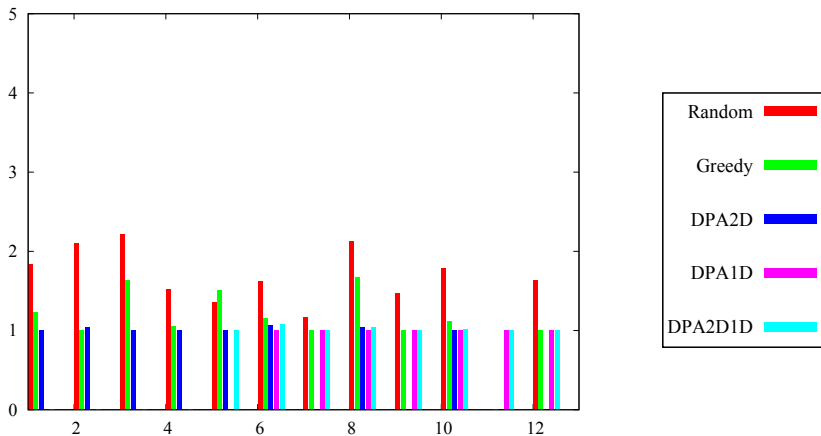


- DPA1D, DPA2D and Greedy better compared to Random
- DPA2D1D crumbles

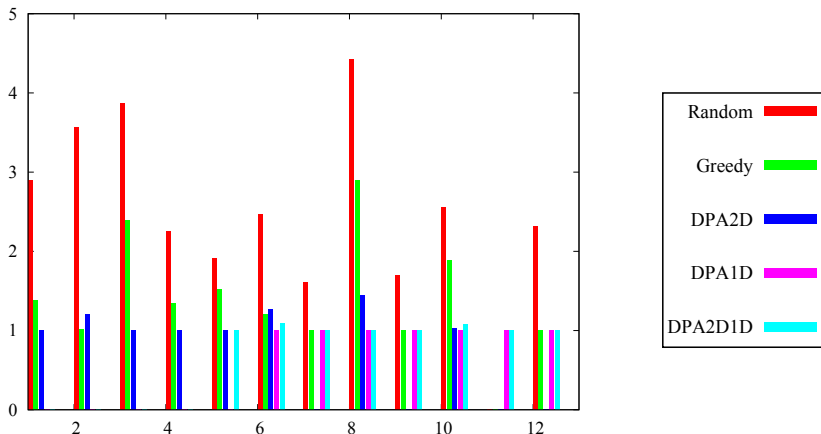
# StreamIT; computation intensive



# StreamIT; balanced



# StreamIT; communication intensive



# Conclusion

- Exhaustive complexity study
- Efficient heuristics for all kinds of SPGs
- Simulations on both randomly generated and real-life SPGs
- Future work: simplified ILP to assess the absolute performance of the heuristics, more accurate power consumption model for communications, multi-path routing



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