Advances in Optimal Task Scheduling

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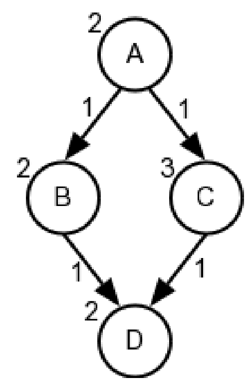
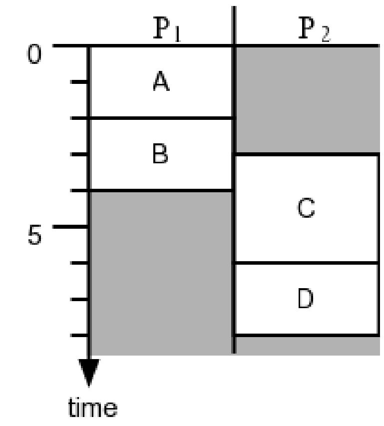
Introduction

Scheduling problem

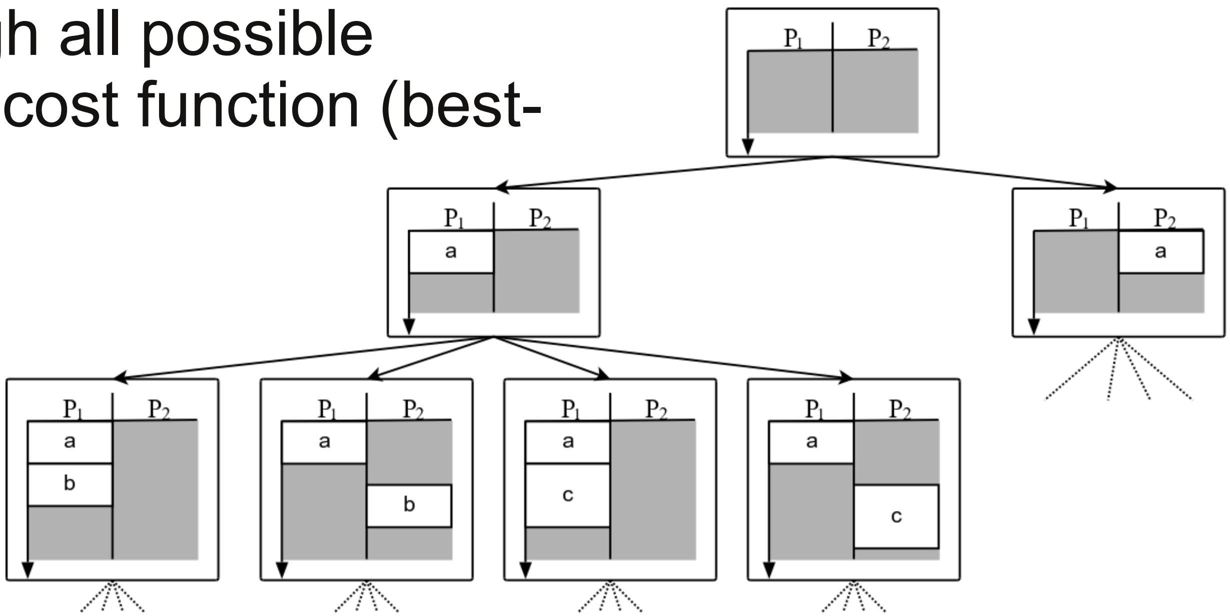
task graph (DAG) with computation and communication costs on *p* processors, minimising makespan → NP-hard





Optimal solution with A*

Extensive search through all possible schedules → guided by cost function (best-first-search)



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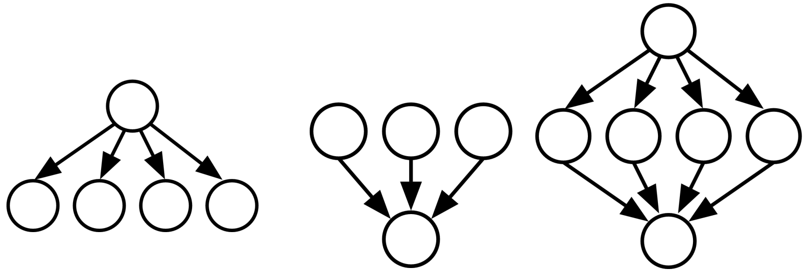
Introduction

Optimal solution with A*

- Good for small graphs (<30tasks), few processors (<8)

Paradox

- Difficult for "simple" scheduling problems! (i.e. task number limit is low)
  - Independent tasks (mostly)
  - Forks or joins
  - Fork-joins



Contribution

- **Novel concepts and techniques to address this and more: reduce search space, accelerate search**

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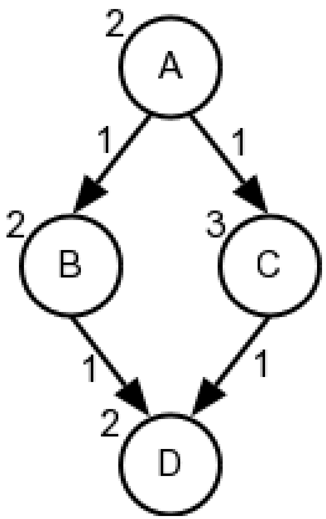
Outline

- Define scheduling problem
- State space formulation and A*
- Pruning techniques
- Some observations
- Equivalent schedule
- Fix order ready list
- First evaluation results
- Conclusions

Scheduling problem: $P|prec,c_{ij}|C_{max}$

Task graph and *p* (identical) processors

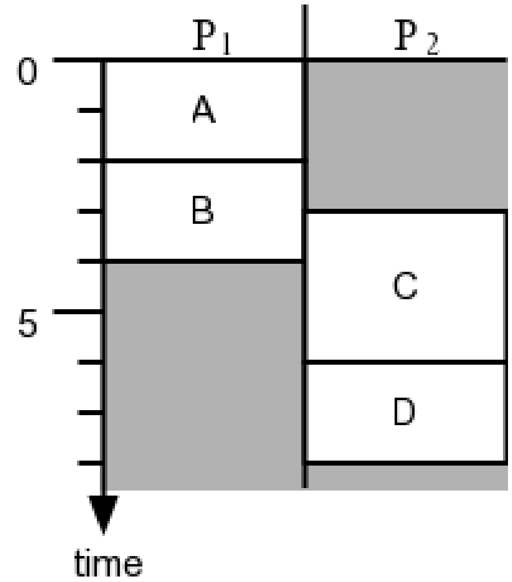


DAG: tasks (*n*) and edges (*e*) with weights (computation cost *w(n)* and communication cost *c(e)*)

- start time: $t_s(n)$; finish time: $t_f(n)=t_s(n)+w(n)$
- processor assignment: *proc(n)*

Objective: minimise makespan

Constraints:



- Processor constraint:

  $proc(n_i)=proc(n_j) \Rightarrow t_s(n_i) \geq t_f(n_j)$ or $t_s(n_j) \geq t_f(n_i)$

- Precedence constraint:

  for all edges e_{ji} of E (from n_j to n_i)

  $t_s(n_i) \geq t_f(n_j) + c(e_{ji})$ if $proc(n_i) \neq proc(n_j)$

  $t_s(n_i) \geq t_f(n_j)$ if $proc(n_i)=proc(n_j)$

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Solution space

Solution

processor allocation and task order

- Allocation problem *plus*
- Permutation problem
- Each problem is NP-hard

Finding optimal solution

- Trying all possible processor allocations with all possible permutations (*naïve*)

$$p^V \times V!$$

- Example: 10 tasks, 3 processors: 219 billion possibilities!

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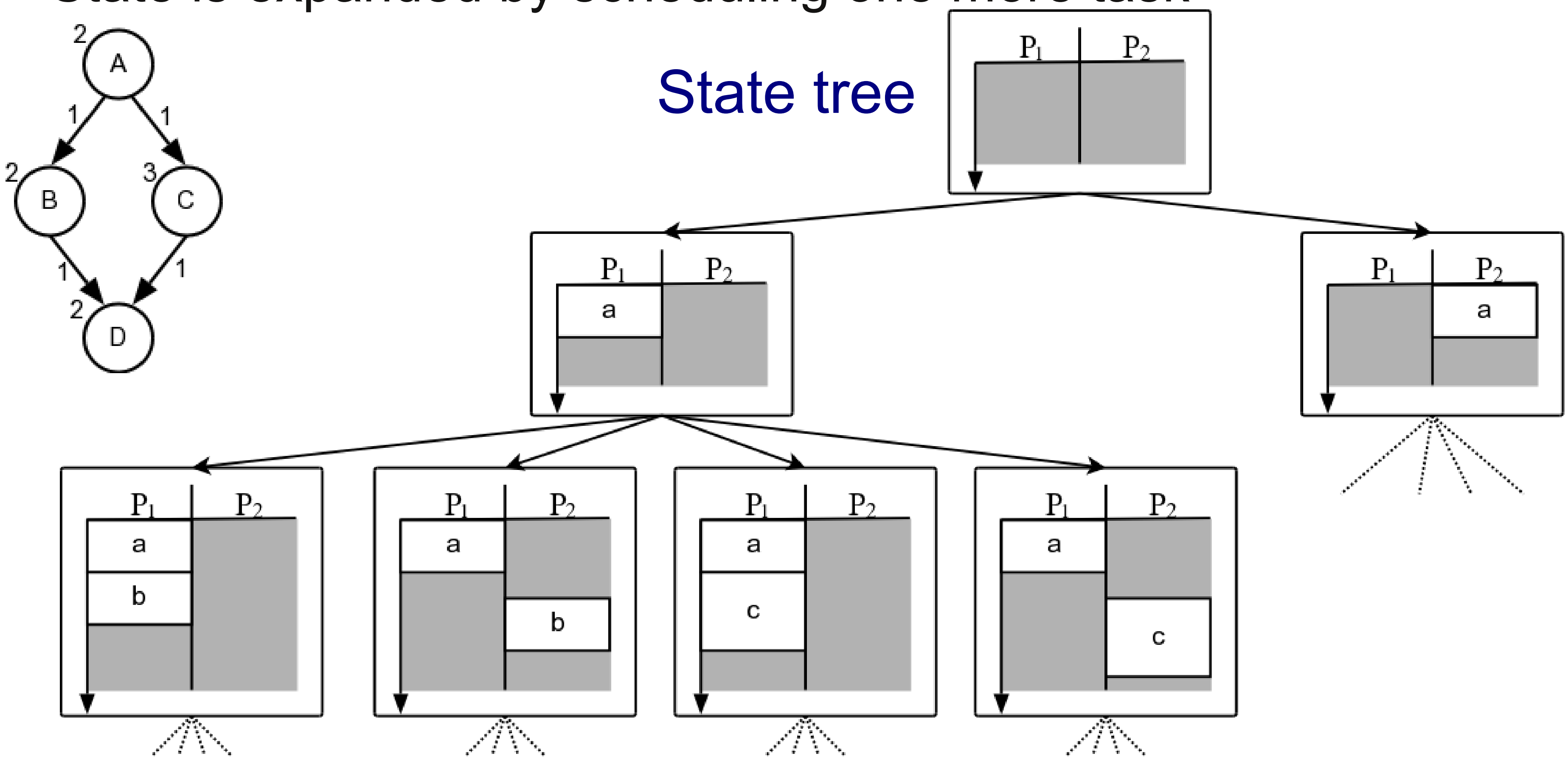
A* for task scheduling

A*: **best first** search, guided by cost function

- State (s) => partial schedule
- Cost function *f(s)* => **underestimate** of schedule length
- State is expanded by scheduling one more task



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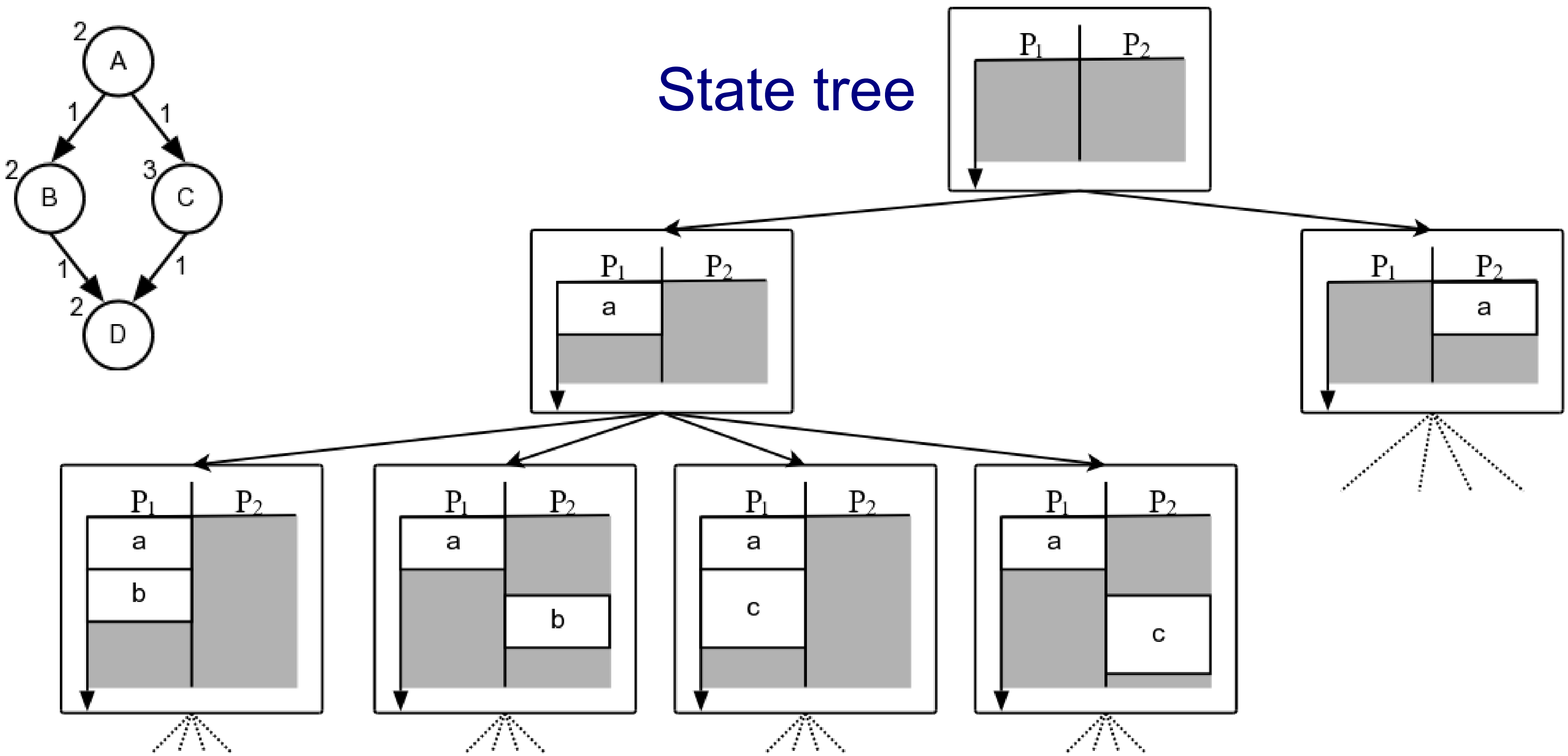
A* for task scheduling

At each step, expand most promising state with best cost *f(s)*

- All free nodes, *free(s)*, scheduled on all *p* processors

=> *free(s)* x *p* new states created, costs calculated



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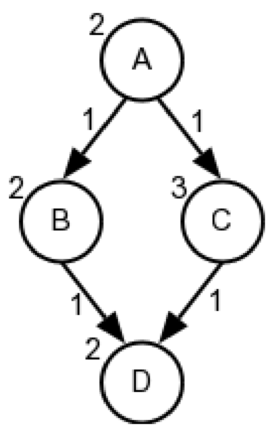
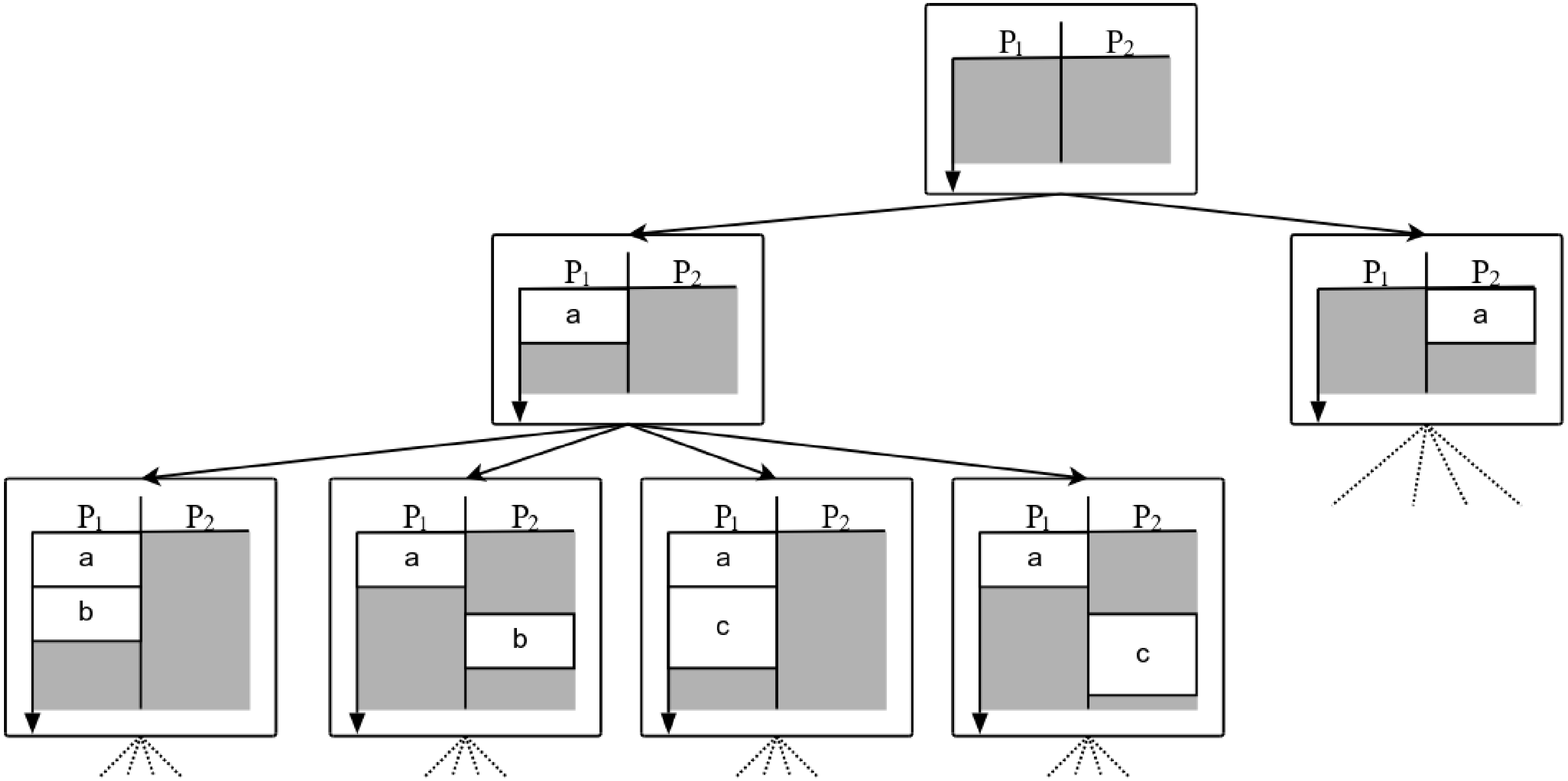
A* algorithm

- Priority queue OPEN for states to be expanded (ordered by *f(s)*)
- Recording already expanded states: duplication detection

=> very, very memory hungry

- Cost function *f(s):* underestimate of minimum cost *f*(s)* of final solution – the tighter the better → less states





Pruning

- State space is tree
- Try to prune entire branches early to limit search space

So far

- Duplication detection
- Processor normalisation
- Identical tasks

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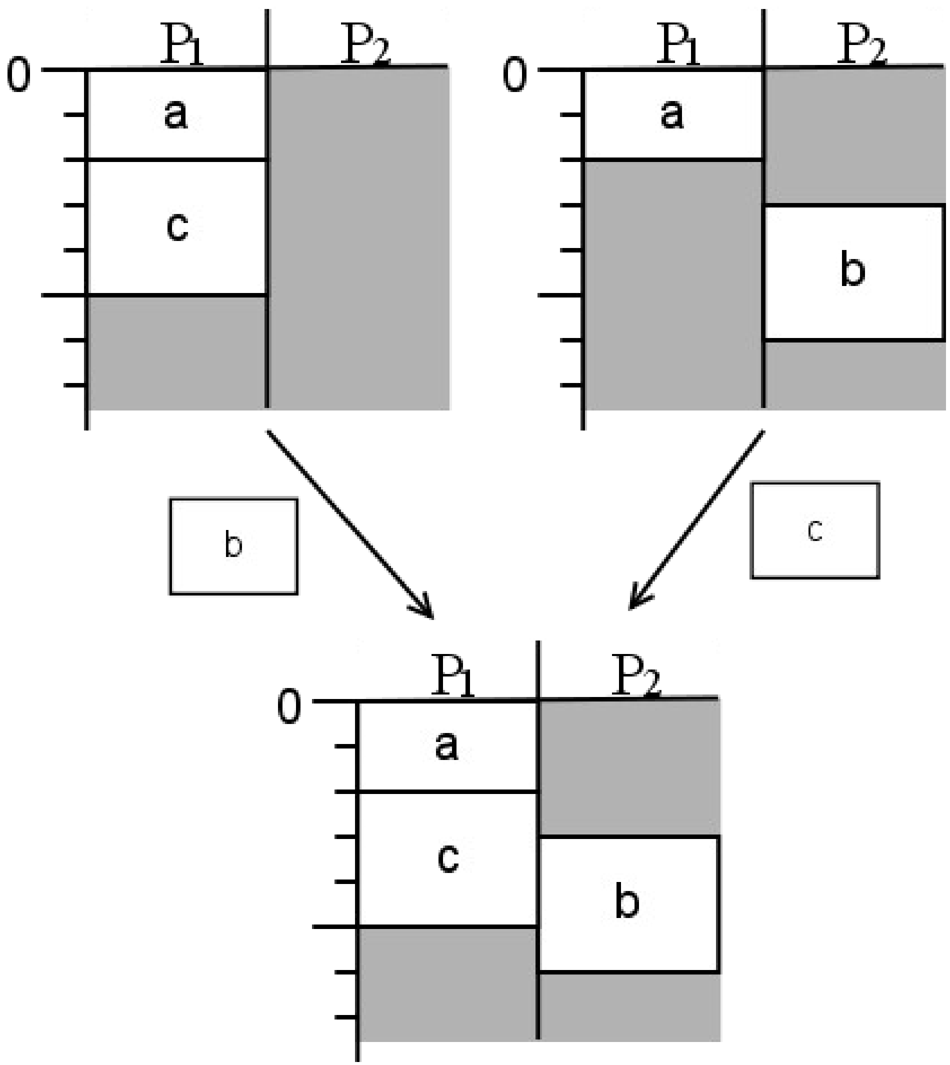
Pruning – duplicated states



- Same schedule created in different ways

=> duplicates

- Duplicates are detected and discarded

Global ordering

→ local ordering

- Permutations per processor with less tasks

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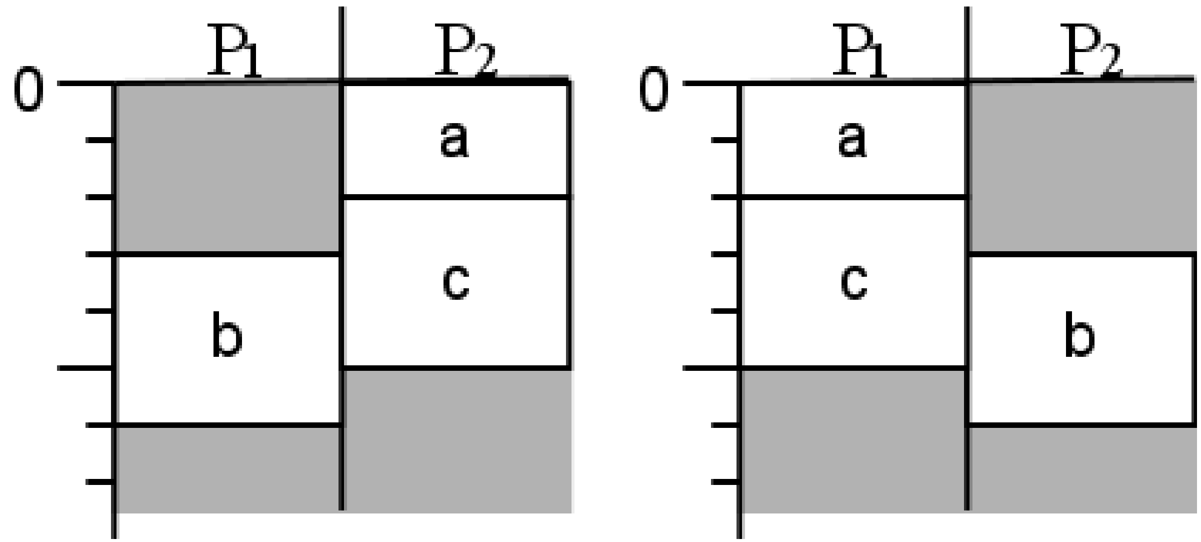
Pruning – processor normalisation



- Equivalent schedules for homogeneous processors

- Normalise schedule/state
    - e.g. processor of task a is always P_1
- Pruned as duplicates
- Processor allocation problem → subset problem

New pruning

- Address task ordering

Observations in the next slides

- Independent tasks
- Fork, Joins
- Mixed graphs

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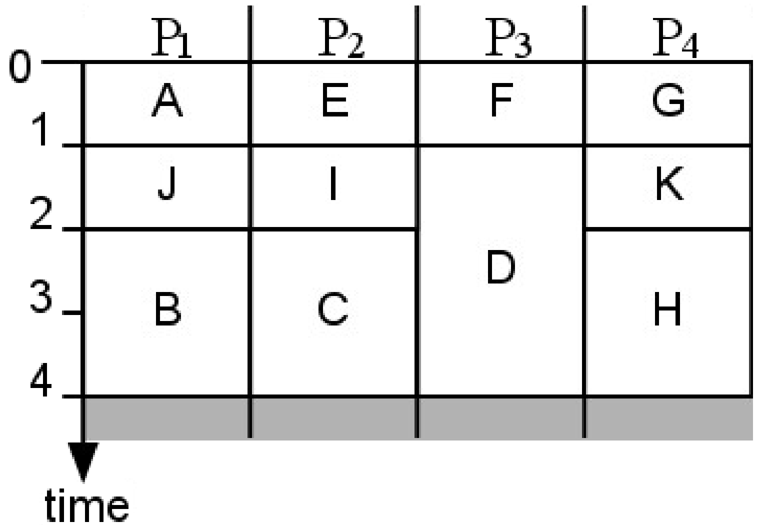
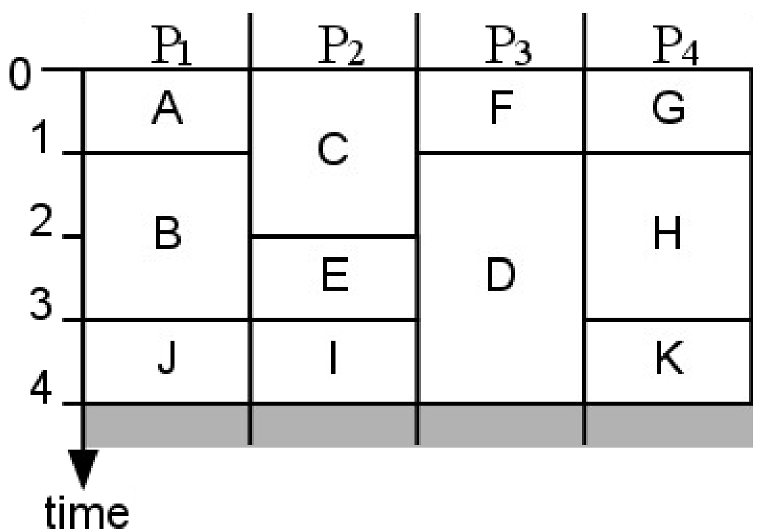
Independent tasks





Two schedules:

- Same length
- Same processor allocation
- Different task order

=> Order does not matter

=> Only allocation problem!

(with processor normalisation: only subset problem)

- Number of states reduced by *V!*

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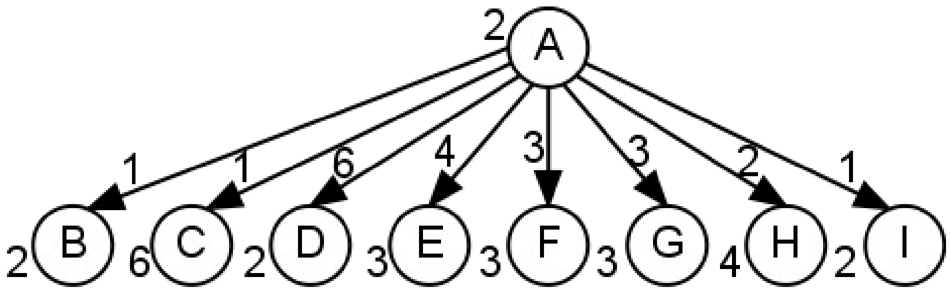
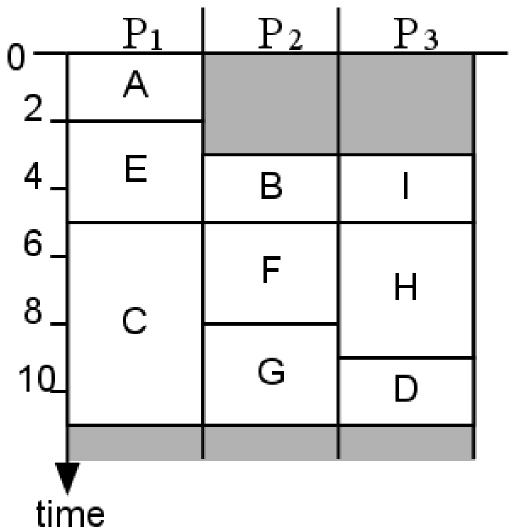
Fork and joins





Fork/Join graphs

- Task order does matter

BUT

- Optimal processor allocation enough
- Ordering by non-decreasing in-edge weight (join: by non-increasing out-edge weight)

=> still only allocation problem (strong NP-hard)

Using observations in A*

- OK, for some graphs order does not matter/can be computed
  - Independent, fork, join
- But how does that help us with general graphs?

=> First, look at mixed graphs

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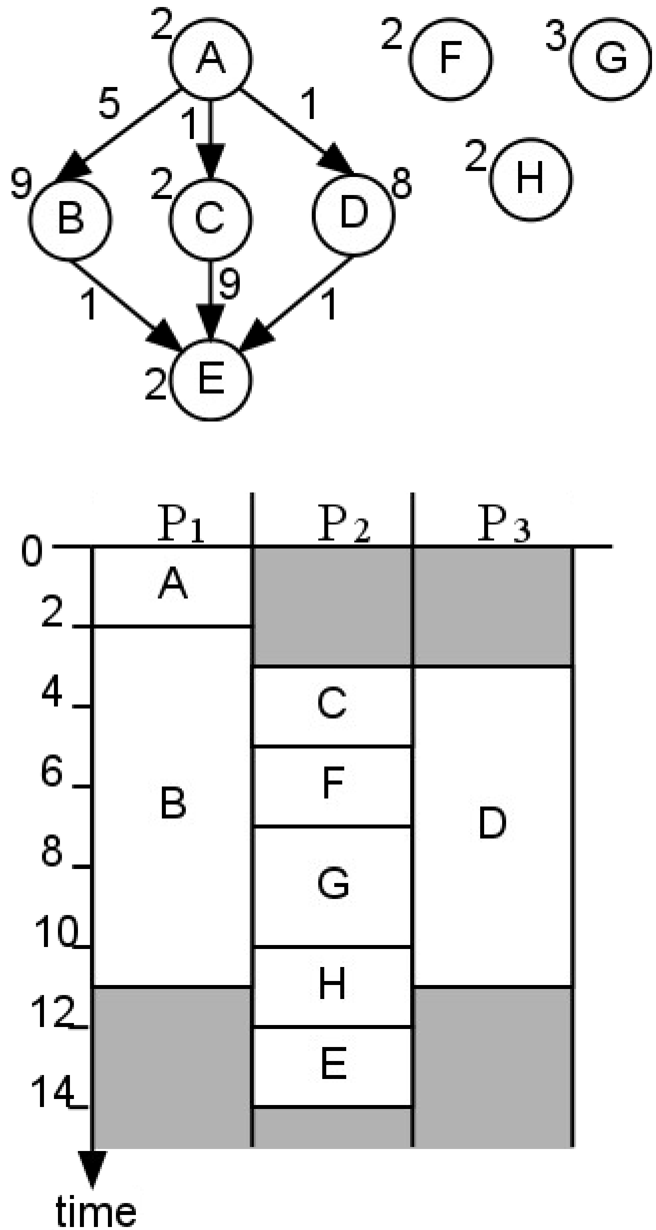
Independent tasks



- Independent tasks F,G,H can be in any order on P_2 (between C and E)

Tasks must be

- Independent
- Consecutive
- On same processor

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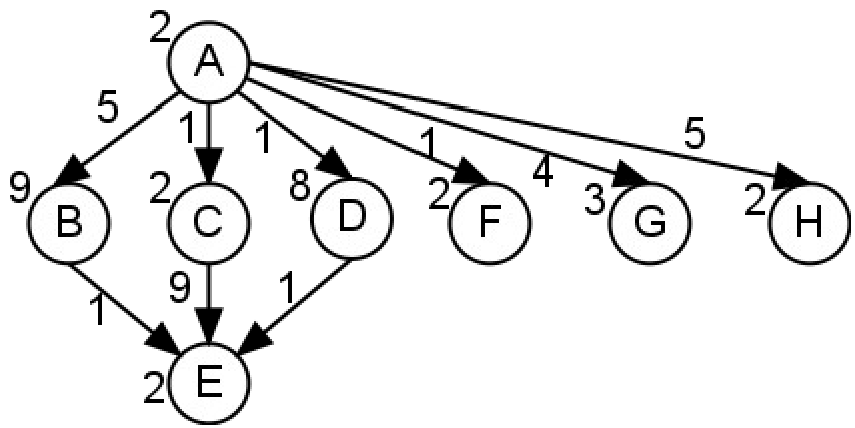
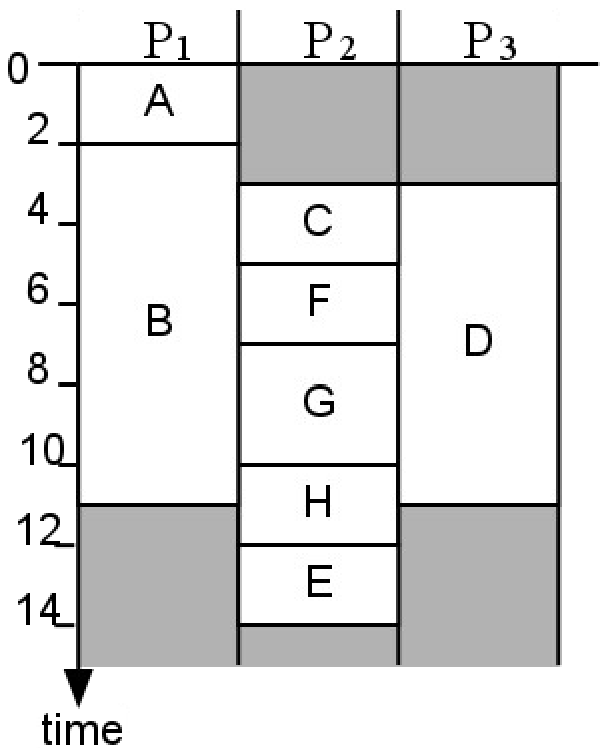
Forks (or sink tasks)





- Tasks F,G,H *might* be reorder on P_2 (between C and E)
  - Depends on data arrival time

Rules

- Last task finishes at the same time/earlier as originally
- Consecutive tasks
- No out edges → no consequence for rest of schedule

Using observations in A*

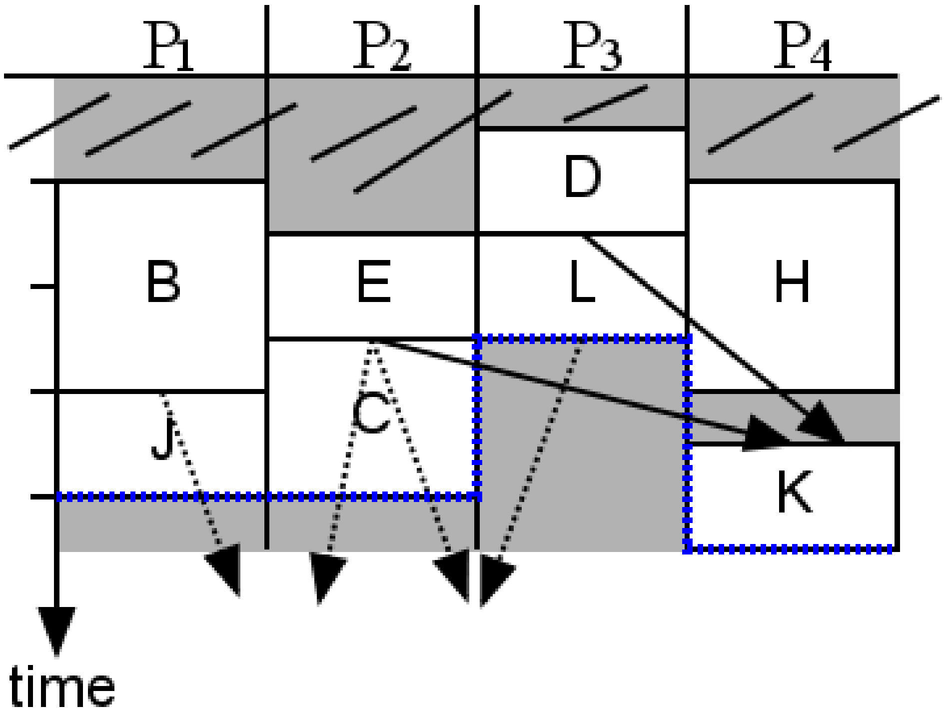
OK, also meaningful for mixed graphs

- But how to use these observations in A*?

=> Scheduling horizon and equivalent schedules concept

Equivalent schedule pruning

Schedule horizon



Partial schedules of same tasks

Only relevant:

- Finish time of processors
- Potential start time of successors

=> Schedule Horizon

- Some partial schedules dominate others (better in every aspect) → discard

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Using schedule horizon

- How to use this in A*?

Remember: To create a new state, one more task is scheduled at the end on one processor

Approach

- Try to “bubble up” the task to bring into certain order (index order/alphabetical order)
    - Accept if horizon does not get worse

=> normalisation

=> duplicate detection

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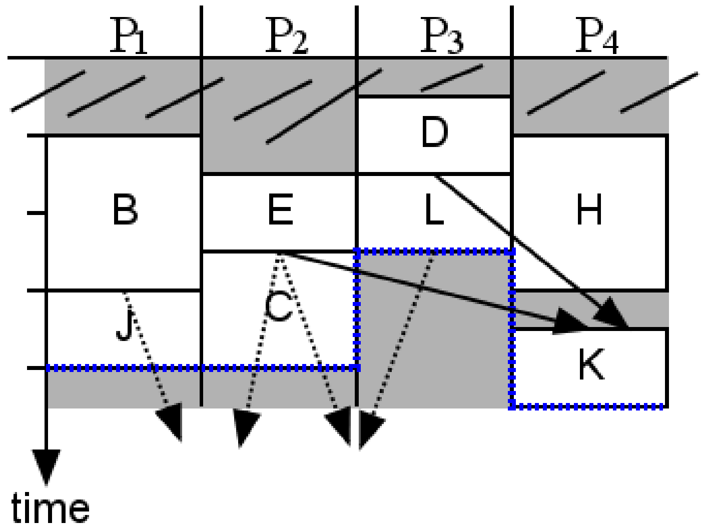
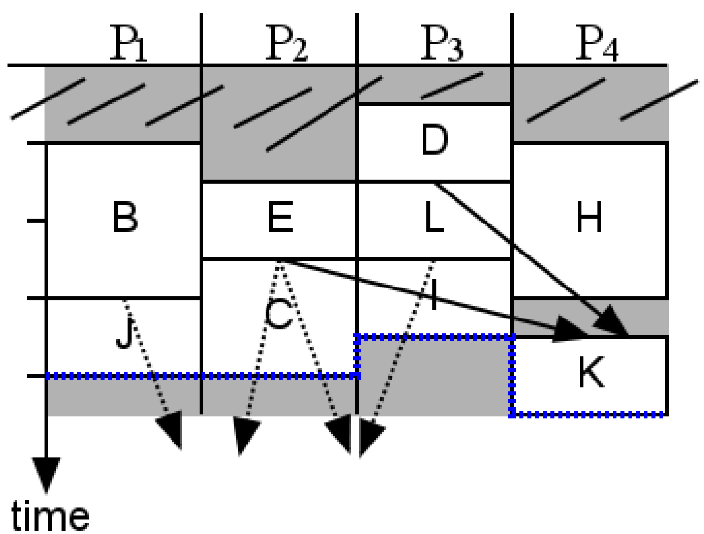
Example using schedule horizon





Top:

partial schedule

Bottom:

- Schedule I on P_3 at the end (after L)

=> new horizon

(here new finish time for P_3)

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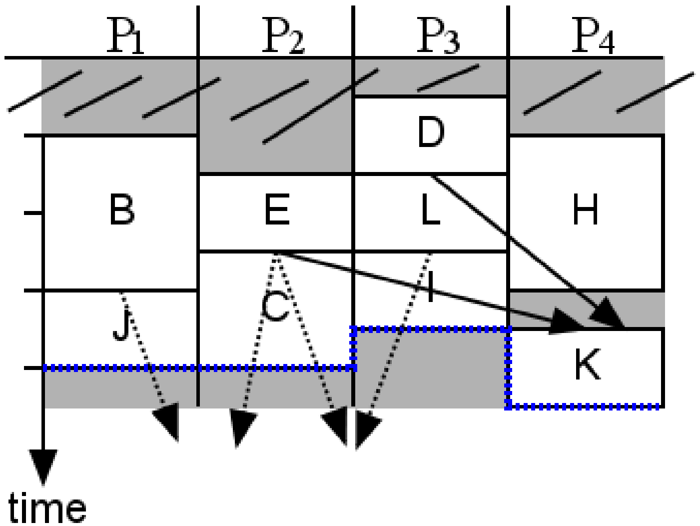
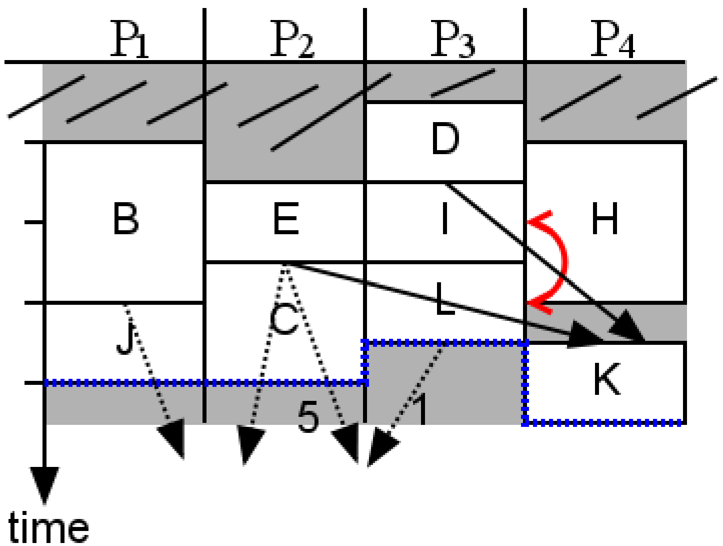
Example using schedule horizon





Try to bubble up I

- Swap of I and L can be made → alphabetical order
- Finish time of P_3 the same
- Communication from L now later, but communication from E dominates

=> Horizon is the same

- Task I could go higher, but already in alphabetical order (Also communication from D might arrive too late on P_4)

Equivalent scheduling pruning

Procedure

- Create new state by scheduling free task *n* on processor *P*
- Try to “bubble up” task *n* until
  - In index order of tasks *or*
  - Schedule horizon gets worse

=> schedules are normalised => duplicates are detected

Index order necessary, otherwise different schedules still possible

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Equivalent scheduling pruning

Testing for outgoing edges of moved tasks not trivial:

- For each moved task, check that child tasks have the same earliest start time on every processor as before
- Some parents of these tasks might not have been scheduled yet (use best case for unknowns)

Discard instead of normalise

- Normalisation and duplication detection problematic for f-value
  - Idle time might be reduced → better estimate
  - Estimate change not allowed (at least problematic) in A*
  - Better discard state immediately

Discard instead of normalise

- Normalisation and duplication detection problematic for f-value
  - Idle time might be reduced → better estimate
  - Estimate change not allowed (at least problematic) in A*
  - Better discard state immediately

- Remember, all processor allocations and all task orders are created
- Can discard state as soon as we can “bubble up” by at least one position
  (because this state will be created anyway)

=> much better than normalisation (less computing)

Fixed order free list

Fixed order free list

- Discarding equivalent schedules is great
- Even better would be not to create them!

We know:

- Certain tasks can be scheduled in certain orders
  - Independent: any order
  - Fork: by non-decreasing in-edge weight

Using this in A*

- When free task list only contains such tasks, fix order
  → effectively reduce list to one element in each step
  → reduces branching factor in tree
  - Safe: without out-edges there will be no new task coming to free list

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Fixed order free list

When free task list only contains such tasks, fix order

Can be extended to

- Join: order tasks by non-increasing out-edge weight
  - Safe, for a single join, as out-edge of all tasks goes to the same task
- Fork-join: order by non-decreasing in-edge weight
  - If this can also be an order by non-increasing out-edge weight → fix order
  - Safe, all out-edges go to same task
- Mixtures of independent, fork, join, fork-join also work
  - Generalised by treating missing edges as zero weight

Combining pruning

- Equivalent schedule pruning
- Fixed free list order

Problem

- Using combination of techniques can destroy assumptions
  - e.g. that all permutations are created

Solution

- When fixing the list order, equivalent schedule pruning is disable (not necessary anyway)

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First experimental results

Graph type	#tasks	processors	States before	States with new pruning
Independent	30	8	>370m	37m
Fork	21	2	>19m	74
Fork	12	2	3.5m	36
Join	16	2	>200m	18358
Join	16	4	>>200m	9.3m
Out-tree	18	2	>17m	488224
In-tree	21	2	-	>180m
In-tree	14	2	>200m	115m
Random (density 2)	30	2	106	28m
Stencil	24	2	15.9m	2.3m
Stencil	24	3	163m	29m

- All random weights, CCR 1.0

=> no node equivalence pruning

Conclusions

New pruning techniques, motivated by A*'s promise, but bad results with simple graphs

- Equivalent schedule pruning
- Fixed order free list

Dramatically improves results for

  - Independent, fork, join
  - Also in/out trees, others

- Goal: do not generate duplicates → get rid of duplication detection
- At the moment: implement memory bounded A*, e.g. SAM*