A Robust AFPTAS for Online Bin Packing with Polynomial Migration

Klaus Jansen   Kim-Manuel Klein

June 30, 2012

Online Bin Packing

Given at time $t \in \mathbb{N}$ an instance I_t of items $I_t = \{i_1, \ldots i_t\}$ and a function $s : I_t \rightarrow [0, 1]$.

Find for each $t \in \mathbb{N}$ a function $B_t : \{i_1, \ldots, i_t\} \rightarrow \mathbb{N}^+$, such that $\sum_{i:B_t(i)=j} s(i) \leq 1$ for all j. Minimize $\max_i \{B_t(i)\}$.

Competitive Ratio of Online Bin Packing

Best known algorithm: Ratio of 1.58889
(Steven S. Seiden. On the online bin packing problem)

Best known lower bound: Ratio of 1.5401
(Andre van Vliet. An improved lower bound for on-line bin packing algorithms)

We need a new model!
Goal: Approximation guarantee $\max_i B_t(i) \leq (1+\epsilon)OPT + f(\frac{1}{\epsilon})$ and bounded migration.

Migration

Migration Factor between B_t and B_{t+1}:

$$\frac{1}{s(i_{t+1})} \sum_{j \leq t : B_t(i_j) \neq B_{t+1}(i_j)} s(i_j)$$

An algorithm is *robust* if the migration factor is bounded by a function $f(\frac{1}{\epsilon})$.
Peter Sanders, Naveen Sivadasan and Martin Skutella. Online scheduling with bounded migration.

Robust Bin Packing

Leah Epstein and Asaf Levin: "A robust APTAS for the classical bin packing problem"

Running time: $\log(t)2^{2^{\mathcal{O}(1/\epsilon)}}$ and migration factor $2^{\mathcal{O}(1/\epsilon)}$

LP-Formulation

Let I be an instance of bin packing with m different item sizes $s_1, \ldots, s_m$. Suppose that for each item $i_k \in I$ there is a size s_j with $s(i_k) = s_j$. A configuration C_i is a multiset of sizes $\{a(C_i, 1) : s_1, a(C_i, 2) : s_2, \ldots a(C_i, m) : s_m\}$ with $\sum_{1 \leq j \leq m} a(C_i, j) s_j \leq 1$, where $a(C_i, j)$ denotes how often size s_j appears in configuration C_i.

$$\min \|x\|_1$$

$$\sum_{C_i \in C} x_i a(C_i, j) \geq b_j \qquad \forall 1 \leq j \leq m$$

$$x_i \geq 0 \qquad \forall 1 \leq j \leq n$$

Sensitivity Analysis

Problem: Let x' be a solution of $\min \{\|x\|_1 \,|Ax \geq b', x \geq 0\}$. Find a solution x'' of $\min \{\|x\|_1 \,|Ax \geq b'', x \geq 0\}$ such that $\|x'' - x'\|_1$ is small.

Theorem of Cook et al.: There exists a x'' satisfying the LP and $\|x'' - x'\|_\infty \leq n\Delta \|b'' - b'\|_\infty$

Our Results:

Running time $\mathcal{O}(\log(t)\frac{1}{\epsilon^9})$ and migration factor $\mathcal{O}(1/\epsilon^4)$

Theorem

Consider the LP $\min\{\|x\|_1 \,|Ax \geq b, x \geq 0\}$ *and an approximate solution* x' *with* $\|x'\|_1 = (1+\delta)OPT$ *for some* $\delta > 0$*. There exists a solution* x'' *of the LP having value of at most* $\|x''\|_1 \leq (1+\delta)OPT - \alpha$ *and* $\|x' - x''\|_1 \leq (2/\delta + 2)\alpha$*.*

Improve packing:

Let B_t be a packing of instance I_t with $\max_i B_t(i) \leq (1+\epsilon)OPT$.
Find a packing B'_t with $\max_i B'_t(i) \leq (1+\epsilon)OPT - \alpha$.

We prove feasibility of the following LP 1.

$$\begin{aligned} Ax &\geq b \\ x &\geq 0 \\ x &\leq x' + \frac{\alpha(1/\delta + 1)}{\|x'\|_1} x^{OPT} \\ x &\geq x' - \frac{\alpha(1/\delta + 1)}{\|x'\|_1} x' \\ \sum x_i &\leq (1+\delta)OPT - \alpha \end{aligned} \tag{LP 1}$$

Algorithm

Let x' be a LP solution with $\|x'\| \leq (1+\delta)OPT$

- *Set* $x^{var} := \frac{\alpha(1/\delta+1)}{\|x'\|}x'$, $x^{fix} := x' - x^{var}$ *and* $b^{var} := b - A(x^{fix})$
- *Solve the LP* $\hat{x} = \min\{\|x\|_1 \,|Ax \geq b^{var}, x \geq 0\}$
- *Generate a new solution* $x'' = x^{fix} + \hat{x}$

Problems:

- Given integral solution y' with $\|y'\|_1 \leq (1+\delta)OPT$.
  Compute integral solution y'' with $\|y'\|_1 \leq (1+\delta)OPT - \alpha$ such that $\|y'' - y''\|_1$ is small.
- Keep the number of non-zero components small
- Dynamic rounding

Open questions:

- Smaller migration factor and running time
- Lower bounds for migration?
- Dynamic bin packing (allow departing of items)
- Use LP-techniques for other online problems (i.e. scheduling)

Thank you!