Towards a General Theory of Barbs, Contexts and Labels*

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Abstract. Barbed bisimilarity is a widely-used behavioural equivalence for interactive systems: given a set of predicates (denoted "barbs", and representing basic observations on states) and a set of contexts (representing the possible execution environments), two systems are deemed to be equivalent if they verify the same barbs whenever inserted inside any of the chosen contexts. Despite its flexibility, this definition of equivalence is unsatisfactory, since often the quantification is over an infinite set of contexts, thus making barbed bisimilarity very hard to be verified.

Should a labelled operational semantics be available for our system, more efficient observational equivalences might be adopted. To this end, a series of techniques have been proposed to derive labelled transition systems (LTSs) from unlabelled ones, the main example being Leifer and Milner's reactive systems. The underlying intuition is that labels are the "minimal" contexts that allow for a reduction to be performed.

We introduce a framework that characterizes (weak) barbed bisimilarity via transition systems whose labels are (possibly minimal) contexts. Differently from other proposals, our theory is not dependent on the way LTSs are built, and it relies on a simple set-theoretical presentation. To provide a test-bed for our formalism, we instantiate it by addressing the semantics of mobile ambients and HoCore, recasting the (weak) barbed bisimilarities of these calculi via label-based behavioural equivalences.

Keywords: Barbed bisimilarity, contexts as labels, reactive systems.

1 Introduction

The operational semantics of process calculi was usually given in terms of labelled transition systems (LTSs), i.e., a set of possible states, plus a labelled transition relation among them, describing the possible evolutions of the computation. The labels express some kind of basic statement about the evolutions themselves, and thus they allow for an easy and fruitful way to provide meaningful behavioural semantics based on observations, i.e., basically looking at the labels of the state evolutions available for a calculus.

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More recently, however, the growing syntactical complexity of these calculi made almost customary to present their behaviour by a reduction semantics: an unlabelled relation, defined modulo a congruence that equates those processes intuitively representing the same system specification. This paradigmatic shift stimulated the adoption of *barbed equivalences* [13]: behavioural semantics based on a family of state predicates, called *barbs*, intended to capture the ability of a process of performing an interaction with the environment.

The trade-off to the relative easiness in defining barbed congruences even for operationally rich calculi is given by the difficulty of formally checking them: indeed, the verification usually requires to evaluate the barbs of a system with respect to all the contexts it can be possibily inserted in. Hence, the ingenuity of the researcher focussed on devising suitable labelled semantics from reduction ones, cases at hand being the calculus of mobile ambients (MAs) [7] and the core calculus for higher-order concurrency (HoCore) [10]. Indeed, a series of techniques [9,17,8,1,16] have been proposed to address the automatic derivation of an LTS starting from a reduction semantics, in order to distill behavioural equivalences that are congruences. Among these proposals, the most renowned one is Leifer and Milner's theory of reactive systems (RSs): the underlying intuition is that the labels of the derived transition system (called IPO LTS) are the "minimal" contexts that allow for a reduction to be performed, where minimality is captured by the categorical notion of relative pushout [11].

Our paper moves from Leifer and Milner's seminal work: our aim is to identify suitable conditions under which (weak) barbed bisimilarity can be characterized in terms of a behavioural equivalence over a suitably labelled transition system. Differently from the original theory of RSs, though, as well as from more recent contributions, the present work does not focus on devising novel techiques for distilling an adequate LTS, possibly for open systems [9], or for identifying what the "right" notion of barbs should be [14]: on the contrary, it tries and identifies a class of transition systems (with contexts as labels) that may represent a meaningful abstraction of an underlying reduction semantics. Here, we assume that "meaningful abstraction" precisely means that we may recast barbed bisimilarity using the bisimulation game in the LTS.

To this end, we introduce context LTSs, a general notion of which IPO LTSs are an instance. We then present weak context bisimilarity as the equivalence for these LTSs adopting the standard bisimulation game, and provide conditions ensuring that it is a congruence. We also use contexts as labels to define weak L-bisimilarity. It adopts a bisimulation game that is asymmetric with respect to a set L of contexts, yet it is a congruence and, depending on L, it may represent an efficient characterisation for barbed bisimilarity, which avoids to consider all contexts. Finally, in order to properly establish the adequacy of our theory, we check it against suitable case studies. So, we instantiate our proposal over MAs and HoCore, addressing their weak semantics: the former has a notably complex barbed bisimilarity, resilient until recently to a labelled characterization [12]; and only the strong semantics was so far considered for the latter. Their complementary features allow for testing the expressiveness of our framework.

Related Works. Our theory should be considered as an outcome of the stream of research born out of Leifer and Milner's theory [11], part of the contexts as labels approach for LTSs. However, the emphasis is not on the procedure for distilling the right class of contexts to be chosen as labels, but in the identification of a family of LTSs that are able to characterize barbed bisimilarity. Thus, our sound and complete context LTSs (Definition 10) are a general notion of which IPO LTSs are an instance [11], in the same way that Definition 13 subsumes the property of *having redex RPOs*. Similarly, Theorem 2 extends the congruence result for RSs [11, Theorem 1] to the new setting and for the weak semantics. The notion of (weak) barbed semantics for RSs was presented in [4], and strong L-bisimilarity discussed in [3]. Properties (1) and (2) of Definition 10 were exploited in [5] under the name of soundness and completeness for an LTS.

Synopsis. § 2 summarizes the main notions concerning our case studies, namely MAs (§ 2.1) and HoCore (§ 2.2). § 3 introduces our framework: the standard notions of RSs (albeit recast in a novel, set-theoretical way) and of weak barbed semantics (with a new result on non-discriminating contexts, § 3.1), while the labelled semantics that we propose (weak context bisimilarity, § 3.2) exploits instead the original notion of sound and complete context LTS. § 4 proposes weak L-bisimilarity, the proof that (under mild conditions on L) it is a congruence, and its correspondence with weak barbed semantics. § 5 and 6 show how our theory captures weak bisimilarity for MAs and HoCore, respectively. Finally, § 7 draws some conclusions and outlines directions for further research.

2 Two Case Studies

2.1 Mobile Ambients

This section introduces the finite, communication-free fragment of Mobile Ambients (MAs): its reduction semantics and behavioural equivalence [7], and the labelled transition system (LTS) for the calculus proposed in [2].

The syntax is shown in Fig. 1(a). We assume a set $\mathcal{N}$ of *names* ranged over by $m, n, o, \ldots$ and we let $P, Q, R, \ldots$ range over the set $\mathcal{P}_M$ of processes. The *free names* of a process P (denoted by $fn(P)$) are defined as usual. Processes are taken up to a *structural congruence*, axiomatised in Fig. 1(b) and denoted by $\equiv$. The *reduction relation* $\rightsquigarrow_M$, describes process evolution: it is the least relation $\rightsquigarrow_M : \mathcal{P}_M \times \mathcal{P}_M$ closed under $\equiv$ and generated by the rules in Fig. 1(c).

A *strong barb* o is a predicate over processes, with $P \downarrow_o$ denoting that P satisfies o. In MAs, $P \downarrow_n$ denotes the presence at top-level of an unrestricted ambient n. Formally, $P \downarrow_n$ if $P \equiv (\nu A)(n[Q]|R)$ and $n \notin A$, for some processes Q and R and a set of restricted names A. A process P satisfies the *weak barb* n (denoted as $P \Downarrow_n$) if there exists a process P' such that $P \rightsquigarrow_M^* P'$ and $P' \downarrow_n$, where $\rightsquigarrow_M^*$ is the transitive and reflexive closure of $\rightsquigarrow_M$.

Strong and weak barbs are exploited to define the standard equivalence for MAs: *weak reduction barbed congruence*, of which a labelled characterization is in [12]. Before presenting it, we introduce MAs *contexts*: they are MAs processes with a hole $-$, formally generated by the following grammar (for $R \in \mathcal{P}_M$)

(a)	$P ::= \mathbf{0}, n[P], M.P, (\nu n)P, P_1 \vert P_2$	$M ::= in\ n, out\ n, open\ n$
(b)	$P\vert Q \equiv Q\vert P$ $(P\vert Q)\vert R \equiv P\vert(Q\vert R)$ $P\vert\mathbf{0} \equiv P$ $(\nu n)(\nu m)P \equiv (\nu m)(\nu n)P$	$(\nu n)(P\vert Q) \equiv P\vert(\nu n)Q$ if $n \notin fn(P)$ $(\nu n)m[P] \equiv m[(\nu n)P]$ if $n \neq m$ $(\nu n)M.P \equiv M.(\nu n)P$ if $n \notin fn(M)$ $(\nu n)P \equiv (\nu m)(P\{^m/_n\})$ if $m \notin fn(P)$
(c)	$n[in\ m.P\vert Q]\vert m[R] \rightsquigarrow_M m[n[P\vert Q]\vert R]$ $m[n[out\ m.P\vert Q]\vert R] \rightsquigarrow_M n[P\vert Q]\vert m[R]$ $open\ n.P\vert n[Q] \rightsquigarrow_M P\vert Q$	if $P \rightsquigarrow_M Q$ then $(\nu n)P \rightsquigarrow_M (\nu n)Q$ if $P \rightsquigarrow_M Q$ then $n[P] \rightsquigarrow_M n[Q]$ if $P \rightsquigarrow_M Q$ then $P\vert R \rightsquigarrow_M Q\vert R$

Fig. 1. (a) Syntax, (b) structural congruence and (c) reduction relation of MAs

$$C[-] ::= -,\ n[C[-]],\ M.C[-],\ (\nu n)C[-],\ C[-] \mid R.$$

Definition 1 (Weak Reduction Barbed Congruence). Weak reduction barbed congruence $\approx_M$ *is the largest symmetric relation* $\mathcal{B}$ *s.t.* $P\,\mathcal{B}\,Q$ *implies*

- *if* $P \downarrow_n$ *then* $Q \Downarrow_n$*;*
- *if* $P \rightsquigarrow_M P'$ *then* $Q \rightsquigarrow_M^* Q'$ *and* $P'\,\mathcal{B}\,Q'$*;*
- $\forall C[-], C[P]\,\mathcal{B}\,C[Q]$.

An LTS for MAs. In Fig. 2 we present the LTS M for MAs proposed in [2], and obtained by a suitable application of Leifer and Milner's theory [17] (hence, its transition labels are the "minimal" contexts allowing a reduction to occur). Note that we assume that the LTS is closed with respect to structural congruence.

The rule Tau represents the τ-actions modeling reduction of the process. The rule ExtRed represents three rules, one for each axiom of the reduction relation (see Fig. 1(c), left): the process L of the label represents the left hand side of the axiom and the process R of the target state represents the right hand side (thus, ExtRed model the reduction performed by the environment).

The other rules instead model the interactions of a process with its environment. Note that in the conclusions of some rules there are names and processes (denoted by o, S_1, S_2) that do not appear in the premises. These represent ambient names and processes that are provided by the environment and thus we must always assume that $(\{o\} \cup fn(S_1) \cup fn(S_2)) \cap A = \emptyset$. For instance, the rule Open enables a proces to open an ambient provided by the environment, while the rule CoOpen allows the environment to open an ambient of the process. For a description of the other rules, we refer the reader to [2].

$$\text{(Tau)}\ \frac{P \rightsquigarrow_M Q}{P \xrightarrow{-} Q} \qquad \text{(ExtRed)}\ \frac{L \rightsquigarrow_M R}{P \xrightarrow{-|L} P|R} \qquad \text{(Out)}\ \frac{P \equiv (\nu A)(out\ m.P_1|P_2) \quad m \notin A}{P \xrightarrow{m[o[-|S_1]|S_2]} (\nu A)(m[S_2]|o[P_1|P_2|S_1])}$$

$$\text{(In)}\ \frac{P \equiv (\nu A)(in\ m.P_1|P_2) \quad m \notin A}{P \xrightarrow{o[-|S_1]|m[S_2]} (\nu A)m[o[P_1|P_2|S_1]|S_2]} \qquad \text{(OutAmb)}\ \frac{P \equiv (\nu A)(n[out\ m.P_1|P_2]|P_3) \quad m \notin A}{P \xrightarrow{m[-|S_1]} (\nu A)(m[P_3|S_1]|n[P_1|P_2])}$$

$$\text{(InAmb)}\ \frac{P \equiv (\nu A)(n[in\ m.P_1|P_2]|P_3) \quad m \notin A}{P \xrightarrow{-|m[S_1]} (\nu A)(m[n[P_1|P_2]|S_1]|P_3)} \qquad \text{(Open)}\ \frac{P \equiv (\nu A)(open\ n.P_1|P_2) \quad n \notin A}{P \xrightarrow{-|n[S_1]} (\nu A)(P_1|P_2|S_1)}$$

$$\text{(CoIn)}\ \frac{P \equiv (\nu A)(m[P_1]|P_2) \quad m \notin A}{P \xrightarrow{-|o[in\ m.S_1|S_2]} (\nu A)(m[o[S_1|S_2]|P_1]|P_2)} \qquad \text{(CoOpen)}\ \frac{P \equiv (\nu A)(n[P_1]|P_2) \quad n \notin A}{P \xrightarrow{-|open\ n.S_1} (\nu A)(P_1|S_1|P_2)}$$

Fig. 2. The LTS M, for $L \rightsquigarrow_M R$ ranging over the three axioms of $\rightsquigarrow_M$

2.2 HoCore

HoCore [10] has been introduced as a core language for higher-order concurrency (where processes may exchange messages containing processes). Its main peculiarities consist in the fact that (1) several different notions of strong behavioural equivalence coincide, and (2) they are all computable, even if the formalism is Turing complete. We consider here weak equivalence: it was not studied before, even if it is immediate to note that (1) does not hold anymore.

The syntax is shown in Fig. 3(a). We assume a set $\mathcal{N}$ of *names* ranged over by $a, b, c, \ldots$ and a set $\mathcal{V}$ of *process variables* ranged over by $x, y, z, \ldots$, requiring $\mathcal{N}$ and $\mathcal{V}$ to be disjoint. The set of names of a process P (denoted by $n(P)$) is defined as usual. The input $a(x).P$ binds the free occurrences of the variable x in P. We write $fv(P)$ for the set of free variables of P, and we identify processes up-to the renaming of bound variables. A process is *closed* if it does not have free variables. To make the presentation lighter, we say *process* to mean closed process, and we write *open process* whenever we mean process that might not be closed. We let $P, Q, R, \ldots$ range over the set $\mathcal{P}_H$ of closed processes.

HoCore *contexts* are closed processes with a hole $-$, formally generated by the following grammar (for $Q \in \mathcal{P}_H$)

$$C[-] ::= -,\ a(x).C[-],\ C[-] \mid Q.$$

The structural congruence $\equiv$ is the smallest congruence induced by the axioms in Fig. 3(b). The behaviour of a process P is then described as the reaction relation $\rightsquigarrow_H$ over processes up to $\equiv$, obtained by the rules in Fig. 3(c).

Barbs are defined as follows: $P \downarrow_{\bar{a}}$ if $P \equiv \overline{a}P_1|P_2$, for processes P_1 and P_2. A process P satisfies the *weak barb* $\bar{a}$ (denoted as $P \Downarrow_{\bar{a}}$) if there exists a process P' such that $P \rightsquigarrow_H^* P'$ and $P' \downarrow_{\bar{a}}$, where $\rightsquigarrow_H^*$ is the transitive and reflexive closure of $\rightsquigarrow_H$. The notion of barb is used in [10] to give the definition of *asynchronous barbed congruence*. Here we straightforwardly extend it to the weak case.

Definition 2 (Weak Asynchronous Barbed Congruence). Weak asynchronous barbed congruence $\sim_H^B$ *is the largest symmetric relation* $\mathcal{B}$ *s.t.* $P\,\mathcal{B}\,Q$ *implies*

(a) $P ::= \mathbf{0},\ \overline{a}P,\ a(x).P,\ x,\ P_1 \vert P_2$	(b) $P\vert\mathbf{0} \equiv P\ (P\vert Q)\vert R \equiv P\vert(Q\vert R)\ P\vert Q \equiv Q\vert P$
(c) $\overline{a}Q\vert a(x).P \rightsquigarrow_H P\{^Q/_x\}$	if $P \rightsquigarrow_H Q$ then $P\vert R \rightsquigarrow_H Q\vert R$

Fig. 3. (a) Syntax, (b) structural congruence and (c) reduction relation of HoCore

- *if* $P \Downarrow_{\bar{a}}$ *then* $Q \Downarrow_{\bar{a}}$*;*
- *if* $P \rightsquigarrow_H^* P'$ *then* $Q \rightsquigarrow_H^* Q'$ *and* $P' \mathcal{B} Q'$*;*
- $\forall C[-], C[P] \mathcal{B} C[Q]$*.*

For instance, $a(x).\overline{a}x \sim_H^B \mathbf{0}$. This is intuitively understood by observing that the former process can interact only with a context of the shape $-|\overline{a}P_1|P_2$. Moreover, the interaction of $a(x).\overline{a}x$ with $-|\overline{a}P_1|P_2$ reduces to the process $\overline{a}P_1|P_2$, which is clearly equivalent to $\mathbf{0}|\overline{a}P_1|P_2$ (i.e., $\mathbf{0}$ inside the context $-|\overline{a}P_1|P_2$).

Concerning the equalities above, it is worth noting that $a(x).\overline{a}x$ and $\mathbf{0}$ are not equivalent with respect to the *strong* asynchronous barbed congruence (obtained by replacing $\Downarrow_{\bar{a}}$ with $\downarrow_{\bar{a}}$ and $\rightsquigarrow^*$ by $\rightsquigarrow$ in the above definition).

Two LTSs for HoCore. Fig. 4 shows the LTS semantics of HoCore introduced in [10] for open processes (the symmetric variants of the two rightmost rules are omitted). We let α denote any label of a transition and $bv(\alpha)$ the set of its bound variables (for $bv(a(x)) = \{x\}$ and $bv(\alpha) = \emptyset$ for $\alpha \neq a(x)$).

This LTS is used in [10] to define a few alternative notions of equivalence that (when restricted to closed processes) are shown to coincide with *strong* asynchronous barbed congruence. What is noteworthy is that these equivalences avoid the quantification over all contexts of asynchronous barbed congruence, and they are thus proved to be computable.

These behavioural equivalences (making use of the LTS) cannot be straightforwardly extended to the weak case, and indeed their "naive weak extension" would not coincide with $\sim_H^B$. This can be straightforwardly observed by noting that all the definitions in [10] are "synchronous", i.e., they require that every input transition is matched by another input transition. On the contrary, any equivalence defined in such a manner would never equate the processes $a(x).\overline{a}x$ and $\mathbf{0}$, which instead are related, as discussed before, by $\sim_H^B$.

At the end of this paper, we will apply our theoretical framework to derive a labelled characterization of $\sim_H^B$. We will make use of the LTS H shown in Fig. 5 where, once more, labels are (minimal) contexts, as in the LTS for MAs in Fig. 2.

The Tau rule models internal computations. The In rule models the communication over a channel a, where the environment send a process S to the process P. Vice versa, in the Out rule the process P send a process P_1 to the the environment: the sent process P_1 is substituted in the continuation S of the inputting environment. The rule ExtRed models transitions where the redex is fully offered by the environment. It represents the axiom of the reduction relation (see Fig. 3(c), left): the process L of the label and the process R

$$(\text{OUT})\ \overline{a}Q \xrightarrow{\overline{a}Q} \mathbf{0} \quad (\text{IN})\ a(x).P \xrightarrow{a(x)} P \quad (\text{COM})\ \frac{P_1 \xrightarrow{\overline{a}Q} P_1' \quad P_2 \xrightarrow{a(x)} P_2'}{P_1|P_2 \xrightarrow{\tau} P_1'|P_2'\{^Q/_x\}} \quad (\text{PAR})\ \frac{P_1 \xrightarrow{\alpha} P_1' \quad bv(\alpha)\cap fv(P_2)=\emptyset}{P_1|P_2 \xrightarrow{\alpha} P_1'|P_2}$$

Fig. 4. The ordinary LTS of HoCore

$$(\text{OUT})\ \frac{P\equiv\overline{a}P_1|P_2}{P \xrightarrow{-|a(x).S} P_2|S\{^{P_1}/_x\})} \quad (\text{IN})\ \frac{P\equiv(a(x).P_1)|P_2}{P \xrightarrow{-|\overline{a}S} P_1\{^S/_x\}|P_2} \quad (\text{TAU})\ \frac{P \rightsquigarrow_H Q}{P \xrightarrow{-} Q} \quad (\text{EXTRED})\ \frac{L \rightsquigarrow_H R}{P \xrightarrow{-|L} P|R}$$

Fig. 5. The LTS H, for $L \rightsquigarrow_H R$ ranging over the three axioms of $\rightsquigarrow_H$

of the target state represent the left-hand and the right-hand side of the axiom, respectively.

Even if we do not state it formally, it is immediate to note the tight correspondence between the ordinary LTS (Fig. 4) and our own H, modulo the structural congruence: if $P \xrightarrow{\tau} Q$ then $P \xrightarrow{-} Q$, and vice versa if $P \xrightarrow{-} Q$ there exist P', Q' s.t. $P' \xrightarrow{\tau} Q'$ and $P \equiv P'$, $Q \equiv Q'$ (and similarly for $P \xrightarrow{a(x)} Q$ and $P \xrightarrow{\overline{a}P_1} Q$ with respect to $P \xrightarrow{-|\overline{a}S} Q\{^S/_x\}$ and $P \xrightarrow{-|a(x).S} Q|S\{^{P_1}/_x\}$, respectively).

Remark 1. It is worth remarking here that the ordinary LTS is finite, while H is not because S represents any possible process provided by the environment. This kind of problem occurs also for the LTS M that we have shown for MAs and for other LTSs that have been proposed in analogous works (see e.g. [15,16]). It is challenging to devise a general solution to this problem, but some suggestions come from *open* reactive systems [9]: instead of considering only closed process and contexts, one might take into account *open* processes, contexts and *variable substituions*. In the special case of HoCore, it would be interesting to develop (for the weak case) a technique analogous to the *normal bisimulation* of [10]: instead of considering all the possible S in the transitions $P \xrightarrow{-|\overline{a}S}$ and $P \xrightarrow{-|a(x).S}$, we might take only $S = \overline{m}\mathbf{0}$ and $S = m(y).x$, respectively, for a fresh name m. This is out of the scope of the present paper and is left for future work.

3 Reactive Systems and Context LTSs

This section introduces a framework that encompasses MAs and HoCore, aiming at a general theory for modeling the weak (barbed) semantics of interactive formalisms. After providing the (set-theoretical) definition of reactive system, we introduce a barbed (§ 3.1) and a labelled semantics for these systems (§ 3.2), showing that the latter is, under suitable conditions, a congruence (§ 3.3).

We first define a notion of *system theory*, denoting how the states of a system are built. Recall that a monoid $\mathbb{M} = (\mathcal{M}, \otimes, 1)$ consists of a set $\mathcal{M}$, an associative binary operator $\otimes : \mathcal{M} \times \mathcal{M} \to \mathcal{M}$, and the identity element $1 \in \mathcal{M}$. Given a monoid $\mathbb{M}$ and a set X, a monoid action of $\mathbb{M}$ on X is an operation $\cdot : \mathcal{M} \times X \to X$ compatible with the monoid operation, i.e., such that for each $m_1, m_2 \in \mathcal{M}$ and $x \in X$, $m_1 \cdot (m_2 \cdot x) = (m_1 \otimes m_2) \cdot x$ and for each $x \in X$, $1 \cdot x = x$.

Definition 3 (System Theory). *A* system theory *is a triple* $\mathbb{S} = \langle \mathcal{P}, \mathbb{C}, \cdot \rangle$ *such that* $\mathcal{P}$ *is a set of* processes*, ranged over by* $P, Q, R, \ldots$, $\mathbb{C} = (\mathcal{C}, \circ, -)$ *a monoid of* contexts*, ranged over by* $C[-], C_1[-], \ldots$*, and* $\cdot : \mathcal{C} \times \mathcal{P} \to \mathcal{P}$ *a monoid action.*

We usually denote context composition $C_1[-] \circ C_2[-]$ as $C_2[C_1[-]]$ and the action $C[-] \cdot P$ as $C[P]$. The chosen notation supports the intuition that the monoid operation represents the functional composition of unary contexts, while the action is just the insertion of a process into a context. It allows for an easier comparison with the process calculi notation adopted in later sections.

Definition 4 (Reactive System). *A* reactive system *(RS) is a triple* $\mathcal{R} = \langle \mathbb{S}, \rightsquigarrow, O \rangle$ *where* $\mathbb{S}$ *is a system theory,* $\rightsquigarrow \subseteq \mathcal{P} \times \mathcal{P}$ *a transition relation and* O *a set of predicates on* $\mathcal{P}$*.*

We write $P \rightsquigarrow Q$ to mean $\langle P, Q \rangle \in \rightsquigarrow$, and we denote by $\rightsquigarrow^*$ the reflexive and transitive closure of $\rightsquigarrow$. The predicates in O are called *barbs* and they represent basic observations on the state of a system. We write $P \downarrow_o$ if P satisfies $o \in O$. Analogously, the state P satisfies the weak barb o (written $P \Downarrow_o$) if there exists a state P' such that $P \rightsquigarrow^* P'$ and $P' \downarrow_o$.

3.1 Barbed Saturated Semantics

With the ingredients offered by our theory, we can define a behavioural equivalence which equates two processes if these cannot be distinguished by an observer that can insert a process into any context and check the exposed barbs.

In the following, we fix an RS $\mathcal{R} = \langle \mathbb{S}, \rightsquigarrow, O \rangle$, with $\mathbb{S} = \langle \mathcal{P}, \mathbb{C}, \cdot \rangle$.

Definition 5 (Weak Barbed Saturated Bisimilarity). Weak barbed saturated bisimilarity $\approx^{BS}$ *(for* $\mathcal{R}$*) is the largest symmetric relation* $\mathcal{B}$ *such that* $P \,\mathcal{B}\, Q$ *implies* $\forall C[-] \in \mathcal{C}$

- *if* $C[P] \Downarrow_o$ *then* $C[Q] \Downarrow_o$*;*
- *if* $C[P] \rightsquigarrow^* P'$ *then* $C[Q] \rightsquigarrow^* Q'$ *and* $P' \,\mathcal{B}\, Q'$*.*

Weak barbed saturated bisimilarity (which is a congruence) is general enough to encompass the standard behavioural equivalences of many process calculi. The main drawback of this kind of definition is the quantification over all contexts that makes hard the proofs of equivalence. In this paper, we provide a general proof technique for $\approx^{BS}$ that avoids the quantification over all possible contexts by relying on LTSs like those derived by Leifer and Milner's theory [11].

It is sometimes possible to restrict the set of contexts to be checked against, so obtaining again the same barbed equivalence. This can be accomplished by identifying those contexts whose presence does not really influence $\approx^{BS}$.

Definition 6 (Non-Discriminating Context). *Let $E[-] \in \mathcal{C}$ be a context. It is* non-discriminating *(for $\mathcal{R}$) if $\forall P \in \mathcal{P}$, $\forall C[-] \in \mathcal{C}$ and $\forall o \in \mathcal{O}$*

1. *if $C[E[P]] \downarrow_o$ then $\forall Q \in \mathcal{P}.\ C[E[Q]] \downarrow_o$;*
2. *if $E[P] \rightsquigarrow P'$ then $P' = E'[P]$ and $\forall Q \in \mathcal{P}.\ E[Q] \rightsquigarrow E'[Q]$.*

For instance, the context $M.-$ of MAs is non-discriminating, since it hides the strong barbs of the process that is inserted inside it, and it inhibits its transitions. For the same reason, the context $a(x).-$ of HoCore is non-discriminating as long as it is applied to closed processes (thus, no variable is actually bound).

Now, starting from an RS $\mathcal{R}$ we can build a new reactive system $\mathcal{R}'$ by removing some (possibly all) non-discriminating contexts. The following proposition ensures that the barbed saturated bisimilarity in the two systems coincide.

Proposition 1. *Let $\mathbb{C}'$ be a submonoid of the context monoid $\mathbb{C}$, $\mathbb{S}' = \langle \mathcal{P}, \mathbb{C}', \cdot \rangle$ the system theory derived from $\mathbb{S}$, and $\mathcal{R}' = \langle \mathbb{S}', \rightsquigarrow, O \rangle$ the RS derived from $\mathcal{R}$. If all contexts in $\mathbb{C}$ but not in $\mathbb{C}'$ are non-discriminating in $\mathcal{R}$, then $\approx^{BS}_{\mathcal{R}} = \approx^{BS}_{\mathcal{R}'}$.*

3.2 Weak Context Bisimilarity

In order to equip RSs with a labelled equivalence, we need to introduce a more basic notion, identifying those contexts that always allow a reaction.

Definition 7 (Reactive Context). *Let $C[-] \in \mathcal{C}$ be a context. It is* reactive *(for $\mathcal{R}$) if $\forall P \in \mathcal{P}$, if $P \rightsquigarrow P'$ then $C[P] \rightsquigarrow C[P']$. The set (actually, submonoid) of reactive contexts is denoted $\mathbb{R}$.*

For instance, the contexts $-|R$ and $(\nu a)-$ of MAs and $-|R$ of HoCore are reactive, while the prefixes ($a(x).-$ in HoCore and $M.-$ in MAs) are not.

Definition 8 (Context LTS). *A* context *LTS is a triple $\mathbb{D} = \langle \mathcal{R}, \mathcal{D}, \rightarrow_D \rangle$ such that $\mathcal{D} \subseteq \mathcal{C}$ and $\rightarrow_D \subseteq \mathcal{P} \times \mathcal{D} \times \mathcal{P}$.*

As usual, we write $P \xrightarrow{C[-]}_D Q$ to mean that $\langle P, C[-], Q \rangle \in \rightarrow_D$. Note moreover that the set of labels $\mathcal{D}$ is a subset of the set of all contexts $\mathcal{C}$. For instance, the LTSs M and H (in Figs. 2 and 5) are two context LTSs. The set of labels of H is $\{-, -|\overline{a}S, -|a(x).S \mid a \in \mathcal{N} \text{ and } S \in \mathcal{P}_H\}$.

In order to characterize the class of LTSs on which the labelled semantics for RSs is based, we need to introduce the following definition.

Definition 9 (Decomposition Pair). *Let $\langle C_1[-], C_2[-] \rangle$ a pair belonging to $\mathcal{D} \times \mathbb{R}$. It is a decomposition pair for $C[P] \rightsquigarrow P'$ if there exists a process P'' such that $P \xrightarrow{C_1[-]}_D P''$, $C_2[C_1[-]] = C[-]$ and $C_2[P''] = P'$.*

For instance, consider the MAs process $P = open\, n.P_1$ and the context $C[-] = -|n[P_2]|P_3$. A decomposition pair for $C[P] \rightsquigarrow_M P_1|P_2|P_3$ is $\langle -|n[P_2], -|P_3 \rangle$. Indeed, it is easy to check that $P \xrightarrow{-|n[P_2]} P_1|P_2$; moreover, if we compose $-|n[P_2]$

with $-|P_3$ we exactly obtain $C[-]$, and finally the composition between $P_1|P_2$ and $-|P_3$ gives $P_1|P_2|P_3$.

Now we can characterize the class of LTSs we are interested in: they are context LTSs satisfying suitable soundness and completeness properties.

Definition 10 (Sound and Complete Context LTS). *A context LTS is* sound *and* complete *if it satifies the properties*

1. *if $P \xrightarrow{C[-]}_D P'$ then $C[P] \rightsquigarrow P'$;*
2. *each $C[P] \rightsquigarrow P'$ has a decomposition pair $\langle C_1[-], C_2[-] \rangle$.*

The LTS M for MAs and the LTS H for HoCore are both sound and complete, as we discuss later in § 5 and § 6, respectively.

In the following, we fix a sound and complete context LTS $\mathbb{D}$. Moreover, we use $P \xrightarrow{C[-]} *_D P'$ to denote $P \rightsquigarrow^* \bullet \xrightarrow{C[-]}_D \bullet \rightsquigarrow^* P'$.

Definition 11 (Weak Context Bisimilarity). Weak context bisimilarity $\approx^D$ *(for $\mathbb{D}$) is the largest symmetric relation $\mathcal{B}$ such that $P \,\mathcal{B}\, Q$ implies*

– *if $P \xrightarrow{C[-]} *_D P'$ then $Q \xrightarrow{C[-]} *_D Q'$ and $P' \,\mathcal{B}\, Q'$.*

Note that the definition above requires that when P performs what is intuitively a τ-transition, i.e., when $P \xrightarrow{-}_D P'$, then a bisimilar process Q has to reply either with *one* or more τ-moves. Usually, the process Q might instead reply with *zero* or more τ-moves. This fact is discussed at the end of § 4.1.

3.3 Congruence Property

We now focus on adding constraints over $\mathbb{D}$ in order to make $\approx^D$ a congruence.

Definition 12. *Let $\langle C_1[-], C_2[-] \rangle$ be a decomposition pair for $C'[C[P]] \rightsquigarrow P'$. It is* universal *if $\forall Q.\ Q \xrightarrow{C_1[-]}_D Q'' \Rightarrow C[Q] \xrightarrow{C'[-]}_D Q' \wedge Q' = C_2[Q'']$.*

Consider the MAs process $P = open\, n.P_1$ and the context $C[-] = -|P_3$ and $C'[-] = -|n[P_2]$. The decomposition pair $\langle -|n[P_2], -|P_3 \rangle$ of $C'[C[P]] \rightsquigarrow_M P_1|P_2|P_3$ is universal: for all Q s.t. $Q \xrightarrow{-|n[P_2]} Q''$ it holds $Q|P_3 \xrightarrow{-|n[P_2]} Q|P_3$.

Definition 13. *A context LTS $\mathbb{D}$ is* decomposable *if whenever $C[P] \xrightarrow{C'[-]}_D P'$ there exists a universal decomposition pair $\langle C_1[-], C_2[-] \rangle$ for $C'[C[P]] \rightsquigarrow P'$.*

The decomposition property introduced above is only required to prove the congruence property of the context bisimilarity (Proposition 2) and of the weak L-bisimilarity introduced in the next section (Proposition 3). The main result of our theory (namely, the labelled characterization of barbed bisimilarity stated in Theorem 1) just requires that the LTS is sound and complete.

Proposition 2. *If all contexts in $\mathcal{C}$ are reactive and $\mathbb{D}$ is decomposable, then $\approx^D$ is a congruence.*

Unfortunately, $\approx^D$ is often too fine-grained. Consider e.g. HoCore and the associated LTS H: $\approx^D$ is too discriminating for H. Indeed, as shown in § 2.2, the HoCore processes $a(x).\overline{a}x$ and $\mathbf{0}$ are asynchronously barbed congruent but they are obviously distinguished by $\approx^D$.

The right semantics is often represented by the barbed saturated one, whose flexibility allows for recasting a wide variety of observational, bisimulation-based equivalences. The main drawback of this semantics is the fact that in an equivalence proof all contexts must be tackled. For this reason, the next section introduces an alternative bisimilarity that efficiently characterizes the barbed saturated one, since it allows reasoning about it without considering all contexts.

4 Weak *L*-Bisimilarity

This section introduces weak L-bisimilarity, the weak counterpart of the bisimilarity proposed in [3]. It is parametric with respect to a set of contexts (also referred to as labels) L and it is a congruence, should L satisfy a closure property. More importantly, it can be used as an alternative proof technique of $\approx^{BS}$.

Definition 14 (Weak L-Bisimilarity). *Let $L \subseteq \mathcal{D}$ be a set of contexts.* Weak L-bisimilarity $\approx^L$ *(for $\mathbb{D}$) is the largest symmetric relation $\mathcal{B}$ s.t. $P \,\mathcal{B}\, Q$ implies*

$$\text{if } P \xrightarrow{C[-]}_D P' \text{ then } \begin{cases} Q \xrightarrow{C[-]} *_D\, Q' \text{ and } P' \,\mathcal{B}\, Q', \text{ if } C[-] \in L; \\ C[Q] \rightsquigarrow^* Q' \text{ and } P' \,\mathcal{B}\, Q', \text{ otherwise.} \end{cases}$$

Note that $\approx^L$ generalizes $\approx^D$. Indeed, it is easy to prove that they coincide, when L is actually the whole set $\mathcal{D}$. In § 4.1 we show that for some L, weak L-bisimilarity also coincides with weak barbed saturated bisimilarity. We now show that $\approx^L$ is a congruence, under suitable conditions on L.

Definition 15. *A set of contexts $L \subseteq \mathcal{D}$ in a decomposable $\mathbb{D}$ is* closed under decomposition *if whenever $\langle C_1[-], C_2[-]\rangle$ is a universal decomposition pair for $C'[C[P]] \rightsquigarrow P'$ (see Definition 13) and $C'[-] \in L$ then $C_1[-] \in L$.*

Proposition 3. *Let L be a set of contexts in a decomposable $\mathbb{D}$. If all contexts in $\mathcal{C}$ are reactive and L is closed under decomposition, then $\approx^L$ is a congruence.*

4.1 Weak Barbed Saturated Bisimilarity via Weak *L*-Bisimilarity

Here we show that weak L-bisimilarity can characterize weak barbed saturated bisimilarity. It generalizes the correspondence between the strong version of the semantics, as it holds for Leifer and Milner's RSs [3]. This result is later used to prove that L-bisimilarity captures the right equivalences for MAs and HoCore.

The following definitions are needed to ensure that $\approx^L \subseteq \approx^{BS}$.

Definition 16 (Weak Contextual Barbs). *A barb o is a* weak contextual barb *if whenever $P \downarrow_o$ implies $Q \Downarrow_o$ then $\forall C[-]$, $C[P] \downarrow_o$ implies $C[Q] \Downarrow_o$.*

Definition 17. *Let L be a set of contexts. We say that L is O-*capturing *(for $\mathbb{D}$) if for each barb $o \in O$ there exists a context $C[-] \in L$ such that for each process P we have $P \downarrow_o$ if and only if $P \xrightarrow{C[-]}_D P'$.*

The next two definitions are instead needed to ensure that $\approx^{BS} \subseteq \approx^L$.

Definition 18. *Let $\mathcal{B}$ be a relation on processes. A predicate $\mathcal{P}(X, Y)$ on processes is* stable under $\mathcal{B}$ *(for $\mathbb{D}$) if for any two processes P, Q whenever $P\mathcal{B}Q$ and $\mathcal{P}(P, P')$ there exists Q' such that $\mathcal{P}(Q, Q')$ and $P'\mathcal{B}Q'$. A context $C[-]$ is* weakly stable under $\mathcal{B}$ *(for $\mathbb{D}$) if $\mathcal{P}(X, Y) = X \xrightarrow{C[-]} *_D Y$ is stable under $\mathcal{B}$.*

In § 5 and 6, we show some contexts of MAs and HoCore that are weakly stable under $\approx^{BS}$. Note that the identity context $-$ is usually *not* stable under $\approx^{BS}$: for $P \approx^{BS} Q$ and $P \xrightarrow{-} P'$ (i.e., $P \rightsquigarrow P'$), it is not guaranteed that $Q \xrightarrow{-}$.

We may now state one of the main correspondence results for our theory, witnessing the usefulness of L-bisimilarity.

Theorem 1. *Let O be a set of weak contextual barbs and $L \subseteq \mathcal{D}$ a set of contexts in a sound and complete $\mathbb{D}$. If L is O-capturing and all its contexts are reactive and weakly stable under $\approx^{BS}$ (for $\mathbb{D}$), then $\approx^{BS} = \approx^L$.*

Theorem 1 requires that only the context of a subset L of $\mathcal{D}$ are reactive. However, sometimes it may be useful to consider only reactive contexts (like done implicitly in the graphical encodings of process calculi, see e.g [6]). This can be easily done, by observing that, as hinted above for MAs and HoCore, in most process calculi those contexts that are not reactive are also non-discriminating.

Remark 2. Note that weak L-bisimilarity coincides with weak context bisimilarity (Definition 11) when taking $L = \mathcal{D}$. Thus, as a consequence of the previous theorem, weak context bisimilarity coincides with weak barbed bisimilarity, whenever all the contexts in $\mathcal{D}$ are O-capturing and weakly stable under $\approx^{BS}$. However, this is usually not the case because, as discussed above, the identity context $-$ is not weakly stable. Indeed, in standard weak bisimilarities (e.g., the one for CCS), when one process perform one internal τ-transition (corresponding to $\xrightarrow{-}$), the equivalent process is not forced to perform any transition. By taking $L = \mathcal{D} \setminus \{-\}$, we capture exactly these standard weak bisimilarities (it suffices to observe that if $P \approx^L Q$ and $P \xrightarrow{-}_D P'$ then $Q \rightsquigarrow^* Q'$, since $- \notin L$) and via Theorem 1 we can check when they coincide with saturated barbed bisimilarity.

5 Labelled Characterization for MAs Weak Semantics

This section proposes a labelled characterization of the weak reduction barbed congruence for MAs by means of the weak L-bisimilarity over the LTS M.

First of all, we show that MAs fit in the theory of § 3. We consider the system theory $\mathbb{S}_M = \langle \mathcal{P}_M, \mathbb{C}_M, \cdot \rangle$, where $\mathbb{C}_M$ is the monoid $(\mathcal{C}_M, \circ, -)$, with $\mathcal{C}_M$ the set of unary MAs contexts presented in § 2.1. The calculus can thus be seen as an RS $\mathcal{R}_M = \langle \mathbb{S}, \rightsquigarrow_M, O_M \rangle$, where O_M represents the set of MAs barbs.

It is easy to see that barbed saturated semantics for $\mathcal{R}_M$ coincides with the weak reduction barbed congruence for MAs.

Proposition 4. $\approx_M = \approx_M^{BS}$ *(for $\mathcal{R}_M$).*

The equivalence is thus a congruence with respect to all MAs contexts. However, in order to exploit the L-bisimilarity, we also have to show that M is a sound and complete context LTS. Instead of proving it directly, we consider a simpler RS, distilled in [2] by exploiting Leifer and Milner's theory. It differs from $\mathcal{R}_M$ only with respect to the monoid of contexts: $\mathcal{C}'_M$ contains the unary MAs contexts generated by the following grammar (for $R \in \mathcal{P}_M$)

$$C[-] ::= -,\ C[-]|R,\ (\nu n)C[-],\ n[C[-]].$$

We therefore consider the MAs system theory $\mathbb{S}'_M = \langle \mathcal{P}_M, \mathbb{C}'_M, \cdot \rangle$, where $\mathbb{C}'_M$ is the monoid $(\mathcal{C}'_M, \circ, -)$. The RS modelling MAs is now $\mathcal{R}'_M = \langle \mathbb{S}', \rightsquigarrow_M, O_M \rangle$ and M is a sound and complete context LTS based on $\mathcal{R}'_M$: its labels are "minimal" contexts, according to Leifer and Milner's terminology, and this fact ensures that both Property 1 (thanks to [11, Proposition 3]) and Property 2 (thanks to [11, Proposition 1]) of Definition 10 are satisfied.

Now, since the contexts of the shape $M.C[-]$ are non-discriminating, thanks to our Propositions 1 and 4, we can state the proposition below.

Proposition 5. $\approx_M = \approx_M^{BS}$ *(for $\mathcal{R}'_M$).*

As shown in § 4.1, we can characterize the weak barbed semantics on a set of weak contextual barbs O through a sound and complete context LTS and a set of reactive contexts L. As required by Theorem 1, L must be O-capturing and each $C[-] \in L$ must be weakly stable under weak barbed saturated bisimilarity.

Proposition 6. *Let L_M be the set of labels having the shape $-|open\ n.T_1$, for n an ambient name and T_1 an MAs process. Then, L_M is O_M-capturing. Moreover all the barbs in O_M are weak contextual.*

The proof of the proposition above occurs in [3, Proposition 8] ("L_M is O_M-capturing") and [4, Proposition 5] ("O_M are weak contextual"). It is easy to note that the contexts in L_M are reactive: it is now needed to prove that they are weakly stable under $\approx_M^{BS}$. To this end, we exploit the predicate in Fig. 6 that is equivalent to $\mathcal{P}(X,Y) = X \xrightarrow{C[-]} *_D\ Y$ and we show that it is stable under $\approx_M^{BS}$.

Lemma 1. *Let $\mathcal{P}^{-|open\ n.T_1}(X,Y)$ be the predicate on MAs processes shown in Fig. 6, for n ambient name and T_1 a process. Then, for any two processes P and P' we have $\mathcal{P}^{-|open\ n.T_1}(P,P')$ if and only if $P \xrightarrow{-|open\ n.T_1} *_M\ P'$.*

Proposition 7. *All labels in L_M are weakly stable under $\approx_M^{BS}$.*

From the previous results, the following theorem follows immediately.

Theorem 2. $\approx_M^{BS} = \approx^{L_M}$ *(for $\mathcal{R}'_M$).*

6 Labelled Characterizations for HoCore Semantics

As for MAs, we first show that HoCore fits in the theory of §3. We consider the HoCore system theory $\mathbb{S}_H = \langle \mathcal{P}_H, \mathbb{C}_H, \cdot \rangle$, where $\mathbb{C}_H$ is the monoid context $(\mathcal{C}_H, \circ, -)$, with $\mathcal{C}_H$ the set of unary HoCore contexts presented in § 2.2. The calculus can thus be seen as an RS $\mathcal{R}_H = \langle \mathbb{S}_H, \rightsquigarrow_H, O_H \rangle$, where O_H is the set of HoCore barbs defined in § 2.2.

It is easy to see that the weak barbed saturated semantics for $\mathcal{R}_H$ coincides with the weak asynchronous barbed congruence (Definition 2).

Proposition 8. $\sim_H^B = \approx_H^{BS}$ *(for $\mathcal{R}_H$).*

The LTS H (Fig. 5) is a sound and complete context LTS. Indeed, as for MAs, its labels are minimal contexts, according to Leifer and Milner's theory. For the sake of space we do not report the distillation procedure, but it is analogous to the one for CCS shown in [6]. However, as for MAs, the RS employed for distilling H slightly differs from $\mathcal{R}_H$, since there is no context of the shape $a(x).C[-]$.

We thus define $\mathcal{C}'_H$ as the set of contexts generated by $C[-] ::= -, C[-]|R$ (for $R \in \mathcal{P}_H$) and we consider the system theory $\mathbb{S}'_H = \langle \mathcal{P}_H, \mathbb{C}'_H, \cdot \rangle$, where $\mathbb{C}'_H$ is the monoid context $(\mathcal{C}'_H, \circ, -)$. Therefore, the RS modelling HoCore is now $\mathbb{R}'_H = \langle \mathbb{S}'_H, \rightsquigarrow_H, O_H \rangle$. Since contexts of the shape $a(x).C[-]$ are non-discriminating, Propositions 1 and 8 allow to state the following result.

Proposition 9. $\sim_H^B = \approx_H^{BS}$ *(for $\mathcal{R}'_H$).*

We now characterize $\sim_H^B$ via the LTS H (in Fig. 5) by choosing a set of contexts L_H and showing that the barbs in O_H are weak contextual and that L_H is O_H capturing and its contexts are reactive and weakly stable under $\approx_H^{BS}$.

In order to capture the barbs in O_H (defined as $P \downarrow \bar{a}$ iff $P \equiv \overline{a}P_1|P_2$) it suffices to take as set of contexts

$$L_H = \{- \mid a(x).S \text{ s.t. } a \in \mathcal{N},\ S \in \mathcal{P}_H\}.$$

Indeed, by definition of the LTS H, $P \xrightarrow{-|a(x).S}$ iff $P \downarrow \bar{a}$.

Proposition 10. *L_H is O_H-capturing and the barbs in O_H are weak contextual.*

It is easy to note that the contexts in L_H are reactive, so it only remains to prove that they are weakly stable under $\approx_H^{BS}$. To show this, we make use of the predicate in Fig. 7 and of the following lemma.

Lemma 2. *Let $\mathcal{P}^{-|a(x).S}(X, Y)$ be the predicate on* HoCore *processes shown in Fig. 7, for a name a and a process S. Then, for any two processes P and P' we have $\mathcal{P}^{-|a(x).S}(P, P')$ if and only if $P \xrightarrow{-|a(x).S}*_H P'$.*

$\mathcal{P}^{-\mid open\ n.T_1}(X,Y)$	$\exists m \notin fn(X).\ X \mid open\ n.open\ m.T_1 \mid m[\mathbf{0}] \rightsquigarrow^* Y \wedge Y \not\Downarrow_m$

Fig. 6. Predicate for the labels $-|open\ n.T_1$

Proposition 11. *All labels in* L_H *are weak stable under* $\approx_H^{BS}$.

Theorem 3. $\approx_H^{BS} = \sim^{L_H}$ *(for* $\mathbb{H}'_M$*).*

It is interesting to remark that those contexts of the shape $-|\overline{a}Q$ (intuitively corresponding to the labels $a(x)$ in the ordinary LTS in Fig. 4) are not stable under $\approx_H^{BS}$. Indeed, $a(x).\overline{a}x \xrightarrow{-|\overline{a}Q}$ and $\mathbf{0} \not\xrightarrow{-|\overline{a}Q}$, even if $a(x).\overline{a}x \approx_H^{BS} \mathbf{0}$. The latter equivalence can now be formally proved by employing $\sim^{L_H}$: since the context $-|\overline{a}Q$ does not belong to L_H, whenever $a(x).\overline{a}x \xrightarrow{-|\overline{a}Q} \overline{a}Q$, the process $\mathbf{0}$ can reply with $\mathbf{0}|\overline{a}Q \rightsquigarrow^* \mathbf{0}|\overline{a}Q$ and clearly $\overline{a}Q \sim^{L_H} \mathbf{0}|\overline{a}Q$.

$\mathcal{P}^{-\vert a(x).S}(X, Y)$	$\exists i \notin n(X)$ s.t. $X\vert a(x).(S\vert i(x).\mathbf{0})\vert\bar{i}\mathbf{0} \rightsquigarrow^* Y \not\Downarrow_{\bar{i}}$

Fig. 7. Predicates for the labels $-|a(x).S$

7 Conclusions and Future Work

Our paper investigates the notion of weak barbed semantics for RSs. More precisely, it proposes a general approach for the identification of suitable conditions under which weak barbed saturated bisimilarity can be characterized in terms of a behavioural equivalence over a suitably labelled transition system.

Weak barbed saturated bisimilarity generalizes the standard equivalences of calculi such as MAs and HoCore. Indeed, both case studies fall in our framework. In particular, for MAs we proved that our proposal captures its weak reduction barbed congruence, giving an alternative labelled characterization via weak L-semantics. For HoCore, after introducing the weak variant of the strong barbed semantics proposed in the original paper [10], we show that it can be captured via weak L-bisimilarity.

The two case studies also show an interesting problem of our approach that provides guidelines for future research. The LTSs M and H (in Figs. 2 and 5) are infinite, because the environment may provide infinitely many different S, S_1, S_2. Similar problems arise with different approaches (see e.g. [15,16]) and also for even more basic calculi. For instance, for the CCS shown in [6], to an ordinary input transition $\xrightarrow{a}$ correspond infinitely many contextual transitions $\xrightarrow{-|\bar{a}.S}$ and to an output $\xrightarrow{\bar{a}}$ correspond $\xrightarrow{-|a.S}$. However, in this simple case it suffices to take only the contexts where $S = \mathbf{0}$. The case of HoCore is slightly more complex but, similarly to CCS, one can check only few S, as in [10] with *normal bisimulation*. While for MAs, we are not aware of any "finite characterization" of its behavioural equivalence. This is also the case of many new-generation calculi featuring higher-order communication and hierarchical localities, such as the Kell [18]. Devising a general solution to this problem is a challenging task, that we would like to face with *open* reactive systems, in the style of [9].

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