

Self-assembling Tilings of the Whole Plane

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Plan

Definitions

Temperature 1

Temperature 2

Covering the plane

Definition

Let \mathcal{S} be a self-assembling system. \mathcal{S} covers the plane if for any derived supertile a of \mathcal{S} , and any (x, y) , there is a supertile $b \succ a$ such that $(x, y) \in \text{Dom}(b)$.

In a random setting, this means that \mathcal{S} covers the whole plane with probability 1.

Drawing a tiling

Definition

Let \mathcal{S} be a self-assembling system which covers the plane. A pattern $p \in \mathcal{S}^{\mathbb{Z}^2}$ is a limit of \mathcal{S} if there exists a sequence of derived supertiles $(a_n)_{n \in \mathbb{N}}$ with $a_i \rightarrow a_{i+1}$ such that $\bigcup a_i = p$

Given a set of tilings \mathcal{T} , we say that \mathcal{S} assembles \mathcal{T} if there is a tilewise function π such that $\mathcal{T} = \pi(\{\lim \mathcal{S}\})$

The main result

Fact

Temperature 1 assembly yields periodic patterns.

Given a temperature 1 self assembly system \mathcal{S} , there is an equivalence relationship \cong such that the limits of \mathcal{S} are periodic modulo \cong .

The deterministic case

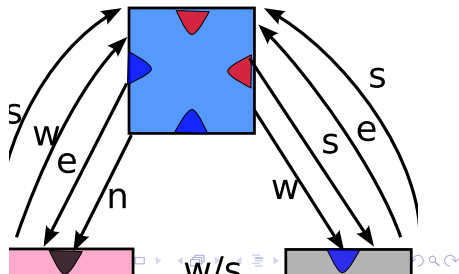
A tiling is *locally deterministic* when for each tile, there is only one possible matching tile on each side.

Theorem

The limit of a locally deterministic is a periodic pattern.

proof

- ▶ Consider the finite automaton A
- ▶ Its output on a path is the tile at the end of the path
- ▶ Apply pumping lemma



The non deterministic case

- ▶ Two tiles are equivalent if they can be exchanged with minimal damages to the tiling.
- ▶ Define an automaton as in the deterministic case.
- ▶ Determinize it: the states now form classes for the equivalence.
- ▶ Hence the patterns are periodic modulo equivalence.

Simulating CAs

Simulating CAs

Quasi periodic patterns: the plan

- ▶ Robinson tiling: historical reasons, easy geometry;
- ▶ Recursive steps with signals;
- ▶ Recursion through timing;
- ▶ Order condition for the analysis.

Order condition

Given a pattern, one can reconstitute the dependencies in the construction.

Definition

there is an order on each pattern whose ideals are the derived supertiles.

Allows for simple, intuitive proofs.

The Signals

Robinson tiling

Scheme of the recursion

Other self-similar patterns ?

- ▶ Non rectangular patterns are tougher
- ▶ Complexity limits ?

The shape of things to come

- ▶ Non assemblable patterns ?
- ▶ Complexity limits ?
- ▶ Synchronization problems (Jordan curves) ?
- ▶ Other Cayley graphs ?
- ▶ Kiitos