Self-assembling Tilings of the Whole Plane

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Plan

Definitions

Temperature 1

Temperature 2
Covering the plane

Definition

Let $S$ be a self-assembling system. $S$ covers the plane if for any derived supertile $a$ of $S$, and any $(x, y)$, there is a supertile $b > a$ such that $(x, y) \in \text{Dom}(b)$.

In a random setting, this means that $S$ covers the whole plane with probability 1.
Definition

Let $S$ be a self-assembling system which covers the plane. A pattern $p \in S^{\mathbb{Z}^2}$ is a limit of $S$ if there exists a sequence of derived supertiles $(a_n)_{n \in \mathbb{N}}$ with $a_i \to a_{i+1}$ such that $\bigcup a_i = p$

Given a set of tilings $\mathcal{T}$, we say that $S$ assembles $\mathcal{T}$ if there is a tilewise function $\pi$ such that $\mathcal{T} = \pi(\{\lim S\})$
The main result

Fact

Temperature 1 assembly yields periodic patterns.

Given a temperature 1 self assembly system $S$, there is an equivalence relationship $\simeq$ such that the limits of $S$ are periodic modulo $\simeq$. 
The deterministic case

A tileset is *locally deterministic* when for each tile, there is only one possible matching tile on each side.

**Theorem**

*The limit of a locally deterministic is a periodic pattern.*

**proof**

- Consider the finite automaton $A$
- Its output on a path is the tile at the end of the path
- Apply pumping lemma
The non deterministic case

- Two tiles are equivalent if they can be exchanged with minimal damages to the tiling.
- Define an automaton as in the deterministic case.
- Determinize it: the states now form classes for the equivalence.
- Hence the patterns are periodic modulo equivalence.
Simulating CAs
Simulating CAs
Quasi periodic patterns: the plan

- Robinson tiling: historical reasons, easy geometry;
- Recursive steps with signals;
- Recursion through timing;
- Order condition for the analysis.
Order condition

Given a pattern, one can reconstitute the dependencies in the construction.

**Definition**

*there is an order on each pattern whose ideals are the derived supertiles.*

Allows for simple, intuitive proofs.
The Signals
Robinson tiling
Scheme of the recursion
Other self-similar patterns?

- Non rectangular patterns are tougher
- Complexity limits?
The shape of things to come

- Non assemblable patterns?
- Complexity limits?
- Synchronization problems (Jordan curves)?
- Other Cayley graphs?
- Kiitos