Cyclic Scheduling Problems for the Synthesis of Digital Signal Processing

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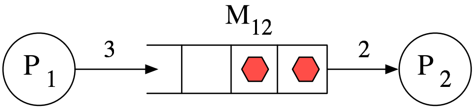
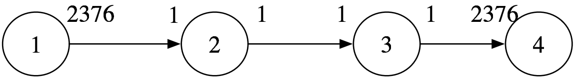
Outline

A buffer between two processes



1. P_1 and P_2 are non synchronous processes continuously executed; l_1 (*resp.* l_2) is the duration of one execution of P_1 (*resp.* P_2) .
2. The buffer M_{12} has exactly one input, one output and a bounded size;
3. At the termination of P_1, 3 data are stored in M_{12};
4. At the beginning of P_2, 2 data are read from M_{12};

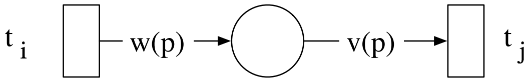
H263 Decoder



Synchronous Data Flow:

1. Circles ≡ processes;
2. bi-valued arcs ≡ buffers.

Generalized Timed Event Graph (GTEG)



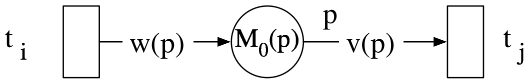
Definition

A Generalized Timed Event Graph is a triple $G = (T, P, l)$

1. T = set of transitions;
2. P = set of places;
3. $l : T \rightarrow$ is the duration function.

Each place has exactly one input and one output transition.

Marked Generalized Timed Event Graph (MTGEG)



Definition

An initial marking is a function $M_0 = P \rightarrow N$ corresponding to the initial number of tokens.
A Marked Generalized Timed Event Graph is a GTEG $G = (T, P, I)$ associated with an initial marking M_0.

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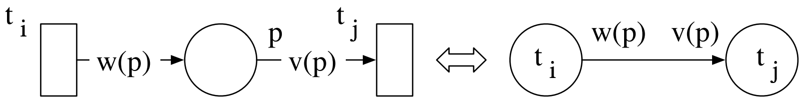
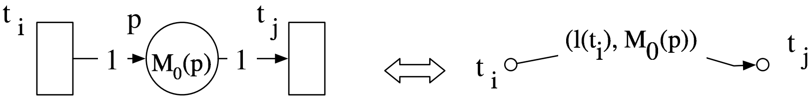
Equivalence between GTEG and SDF



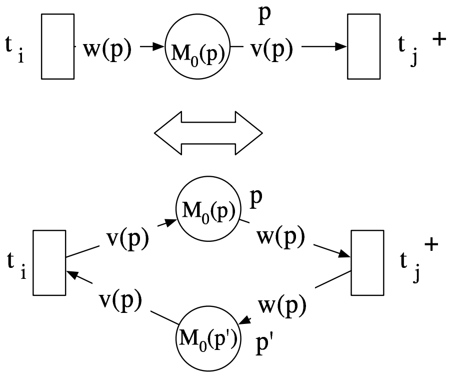
- Transitions $\equiv$ set of processes;
- places $\equiv$ buffers;
- $\forall t \in T, l(t) =$ duration of one execution of t.
- Tokens $\equiv$ data exchanged between processes;

Equivalence between Marked Timed Event Graphs and bi-valued graph



- Transitions $\equiv$ generic tasks;
- place $\equiv$ bi-valued arcs (l_{ij}, h_{ij}) with $l_{ij} = l(t_i)$ and $h_{ij} = M_0(p)$.

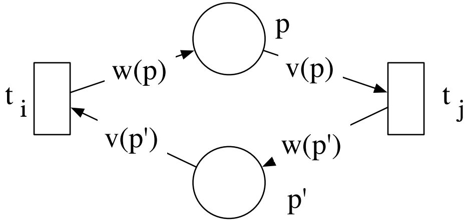
Bounding the size of the buffers



the number of token stored in p is at most k

The initial marking of p and p' verifies $M_0(p)+M_0(p')=k$

Backward places



v(p)=w(p') and w(p)=v(p')

Definition

$p' = (t_j, t_i)$ is a backward place of $p = (t_i, t_j)$ if $v(p) = w(p')$ and $w(p) = v(p')$.

Symmetric GTEG

Definition

A symmetric GTEG is such that every place p has a backward place p'.

For our practical application, the size of the buffers are bounded. So, all the GTEG considered are symmetric.

Liveness

Definition

A MGTEG is live if every transition may be fired infinitely often.

Definition

The weight of a circuit C of a GTEG is $W(c) = \prod_{p \in c} \frac{w(p)}{v(p)}$.

Theorem

If a MGTEG is live, then the weight of every circuit is at least 1.

Unitary GTEG

Definition

A GTEG is unitary if the weight of every circuit is 1.

Theorem

If a symmetric MGTEG G is live, then G is unitary.

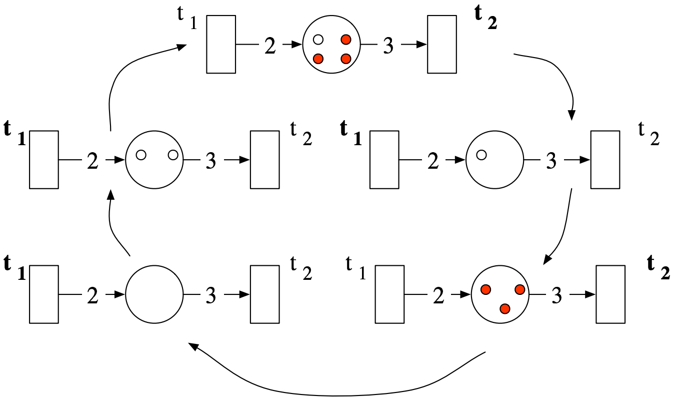
Since the systems considered must be live, we limit our study to unitary symmetric GTEG.

Basic questions

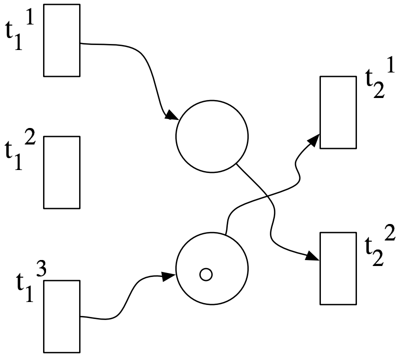
The idea is to minimize the size of the buffers to obtain a given throughput...but the following basic questions are not yet polynomially solved :

1. Is a unitary (symmetric) MGTEG live ?
2. What is the maximum throughput of a marked MGTEG ?

States of a place



Equivalent Timed Event Graph



Precedence constraints induced by a place

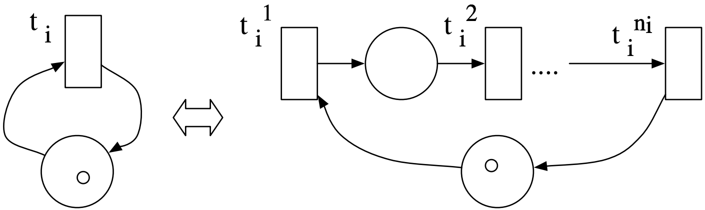
Let $G = (T, P, I, M_0)$ a MGTEG. For any $(t_i, \nu_i) \in T \times N, (t_i, \nu_i)$ is the ν_ith firing of t_i.

Theorem

A place $p = (t_i, t_j)$ will induce a precedence constraint between (t_i, ν_i) and (t_j, ν_j) iff

$$w(p) - M_0(p) > w(p)\nu_i - v(p)\nu_j \geq \max(w(p) - v(p), 0) - M_0(p)$$

Duplicates of a transition



Each transition t_i may be replaced by n_i transitions $t_i^1, \cdots, t_i^{n_i}$;
For any $k \in \{1, \cdots, n_i\}$ and $n \in N^{\star}$, $(t_i^k, n) = (t_i, (n-1)n_i + k)$

A necessary condition on the number of duplicates for a place

Theorem

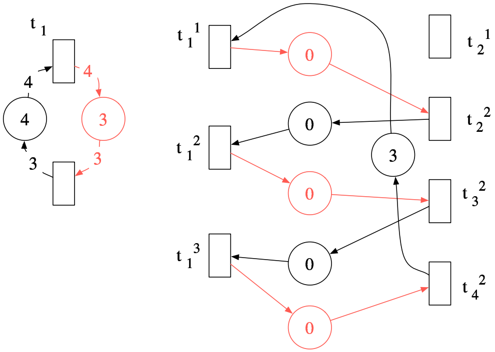
Let $p = (t_i, t_j)$ be a place of a MGTEG. Then p may be expressed using a Timed Event Graphs on the duplicates of t_i and t_j if $\frac{n_i}{v(p)} = \frac{n_j}{w(p)}$.

The condition is sufficient for a place

Theorem

Let $p = (t_i, t_j)$ be a place of a MGTEG. If $\frac{n_i}{v(p)} = \frac{n_j}{w(p)}$, then p may be expressed by a Timed Event Graph using $\min(n_i, n_j)$ places adjacent to duplicates of t_i and t_j.

Expansion of a MGTEG (Example)



Precedences between duplicates are not pictured.

Expansible graph

Definition

A MGTEG is expansible if there exists a vector $(n_1, \cdots, n_{|T|}) \in N^{|T|}$ such that, for every place $p = (t_i, t_j)$, $\frac{n_i}{v(p)} = \frac{n_j}{w(p)}$.

Theorem

Let G be a connected MTGEG. If G is expansible, there exists a minimum vector $(N_1, \cdots, N_{|T|}) \in N^{|T|}$ such that any solution $(n_1, \cdots, n_{|T|})$ of the previous system verifies $(n_1, \cdots, n_{|T|}) = \lambda(N_1, \cdots, N_{|T|})$, $\lambda \in N^\star$.

Expansion of a unitary graph

Lemma

Let G be a strongly connected unitary MTGEG. Then every path between two transitions t_i and t_j has the same weight.

Theorem

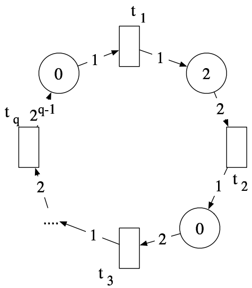
Let G be a strongly connected unitary MTGEG. G is then expansible.

Corollary

Let G be a symmetric MTGEG. G is then expansible.

So, for our practical problem, an expansion may be computed to evaluate the liveness or/and the throughput of our system.

Size of the minimum expansion is not polynomial



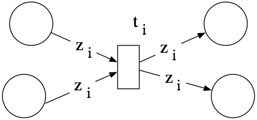
Here, $N_i = 2^{i-1}$ for $i \in \{1, \cdots, p\}$. The number of vertices of the minimum expansion is proportional to $O(2^p)$.
Checking the liveness or/and computing the throughput using the expansion is not polynomial.

Normalized GTEG

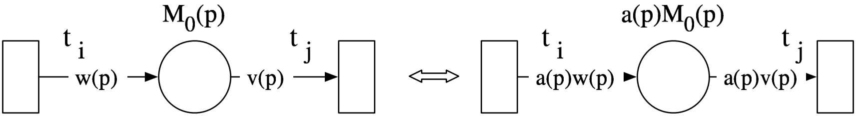
Definition

A GTEG is normalized if, for every transition $t_i \in T$, there exists an integer $z_i \in N^\star$ such that

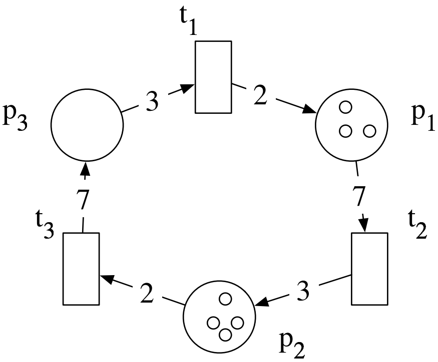
- For every place $p = (t_i, t_j)$, $w(p) = z_i$,
- For every place $p = (t_i, t_j)$, $v(p) = z_i$.



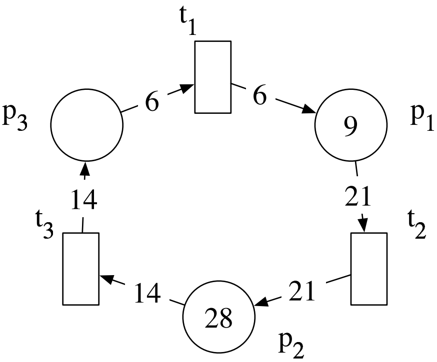
Equivalent places



Normalization of a MGTEG



Equivalent normalized MGTEG



$a(p_1)=3$

$a(p_2)=7$

$a(p_3)=2$

Normalization of a transition $t_i \in T$

Find $a(p) \in N^{\star}$, $p \in P$ and $z_i \in N$,

1. For any place $p = (t_i, t_j) \in P$, $z_i = a(p)w(p)$;
2. For any place $p = (t_j, t_i) \in P$, $z_i = a(p)v(p)$;

Normalization of a symmetric GTEG

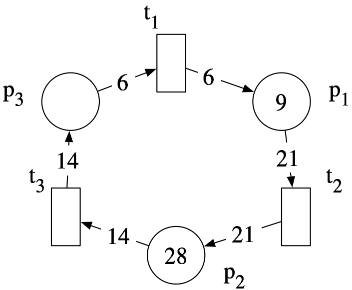
Theorem

If G is a GTEG expansible, then G is normalizable. Moreover, there exists an integer K such that such that $Z_i.N_i = K, \forall t_i \in T$.

Corollary

If G is a symmetric GTEG, then G is normalizable.

A simple remark



If G is a normalized MGEG, the number of tokens is constant in every circuit.

A sufficient condition of liveness

Lemma

We can suppose without loss of generality that the instantaneous marking of any place $p = (t_i, t_j)$ *is a multiple of* $gcd(v(p), w(p))$.

Theorem

Let G be a normalized MGEG. G is live if, for every directed circuit C of G,

$$\sum_{p \in P \cap P} M_0(p) > \sum_{p \in P \cap P} (v(p) - gcd(v(p), w(p)))$$

Circuit of 2 places

Theorem

Let C be a normalized circuit composed by two places p_1 and p_2. C is live iff

$$M_0(p_1) + M_0(p_2) > v(p_1) + v(p_2) - 2gcd(v(p_1), w(p_1))$$

Corollary

The minimum size of the buffer associated with the place $p \in P$ is

$$M^{\star}_{min}(p) = v(p) + w(p) - gcd(v(p), w(p))$$

Problem presentation

G is a symmetric GEG.
The problem is to find an initial marking $M_0 : P \rightarrow N$ such that:

- The marked graph obtained is live;
- The overall marking $\sum_{p \in P} M_0(p)$ is minimum.

Limitation of the initial markings

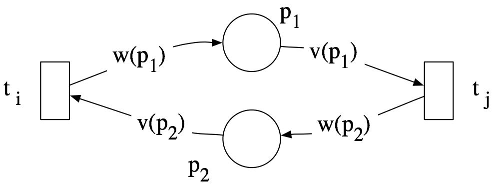
The idea is to consider only $M_0(p) \in \{v(p), v(p) - gcd(v(p), w(p))\}$ such that:

1. For every couple (p_1, p_2) of backward places, $M_0(p_1) + M_0(p_2) = M^{\star}_{min}(p_1)$;
2. Every circuit C of G must verify the sufficient condition of liveness:

$$\sum_{p \in P \cap P} M_0(p) > \sum_{p \in P \cap P} (v(p) - gcd(v(p), w(p)))$$

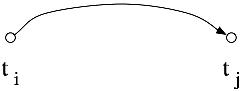
It is equivalent that every circuit of G has at least one place p_1 with $M_0(p_1) = v(p_1)$ and one place p_2 with $M_0(p_2) = v(p_2) - gcd(v(p_2), w(p_2))$.

A (very) simple algorithm

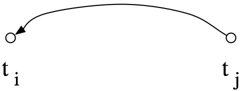


$w(p_1) = v(p_2)$ and $v(p_1) = w(p_2)$

$M\_0(p_1)=v(p_1) - gcd(v(p_1), w(p_1))$
$M\_0(p_2)=v(p_2)$



$M\_0(p_1)=v(p_1)$
$M\_0(p_2)=v(p_2) - gcd(v(p_2), w(p_2))$



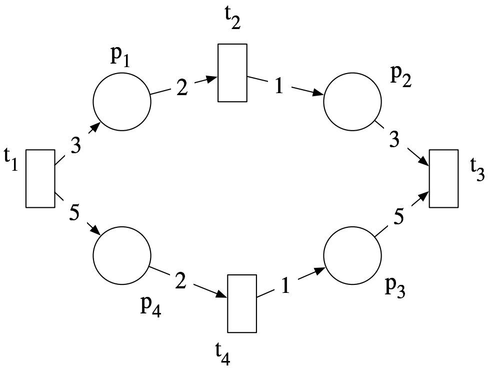
A (very) simple polynomial algorithm

The problem consists then to give an orientation of each couple of backward places such that there is no oriented circuit in the graph obtained.

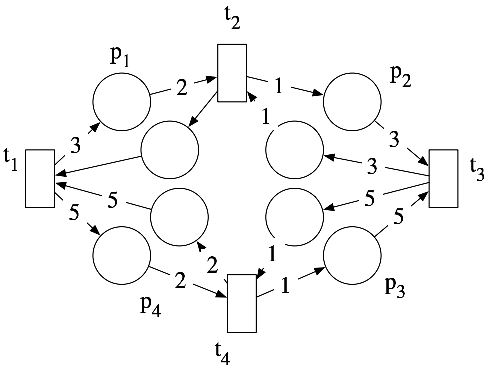
A simple algorithm:

1. Number the transitions of G from 1 to $|T|$.
2. For every couple of (p, p') of backward places with $p = (t_i, t_j)$. If $number(t_i) < number(t_j)$, add an arc (t_i, t_j), otherwise add an arc (t_j, t_i).
3. Compute the initial marking following the arc orientation (see previous slide).

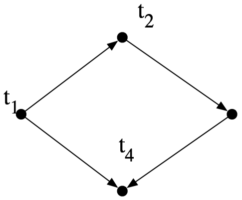
Example

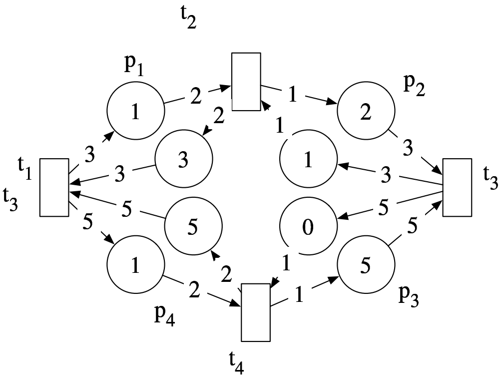


Corresponding symmetric (non nomalized) graph



An initial optimal marking





Conclusion and perspectives

1. Two mathematical tools are available: expansion and normalization.
2. Complexity of the liveness and the determination of the throughput for a unitary GTEG are still open problems.
3. Good feasibility conditions for the liveness and good bounds for the throughput are needed to solve efficiently the optimization problems related.

Alix Munier.
Régime asymptotique optimal d'un graphe d'évènements temporisé généralisé : application à un problème d'assemblage.
RAIRO Automatique Productique Informatique Industrielle, 27:487–513, 1993.

Olivier Marchetti, Alix Munier Kordon
A sufficient condition for the liveness of weighted event graphs.
accepted to European Journal of Operational Research.

Olivier Marchetti, Alix Munier Kordon
Minimizing places storage capacities of a weighted event graph.
Internal report LIP6, submitted.