The internet and the popularization of mathematics

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Abstract. In this paper, "popularization of mathematics" is understood as the attempt to share some of the current mathematical research activity with the general public. I would like to focus on the internet as a powerful tool to achieve this goal. I report on three personal experiences: the making of two animation films available on the web, the participation to a web-journal aimed at a wide audience, and the filming of a 15 minute video clip.

Mathematics Subject Classification (2010). Primary 00A09, 97A80; Secondary 97A40.

Keywords. Popularization of mathematics, Internet.

1. Introduction

Even though the *International Congress of Mathematicians* has been devoting one of its sections to mathematical education for quite some time, the inclusion of "popularization" in its realm is rather recent. Only five talks discussed this topic in previous congresses [11, 20, 21, 22, 25]. Among these contributions, I would like to mention Ian Stewart's article which analyzes in depth the many possible types of media which can be used for popularization. He focuses on magazines, newspapers, books, radio and television but barely mentions the internet. Eight years later, the internet is unavoidable. It has changed our everyday life, be it private or professional. I am convinced that in 2014, the internet should be *the* main tool for the popularization of mathematics and that the mathematical community has the duty of learning how to use this incredible communication instrument. This is not easy and much remains to be done.

I would like to report on three very specific experiences in which I have been involved in recent years: the production of two mathematical films freely available on the web, the creation of a web-based journal aimed at a wide audience and the recording of a very short clip for the web. My intention is to illustrate some of the difficulties that mathematicians can encounter in these kinds of ventures and to propose possible improvements.

This paper is not an attempt to describe in a systematic way all the issues related to mathematics and the internet. My only purpose is to give an account of a very personal experience.

It is a pleasure to thank Jos Leys and Aurélien Alvarez for their collaboration, as well as all the members of the editorial board of *Images des Mathématiques*. I also thank Marie Lhuissier for her very helpful comments.

2. Why popularization?

Amazingly, most articles related to popularization begin with a section trying to explain why this is a honorable occupation. A similar section in a paper dedicated to geometry or topology, for example, would seem inappropriate in the proceedings of the ICM. It is a fact that most of our colleagues are not convinced that popularization is a respectable mathematical activity. There is a need for justification.

My first comment would be that preparing any kind of "popular" presentation is a real challenge, and very frequently forces you to understand much better the topic you want to present: a profitable investment for mathematicians! In [13] Sir Christopher Zeeman explained that after delivering his *Christmas lectures* in 1978 [24], he received a message from the chairman of the British *Science Research Council* who "tickled him off for wasting his time popularizing on TV instead of doing research". Zeeman answered that these lectures had in fact inspired a research paper in dynamical systems.

Let me quote David Hilbert in the introduction of his famous lecture in Paris during the ICM 1900[12][1].

> "A mathematical theory is not to be considered complete until you made it so clear that you can explain it to the man you meet on the street. For what is clear and easily comprehended attracts and the complicated repels us."

Moreover, again from an egocentric point of view, popularization (like teaching) is highly rewarding for the working mathematician. A typical research paper has a few dozens readers (in favorable cases) and this can be frustrating, but a good popular paper can easily be read by thousands of web-users.

At the wider level of mathematics as a whole, Lásló Lovász explains clearly the importance of communication [19]:

> "A larger structure is never just a scaled-up version of the smaller. In larger and more complex animals an increasingly large fraction of the body is devoted to "overhead": the transportation of material and the coordination of the function of various parts. In larger and more complex societies an increasingly large fraction of the resources is devoted to non-productive activities like transportation information processing,

[1] As a matter of fact, Hilbert quotes "un mathématicien français du temps passé" who seems to be Gergonne.

> education or recreation. We have to realize and accept that a larger and larger part of our mathematical activity will be devoted to communication."

Note that this comment primarily applies inside mathematics, with all its subcultures in danger of blowing up into many disconnected components. But it also applies to the communication from inside to outside mathematics, which is the subject of popularization. The ever expanding mathematical body requires more elaborate and stronger links to remain connected to the society at large.

In the same paper, Lovász asks for a special training of our students:

> "While full recognition of expository work is still lacking, the importance of it is more and more accepted. On the other hand, mathematics education does little to prepare students for this. Mathematics is a notoriously difficult subject to talk about to outsiders (including even scientists). I feel that much more effort is needed to teach students at all levels how to give presentations, or write about mathematics they learned. (One difficulty may be that we know little about the criteria for a good mathematical survey)."

From another point of view, the necessity of popularizing mathematics is a direct consequence of the significant decrease of the number of math students, or more generally of scientific students: it is therefore a matter of survival for our discipline. It is our duty to explain to the young generation why mathematics is so beautiful and gratifying, and can be a wonderful option for their careers. A few decades ago, the prestige of science in society was much higher and there was some kind of natural flux coming into mathematics.

Of course, one should emphasize that popularizing mathematics does not only consist in advertising academic careers and in producing more research mathematicians! There is also an obvious utilitarian economic issue since our contemporary world needs more scientists and therefore more mathematicians. If we want more engineers, scientists and mathematicians, we need a general population which is at least aware of the existence of mathematicians. A significant part of the population is indeed convinced that there is nothing more to do in mathematics, and that the field has been closed since the ancient Greeks. Somehow, the most important goal of popularization is not necessarily to convey a specific mathematical content, but to explain that math/science could be a real option for themselves, or for their kids, or at least to show that it is a respectable activity, useful for society at large. More than two thousand years ago, Archimedes wrote *Sand-Reckoner* as a letter to his powerful king. That was a way of expressing the necessity of science for his kingdom. Today, we do not care about kings, but taxpayers want to understand what we are doing with their money and they deserve candid answers.

One should of course not forget the cultural aspect of mathematics, so obvious for professional mathematicians and so unknown to the general population. We have to explain that it is important for the "man on the street" to have some taste for mathematics (or science in general) in the same way as, for instance, it is important to enjoy the arts. Such a taste is not necessarily related to the "usefulness" of mathematics, say for economics or engineering sciences, and does not require a deep understanding of technical details. One should make clear that mathematics can be fun and interesting to everybody, just as literature can be enjoyed at many levels.

The choice of popularizing science is clearly a political and democratic issue. As a historical example, in 1841 François Arago, then director of the Paris observatory, built a large lecture hall in the heart of the main building, entirely dedicated to his weekly lectures on "Popular Astronomy". These lectures, aimed at the general public, were indeed a great popular success (see the marvelous notes [2]). His successor, Urbain Leverrier, decided to transform the observatory into an efficient modern laboratory, fully devoted to research. He demolished Arago's lecture hall[2]:

> "The amphitheater is and will remain purposeless. The Observatory should not compete with the organizations of public instruction located in the very center of Paris, which suffice for their task. An institution which is requested to work at the progress of science [...] must look for the most absolute tranquility" [17].

Two great scientists and two different approaches to the relationship between science and society.

For more on this topic, including a discussion on the history of popularization, I refer to [13].

3. The specificity of the internet

Of course, mathematics is already present at many levels on the internet. One finds thousands of blogs, some of them very popular among... professional mathematicians (for example Tao's blog) but most are not related to popularization. One also finds many websites of teachers sharing their enthusiasm for mathematics but they are usually connected to education and not to mathematical research. The Khan Academy provides a fantastic example of an internet access to education: it contains thousands a short clips covering mathematics from elementary to high school (and even some calculus). Wikipedia is an incredible success in general, and in mathematics in particular, but one should probably not qualify it as popularization. I would like to restrict myself here to websites dedicated to the presentation of some current mathematical developments to the general population (and therefore not aimed at professional mathematicians). Even with this restriction, one finds hundreds of websites, from individual blogs (for instance www.science4all.org) to institutional ones (among many more examples accromath.uqam.ca, plus.maths.org, maddmaths.simai.eu, interstices.info). Many institutions have subsections of their home pages devoted to outreach (for example, www.simonsfoundation.org).

[2] and turned it into a private apartment for his personal use!

The internet is an incredible jungle. Unlike mathematical papers or books, which are more or less built on similar structures, there is no unity on the web. The first mistake would be to try to export our professional habits and to produce webpages which look like mathematical books, with theorems and lemmas. A new tool should not be used to do what we have been doing for many years, even if we can do it faster or more easily : it should instead be used to do something new and more efficient.

Pictures, movies, music, podcasts or apps provide innovative and fascinating instruments to communicate mathematics, in a way which is very different from traditional texts. It is not the purpose of this paper to discuss the potential use of these new tools in research but I mention for instance that some online mathematical journals include short videos by the authors, presenting their own papers[3].

In the domain of popularization, the possibilities are infinite and are still to be explored. As an example, one could easily break the traditional ordering in a mathematical text and let the reader-viewer-listener[4] choose his/her own trajectory inside a rich network of possibilities, according to his/her own background or taste, making him/her more of an actor than a passive reader. This may be the most important paradigmatic shift implied by the internet : from information organized in totally ordered lists to information located in a network. One could almost say that the information is not located on specific places but *coincides* with the network as a whole. A graph is much more than its vertices.

One should realize that when we surf the internet, we hop from webpage to webpage and usually spend a very short time on a given page. The typical "bounce rate" of a website is about 1/2: after viewing the entry page, half of the visitors immediately go somewhere else. Also, web-users do not read linearly, from top to bottom. One could argue that similar facts also apply to mathematical books or papers and that nowadays most of us "read" dozens of preprints at the same time, hopping from theorem to theorem, in the hope of finding something that could be useful for our research. However, the two hopping styles are very different. We should study and understand much better this new reading style on the web, closer to a random walk in a graph than to a motionless scholar reading in a library.

A related aspect of the internet, which is *a priori* in contradiction with the spirit of mathematical research, is its incredible speed and reactivity. Mathematicians usually spend months (or years) writing papers which will be read by a handful of people while web-users spend a few minutes posting tags with an improbable spelling on their *Facebook Wall*. Clearly these are two different communication modes and we should be able to switch from one to the other, keeping in mind their

[3] Could a movie be considered as a bona fide proof of a theorem? Hilbert discusses the status of a picture: "The use of geometrical signs as a means of strict proof presupposes the exact knowledge and complete mastery of the axioms which underlie those figures; and in order that these geometrical figures may be incorporated in the general treasure of mathematical signs, there is necessary a rigorous axiomatic investigation of their conceptual content" [12]. For instance, the movie *Outside In* is very close to an actual proof of Smale's inversion theorem [18].

[4] The internet does not give access to smell, taste and touch... so far!

advantages and drawbacks. Inside the realm of mathematical research, nobody would deny the fundamental importance of long, difficult and carefully written papers. This requires time and is not compatible with "speed science". At some other moments, the researcher needs a quick answer to a specific question and he or she can frequently get immediate answers from MathOverflow : the "blog" style is efficient in these cases. The subject of speed in mathematical research today would require a specific discussion but is outside the scope of this paper.

Is "speed science" compatible with popularization? Does it make sense for graduate students to participate in tournaments like the Three Minute Thesis competition? Even though most of us are reluctant to work at such a speed and look for peace, the answer to these questions has to be yes, if we do not want to lose contact with the younger generation. More importantly, in many cases (but nor all), I believe that a good popularization can be speedy, especially when the expected public has no connection at all with mathematics.

Another important aspect that makes the internet different is related to the validation problem. Everything can be posted on the internet, the best and the worst. No "referees" are present to prevent mistakes. Very often the general public would like to get some kind of certification that the content of a webpage is valid. This should be the role of mathematicians and we have to be creative in this respect. Can we trust the "wisdom of the crowds" and promote some verification in which everyone is encouraged to participate, in the spirit of *Wikipedia*? On the contrary, should we "export" some of our traditional refereeing methods based on anonymity?

The internet is the kingdom of wild plagiarism. It is amazing to see how a given text can travel from place to place, often subject to various "simplifications" or "additions", frequently with no mention of the original author. Mathematicians should understand that it is in some sense a great honor that their contributions are "duplicated" in many places. Of course, ideally, this should be done under the control of the author, but it is much better to accept it as a rule of the game. Trying to prevent this natural diffusion would be fighting a rearguard battle.

All these apparent drawbacks should be seen positively as powerful new opportunities. The ability to get information on almost any aspect of knowledge within a few clicks is of course a revolution. Older mathematicians remember their endless searches in libraries, going through the many (paper) volumes of *Mathematical Reviews*. Today, the published mathematical literature is easily available[5] and arxiv.org provides access to preprints in real time. This high connectivity did not only change the everyday life of researchers. Amateurs surfing the web can now find quickly all kinds of information, for example on popular mathematics... if we know how to create easily accessible quality websites.

In a nutshell, the internet is working in a way which may not always look compatible with our tradition. We have to adapt and to learn how to play this new wonderful instrument.

[5] I don't comment here on the price of mathematical journals.

4. First example: *Dimensions* and *Chaos*

4.1. Genesis of the project. In 2006, as I was preparing slides for a general public talk [6], I wanted to use some mathematical images that I liked on the website www.josleys.com. I therefore asked the webmaster for permission to use them. After my talk, I thanked him and asked for more information concerning his website. Jos Leys is a mechanical engineer who recently retired from a major chemical company. "At last, I can do mathematics!", he told me... Jos' mathematical background is typical for an engineer trained forty years ago: he had mastered pretty well classical analytic and differential geometry, but of course has no idea of contemporary mathematics at a research level. However, he has been interested in fractal geometry and computers since the early 80's. He genuinely loves mathematics. An article in *Pour la Science* portrayed him as an artist-geometer. At the same time, I was preparing a plenary lecture for ICM 2006 and my intention was to present, among other things, a result connecting periodic orbits in the Lorenz attractor and closed geodesics on the modular surface [7]. To my mind, this was a very visual theorem, but I did not know how to transform in practice my imprecise mental images into actual images. I therefore asked Jos for help in producing pictures. We did produce beautiful pictures, some of them being rather intricate, in particular those related to modular forms. Quickly, we realized that in order to explain ideas from dynamical systems, it was in fact best to use pictures in motion: movies! I was quite satisfied with the result and about one third of my talk in Madrid turned out to consist of movies. After the talk, Jos told me: "Now you have to explain to *me* the meaning of the movies *I* prepared with you". I was facing Hilbert's challenge: to make it so clear that you can explain it to the man you meet on the internet.

We first wrote some kind of "visual article", including movies, that we published in the web *Feature column* of the AMS [10]. However, this was not aimed at a "popular level" and Jos wanted something much more elementary. For instance, it was not possible to use complex numbers without explaining what they are... We therefore decided to produce a fully fledged film from scratch, starting at a very elementary level and, hopefully, going to our target: periodic orbits of the Lorenz attractor and closed geodesics on the modular surface. We were very optimistic but we quickly realized that it was not realistic in a single film. Soon, Aurélien Alvarez, who was at the time a graduate student, joined our team. So far, we "only" have produced parts 1 and 2, each two hours long, of a saga which could very well turn out to be infinitely long.

Part 1 is entitled "*Dimensions*". Its main purpose is to provide an introduction to dimension 4. More precisely, it gives a presentation of the 3-sphere inside 4-space and of the Hopf fibration.

Part 2 is entitled "*Chaos*". It is an elementary introduction to dynamical systems. The final chapters try to give a very rough idea of current conjectures on the statistical theory of strange attractors, like the Lorenz butterfly.

We are still far away from the modular surface and its geodesics!

4.2. The making of *Dimensions*. *The first decision* was to produce a film that would be split into "chapters", each being 13 minutes long (which is some kind of time unit in the video world). These chapters had to be as independent of each other as possible, and the mathematical level had to be increasing. Chapter 1 should be understandable by young children and the final chapters by undergraduates. The main idea was to propose to the spectator some kind of menu in which (s)he can select what (s)he wants. Some would only look at the first two chapters, others would only look at the last two and some would only look at chapters 5 and 6, for instance. Of course, this necessitated the careful writing of a scenario, in such a way that the many subsets look (and are) coherent. It would be frustrating for a spectator to see a film which leads him/her to a final chapter which is not understandable to him/her.

Here is the structure of the first movie *Dimensions*.

- *Chapter 1 (dimension two)* is very elementary. It contains the description of the 2-sphere in space, with its parallels and meridians, and shows the stereographic projection.
- *Chapter 2 (dimension three)* is still elementary and is based on the famous popular novel *Flatland* [1].
- *Chapters 3 and 4* get into the fourth dimension. They rely heavily on regular polytopes in dimension 4, seen as drawn on the 3-sphere, and then projected stereographically on 3-space (and then on the 2-dimensional computer screen).
- *Chapters 5 and 6 (complex numbers)* contain a visual introduction to complex numbers. These chapters are completely independent from the others and have been used quite a lot in classrooms.
- *Chapters 7 and 8 (Hopf fibration)* are the hardest parts. We show the linking of Hopf circles and the wonderful Villarceau circles on tori of revolution.
- *Chapter 9 (proof)* is special. It contains the complete proof that the stereographic projection maps circles to circles (or straight lines). This proof uses nothing above the level of secondary school, and we could very well have put this chapter right after chapter 1. We wanted to explain that mathematics is above all a matter of proofs, not only pictures.

For example, we propose the following combinations of chapters: Junior High School (1 or 1-2 or 1-2-9), High School (1-2-3-4-9, or 5-6), Undergraduates (2-3-4-5-6 or 5-6-(7-8-9)), College (7-8), General public (1-2-3-4).

The second decision was to tell a story. Each chapter is "presented" by a famous mathematician, from Hipparchus (for chapter 1), to Heinz Hopf (describing his fibration), along with Adrien Douady (explaining complex numbers). It is well known that the rich and long history of mathematics is a powerful vector for popularization. Naturally, the scenario is not written as a course, in any sense of the term. For instance, our presentation of complex numbers is not intended as a substitute to some kind of tutorial. Many teachers have used it in their classes as a complement or sometimes as an introduction. We explain the general idea of complex numbers, we show their geometric meaning (which unfortunately disappeared from many high school curricula), we deform (conformally!) the portrait of Douady, and we finally illustrate these notions with beautiful pictures of the Mandelbrot set. We try to be precise but never formal. The commentaries and the images are of course supposed to be understandable but we are aware of the fact that some spectators get lost along the way. In this (unwanted but likely) case, the film should be attractive enough to keep the attention.

Technically, *Dimensions* is an animation movie. Most of the 185 000 images have been produced using the (free) software PovRay. This is of course a huge amount of work. *Dimensions* was released in 2008, after 18 months of elaboration.

We quickly realized that many fellow mathematicians were happy to help, in many ways. For instance, we could provide subtitles in 20 languages and soundtracks in 8 spoken languages. The concept of mathematical community is not an abstraction!

We also developed a website www.dimensions-math.org (also in many languages), giving extra information and references.

4.3. The economic model. We believe that *mathematical popularization should be freely accessible on the web*. We therefore decided that all movies could be freely downloaded on our website, under a Creative Commons licence. As a result, we were happy to see that the movies quickly could diffuse all over the web, primarily on *YouTube*.

We also produced a DVD that is sold on the website at a nominal price. This is a non profit activity and all benefits are immediately "invested" to offer DVDs to some organizations (like for instance the *International Mathematical Olympiads*, or *MathEnJeans*, etc.).

4.4. *Chaos*. Our second movie *Chaos* was released in January 2013 and is based on the very same model. We tell the story of dynamical systems, going slowly from periodic motions and limit cycles to chaotic examples, including Smale's horseshoe and the Lorenz attractor.

- *Chapter 1 (Motion and determinism)* is a non technical preview of the whole story, explaining determinism, sensitivity to initial conditions, and giving a hint that one could understand chaotic systems through statistical methods.
- *Chapters 2 and 3 (Vector fields, and Mechanics)* are very basic and can be used in the classroom: they give a very quick introduction to velocity, acceleration and forces. They are independent from the other chapters.
- *Chapter 4 (Oscillations)* gives an introduction to limit cycles.

- *Chapters 5, 6, 7 (Billiards, Horseshoe, Lorenz butterfly)* describe three historical examples of chaotic behavior.
- *Chapters 8, 9 (Statistics, Chaotic or not?)* introduce to the concept of physical measure (Sinai-Ruelle-Bowen) in a very intuitive way and to the general conjecture of Palis describing the statistical behavior of a typical dynamical system.

We could benefit from help not only from friends in the mathematical community all around the world, but also from a famous French actor and Brazilian singer[6], who dubbed the commentaries!

4.5. Assessment. Of course, I would not report on these movies if I were not convinced that this turned out to be a success. It is difficult to quantify the number of viewers or even of downloads. The website *Dimensions* has five mirrors (in Beijing, Mexico, New York, Rio and Tokyo) and the only objective data is that they had more than two million unique visitors, from *all* countries in the world. Obviously, none of my previous productions has been so widely distributed and it was a real pleasure for us to receive congratulations from kids in the middle of China.

We received thousands of emails thanking us for our work, and asking for more. It is not easy to get some clear view of our audience from these emails since their diversities is very impressive, from very young children to people seing improbable connections between the fourth dimension and spirituality... Nevertheless, one could say that many viewers are amateurs in a way or another. They probably found on the web the popular mathematics that they were looking for.

Did we only reach amateurs who were already convinced? We did not have clearly in mind this "target" when we started the project. Clearly, amateurs should not be neglected and one should carefully analyze their requirements. However, the public of those who have no connection at all with mathematics is probably more important and requires a specific approach, with a much weaker mathematical content.

As for the DVD's, we produced 20 000 copies which have been either sold or offered. I am convinced that our choice of *Creative Commons* was the right decision and that no other economic option would have generated such a diffusion for mathematical movies. According to a private publisher that we have contacted at the beginning of the project, there is no market for this kind of film.

From the non positive side, it is clear that a two hour film entirely produced by three persons, with no budget, cannot be compared with a *Pixar* production. Obviously, it is the work of amateurs, with many drawbacks, especially related to the rhythm, which is sometimes too slow. Another difficulty is that we should have planned the scenario and the storyboard in their smallest details before starting the production of the first chapters. It is unclear whether it would have been more

[6] Thierry Lhermitte and Thalma de Freitas.

efficient to develop a much more expensive project and to involve professionals: this would have implied too much of a burden and would have hidden what drives much of us: the fun of doing mathematics.

A successful aspect of the films is the splitting into individual chapters which are more or less independent and can be combined in many possible paths, depending on the viewer. This has been appreciated. However, we have to admit that we did not use the full flexibility of internet. It would have probably been more efficient to produce something more interactive, in which the web-user could make more choices, in the spirit of video games. Of course, this would have been technically much more difficult, probably beyond our capabilities.

One could probably assert that *Dimensions* and *Chaos* deal with mathematics which are easy to popularize: topology, geometry and dynamics. It would be clearly more difficult to produce a film on algebra, number theory or modern algebraic geometry. In these cases, one should choose other internet tools. Even so, it is possible that some domains cannot be shared with the general population. However, this may not be a serious problem. Many aspects of astronomy for instance are too technical to be presented to a wide audience, but astrophysicists have enough beautiful pictures or fascinating stories to popularize their discipline in an exceptional way.

5. Second example: *Images des Mathématiques*

5.1. Genesis of the project. In the 1980's, the French *Centre National de la Recherche Scientifique* (CNRS) decided to publish, once every two years, a volume entitled *Images des Mathématiques* (IdM for short). The idea was to include a dozen articles giving some illustration of recent mathematical progress. The target of this booklet was not clearly defined but instructions were given to the authors that they should not write for their colleagues. A small number of issues appeared but the publication stopped very quickly. This publication was expensive, the published articles were in practice only readable by colleagues, and the 7 000 copies were very badly distributed.

In 2004 and 2006, Jacques Istas and myself edited two more volumes... with the same weaknesses. We realized that many of the printed copies did not go out of the strict circle of mathematical researchers and even that many were not opened at all... Even worse, most articles were not understandable by mathematicians from outside the field of the author. This was a waste of money and energy.

We decided to create a web journal, still hosted by the CNRS, with the same title, dedicated to explaining current mathematical research outside of the circle of research mathematicians, if possible to Hilbert's "man on the street". The main idea was to ask for the help of many colleagues and to create a large editorial board. This would provide an analogue of a daily newspaper, giving "news from the mathematical community" as often as possible, ideally daily... Five years after the opening, in January 2009, about 2000 articles have been published (see below).

Of course, this initiative is not isolated. In 2008, IMU and ICMI commissioned a project to revisit the intent of Felix Klein when he wrote “Elementary Mathematics from an Advanced Standpoint” one hundred years earlier [14]. As explained by the Klein committee: “The aim is to produce a book for upper secondary teachers that communicates the breadth and vitality of the research discipline of mathematics and connects it to the senior secondary school curriculum. The 300-page book, prepared in more than 10 languages, will be written to inspire teachers to present to their students a more informed picture of the growing and interconnected field represented by the mathematical sciences in today’s world. We expect this will be backed up by web, print, and DVD resources.” See the website blog.kleinproject. As one can see, the expected audience of IdM is slightly different since the Klein project is written for teachers. Moreover, the Klein project is more thought as a data base than as a magazine giving information at a continuous pace.

5.2. Structure of IdM. IdM is organized like any research mathematical journal. The editorial board consists of about twenty mathematicians, each being in charge of some section of the journal (see this page). In turn, each section has its own sub-committee taking all editorial decisions relative to this section. The union of the editorial board and all sub-committees contains about sixty colleagues. As examples of sections: history, conjectures, current research, press review etc.

IdM publishes two kinds of contributions, *articles* and *columns*.

Articles are close to research papers in the sense that they are evaluated in a process which is similar to the standard refereeing system. When an article is submitted for publication (authors are almost always invited to contribute by a member of the board), it is deposited on a private page. A few hundred volunteers have agreed to read and comment papers before publication. A dozen of these volunteers are selected for each submitted article and they have access to the private page containing the draft of the paper. Typically, one half of these “referees” are professional mathematicians. These referees can comment the paper in a forum accessible to the author, to the other referees, and to the editors. Note in particular that the referees are not anonymous, even though some of them are only identified through a pseudonym. The process of evaluation then takes the form of a “conversation”, through this forum, between the author and the referees, and this implies a continuous change of the text. When the editor in charge considers that the paper is ready, it can be published. Typically, this process takes about two months. About one thousand such articles have been published in the last five years.

Most articles are original and have been written for IdM. The few exceptions are related to some partnerships with some other journals, agreeing to share some papers. I mentioned earlier the “plagiarism” question. Many blogs do not hesitate to copy parts of articles published elsewhere. Of course, one should criticize this behavior if the original author is not mentioned. However, I am in favor of the idea that a given article might be published in different places, in different forms, for different publics, preferably with the agreement and participation of the author.

Columns are much shorter and usually with much lighter mathematical content. This is somehow the blog part of IdM. A certain number of colleagues have agreed to be columnists and they are encouraged to publish short contributions, of course related to mathematics, but typically from a different point of view. This could be for instance a political opinion, or the review of a book, of a movie, or even a joke... Of course, these columns are not refereed but a small team checks them before their (quick) publication. IdM has now published about one thousand of these columns.

The question of the nature of the public is of course fundamental. IdM is in principle aimed at the general public but clearly a significant part of our readers *are* mathematicians. Many are teachers or students, or have some relationship with mathematics, so that they are mathematicians in some way or another. One of the main difficulties is to ignore research mathematicians, since IdM is not for them! The idea would be to propose something widely accessible (to French readers) but it is of course impossible to write texts which are suitable for *everybody*. We adopted a code inspired by the ski slopes rating colors, from the easy green slope to the black one, and even off-piste. The green slope requires in principle no knowledge in mathematics.

From the financial point of view, IdM is almost cost-free and receives a modest support from CNRS.

5.3. Assessment of IdM. The audience of IdM (as measured with *Google Analytics*) has been steadily increasing since the opening of IdM (with a quasi-periodic modulation, related to weekends, vacations etc.). Today, IdM receives about 4000 visitors a day. This is much less than what we would expect but one should keep in mind that this web journal is only available to French speaking readers (although the project of translating into Spanish is on schedule).

The main difficulty encountered by IdM is to find authors. As a rule, authors are mathematicians and not journalists. Most of our colleagues are under a publication pressure for their own career and, unfortunately, this kind of article is not yet considered valuable enough to be included in their publication list. A possible improvement, giving value to these popularization articles, would be to include them in databases, like *MathSciNet* or *Zentralblatt*[7]. Indeed, from my own experience, the refereeing process in IdM is far more advanced than in most "standard" research journals.

Moreover, potential authors quickly realize that writing such articles is far from easy and requires a lot of work. More often than not, they have great difficulties in understanding that most of the words that they use daily are simply not in the vocabulary of the potential readers. Most mathematicians have a totally wrong idea of the mathematical knowledge of the general population. It is clearly difficult to explain a recent mathematical idea to "the man you meet on the street" and even

[7] As of today, the administrators of these two databases have not answered our proposal for reviewing articles from IdM.

sometimes it may be impossible. The main comment from non-mathematicians about articles from IdM is: "too complex and too long". Our community has to train students in this kind of exercise and this should be included in university curricula. Somehow, one could think of IdM as some kind of laboratory where we practice and improve our ability to write such papers.

One could reasonably question the fact that the authors of IdM are not journalists. Of course, journalists usually know their readers much better than mathematicians do. However, they (usually) do not know mathematics as we know it, from inside. I am convinced that the popularization of mathematics should not be *entirely* delegated to journalists. It is the duty of mathematicians to spread mathematics in the general public. See the article by M. Emmer on the relationship journalists-mathematicians, in [13].

The "semi-public" refereeing system works rather well. As described above, it involves a dozen volunteers for each article who share with the author a private forum. Almost always, the published paper is significantly different from its original version. Professional mathematicians are used to the "dry style" of referees reports. Sometimes, comments from professionals on articles submitted to IdM are expressed so strongly that the non professionals are impressed and hesitate to give their own opinion and remain silent. Usually, non professionals would like to say "I don't understand" and professionals "You forgot to add such and such theorems". As for the authors it is not uncommon that they have difficulties accepting comments on their papers by "referees" who are not experts, even though they represent a good sample of their readers.

Of course visitors are welcome to add comments at the end of articles, after publication. However, we noticed some rather surprising behavior on the part of the readers. Many hesitate a lot before posting a comment by some kind of self censorship. They seem to be "impressed" by the expertise of some authors.

We conducted a survey to get a better understanding of our readers. As we could imagine, a significant minority of our visitors consists of researchers in mathematics. A majority are teachers or students. We still do not reach the very young. Clearly the articles are too long and too difficult. Sadly, it should be noted that 80% of our visitors are male.

Another difficulty is related to the navigation inside IdM. We should use all the possibilities of the internet in order to propose multiple choices to our readers. Unfortunately, most visitors do not understand that behind the home page, there is a large data base of articles. We need keywords, tags and all sorts of modern navigation tools. A web designer is currently analyzing the structure of the "back office" of IdM and will propose solutions. This has of course a cost.

Even though there is still a lot of progress to be made, collaborating with the editorial board of IdM is a challenging and exciting experience.

6. Third example : popular lectures, *les Ernest*

The idea of popular science lectures is certainly not new. For instance, in 1825 Michael Faraday inaugurated the Royal Institution Christmas Lectures aimed at a "juvenile auditory". Since 1967, they are broadcast on the BBC television network and they are very successful. One had to wait until 1978 before one of these series could be dedicated to mathematics (by Christopher Zeeman [24] and Marcus du Sautoy in 2006 [4]).

Nowadays, it has become fashionable for many mathematics departments or institutions to organize popular lectures. It is even common to include them in the program of scientific meetings, including the ICM. The main problem, not always understood by the organizers, is to define the public as clearly as possible and to make sure that it comes! It is impossible for the speaker to prepare a lecture if he or she does not know whether the audience will be "juvenile" or "retired" or consisting of professional mathematicians. All these publics are interesting but very different... Suppose for example that the speaker plans to explain that $\sqrt{2}$ is irrational and discovers that all spectators have a PhD in mathematics. I have personally had several bad experiences of this kind that I will not describe here.

It has also become usual to film these lectures and to post them on the internet. In many cases, the result is a disaster. As explained earlier, the internet is not a new tool for doing what we have been doing for many years. A mathematical lecture filmed with one fixed camera, with no film editing, can be very useful for research mathematics but is certainly not adapted to a popular presentation of mathematics. One problem is the length. Frequently, a live lecture in front of an active public can last one hour and still be a great success. The same lecture posted on the internet will have a very different reception. The web-viewer can (and probably will) hop to some other place with one click. Looking at a static blackboard on a screen quickly becomes boring unless this is a technical research talk and you are really interested in a proof.

One of the standard mistakes from the organizers is to inform the speaker that his/her talk will be recorded one second before the start of the lecture. Theater and cinema are certainly different activities.

For the internet, it is fundamental to enable the spectator to see many different aspects of the lecture. There should be a subtle balance between views of the speaker, of his/her slides, and of the public in the room. This implies a serious editing of the film and a competent technical staff. Everything should be prepared well in advance, in coordination with the speaker.

I would like to report on two personal examples that were quite successful. I gave a public lecture in 2010 in Paris, on the occasion of the Clay Conference in honor of the proof of the Poincaré conjecture [8]. The conditions were optimal: the wonderful amphitheater of the Institut d'Océanographie, a public of high school students (and some distinguished colleagues on the first row), and above all the very professional editing by François Tisseyre, who has a long experience in filming mathematics (see for instance [3]). However, even though the editing seems to me very good, I do not think that the video is adapted to the internet: too long and not directly intended *for* the web.

Les Ernest is an association of young students from the École Normale Supérieure of Paris[8]. They understood that the internet is not just a way of broadcasting standard lectures.

> "One ambition : to offer a format for lectures adapted to the new media. [...] Knowledge should be shared democratically. More than ever, new approaches, frequently interdisciplinary, are necessary to understand our world. Usual lectures are not compatible with the internet code."

Les Ernest are producing films which are very short : 15 minutes. They cover all kinds of subjects, but they seem to have hesitated to include a lecture on mathematics, since I recorded the first one (after a computer scientist) in 2014 [9]. These clips are primarily intended for the internet. However, the organizers are convinced that it is important for the speaker to have a public in front of him or her, but only as a motivation. For instance, the lights are oriented in a way which enables special effects on the web, even though it implies that the speaker barely sees the spectators. The staff uses an impressive number of cameras and they work very hard on the editing. More importantly, they prepare the lecture in advance with the speaker, give him/her useful tips, and describe in great detail the targeted audience. A collaboration between the speaker and the organization team is maybe the key to success.

One of the difficulties with a 15 minute film is that it is short ! We have to know exactly what to say and, above all, what not to say. Should one prepare a detailed speech in advance? I fear that most mathematicians are not actors and this would lead to an artificial tone. We should certainly not improvise in such circumstances. I believe one should prepare some kind of rather precise framework, containing some key sentences, and, of course, rehearse several times in front of a clock.

This association is very close in spirit to the TED Conferences (Technology, Entertainment, Design) which also contain a relatively small number of mathematics lectures. As two model examples of short popular internet lectures, I would recommend [5] and [23]. Note in particular that in these examples, the speakers do not go into any mathematical detail, but both do give a fairly good image of the role of mathematicians.

All these are one-shot videos and one could wonder whether one should not prepare popular internet lectures as one produces a movie, filming many more rushes than necessary for the final product, and spending most of the time in the editing. Again this is the difference between theater and cinema.

[8]Les Ernest is a nickname for the goldfish swimming in a pond of the ENS.

7. Some conclusions and suggestions

Among the many possible communication tools that can be used for popularizing mathematics, the internet is probably the most powerful and efficient. A single individual or a very small group of mathematicians can produce webpages which can be viewed by many web-users, at almost no cost.

We have to learn the language which is adapted to this media and which is very different from the traditional language in mathematics: different in speed, depth and length. The point is not to transmit everything about mathematical research, but something about it. Sometimes, it is even sufficient to transmit *nothing* besides the fact that there exists a very active field of research called mathematics.

The most important mistake that should be avoided is to do on the internet what we are used to do in papers, books, classrooms, lecture halls etc. The internet enables us to develop new concepts.

We have to train the younger generations of mathematicians in these techniques. Almost every mathematician should have some training but we should also encourage some students to specialize in popularization. More importantly, we should consider them as colleagues, with a well defined field of expertise, just like algebraists, geometers or analysts, and we should not consider them contemptuously as "mere journalists".

This implies that popularization has to be evaluated in a rigorous way, just as research papers are refereed. Two centuries ago, the mathematical community was able to develop a system of journals, some of them being specialized, whose "qualities" can be (more or less) compared. There is a need for the creation of mathematical journals specializing in popularization, following strict validation criteria for the acceptance of their published "papers". This will not be easy, since indeed, these papers are never printed on paper... and can take many different forms, far away from our usual introduction-theorem-lemma-proof-conclusion mathematical "literature".

These journals should be considered as "standard" mathematics journals, indexed by the main data bases, supported by the national mathematical societies etc. Published papers should appear proudly in the CVs of mathematicians and should be taken into account by the various hiring or promotion committees.

In short, a mathematician answering the traditional question from a colleague "What's your field?" should not feel anymore ashamed when he or she replies "I work on popularization of mathematics".

En passant, note that almost all references below are freely available on the web...

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