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Actions localement libres du groupe affine. (French) [Locally free actions of the affine group]

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Let GA denote the group of proper affine transformations of $\mathbf{R}$. In this paper the author proves several rigidity theorems of locally free actions of GA on closed, smooth 3-manifolds. The prototypes of such actions are as follows: Let $G = \mathrm{SL}(2,\mathbf{R})$ or let G be the unique (up to isomorphism) nonnilpotent, solvable simply connected Lie group of real dimension three. Then in both cases GA is a subgroup of G. Let $\Gamma \subset G$ be a discrete uniform subgroup. Let $M(\Gamma) = G/\Gamma$. Then $M(\Gamma)$ has a natural locally free smooth action of GA. Such actions are called "homogeneous actions". These actions preserve the normalized Haar measure of $M(\Gamma)$. The author proves, among other things, that any C^r action $(2 \le r \le \omega)$ preserving a volume form of class C^0 is C^{r-1} conjugate to a homogeneous action: Theorem B: Let $\varphi\colon GA \times M^3 \to M^3$ be a locally free action of class C^r $(2 \le r \le \omega)$. Suppose φ preserves a volume form W of class C^0. Then φ is C^{r-1} conjugate to a homogeneous action.

In fact, the author proves that W in Theorem B is actually of class C^{r-2}. This is a corollary of the following: Theorem A: Let M be a closed smooth manifold. Let G be a nonunimodular Lie group and let $\varphi\colon G \times M \to M$ be a locally free action of class C^r $(2 \le r \le \omega)$. Let $\dim G + 1 = \dim M$. Suppose φ preserves a volume form, W, of class C^0. Then W is in fact of class C^{r-2}.

The following theorem gives a sufficient condition for the existence of a C^0 volume form: Theorem D: Let M^3 be a closed smooth 3-manifold such that $H^1(M,\mathbf{R}) = 0$. Let φ be a locally free action of GA on M, of class C^r $(2 \le r \le \omega)$. Then φ preserves a volume form of class C^0. In particular (Theorem B), φ is homogeneous.

As an application of the former results, the author proves the following surprising result which says that the deformations of certain Fuchsian groups (modulo differentiable conjugacy) can be described by a finite number of parameters: Theorem C: Let M be a compact orientable surface of genus $g \ge 2$. Let $\psi\colon \Gamma_g \to \mathrm{PSL}(2,\mathbf{R}) \subset \mathrm{Diff}^r(S^1)$ be the representation of the fundamental group of M which corresponds to a metric of constant negative curvature equal to -1 on M. Let $\psi'\colon \Gamma_g \to \mathrm{Diff}^r(S^1)$ $(5 < r < \omega)$ be a representation which is C^3-close to ψ (in the sense that ψ' is C^3-close in a set of generators of Γ_g). Then there exists $h \in \mathrm{Diff}^{r-3}(S^1)$ such that $h(\psi'(\Gamma_g))h^{-1} \subset \mathrm{PSL}(2,\mathbf{R})$.

In the proofs of the previous theorems the author uses many of the results of the theory of codimension one foliations, the representation theory of GA, and ergodic theory, among other things.

Reviewed by *Alberto Verjovsky*