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Sur la coupe des vêtements: variation autour d'un thème de Tchebychev. (French) [On the cutting of garments: variation on a theme of Chebyshev]

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The first few lines of this paper, as well as a relatively long appendix, contain some history of the subject. One learns that Chebyshev, in order to supplement his income as a university professor, signed a contract with a textile businessman, for a second job where the goal was to find methods of cutting military uniforms (for which there was a huge demand at that time) that would minimize fabric loss. This was Chebyshev's motivation for studying the cutting of garments. It is not clear to what extent Chebyshev helped the profession of clothing design, because he immediately got more interested in the mathematical aspect of the theory. This led Chebyshev to write a paper which bears the title of the paper under review, and a book which seems to be very difficult to locate. Chebyshev's paper, written in French, is reproduced in the appendix of the paper under review. In particular, Chebyshev developed the theory of what are now called "Chebyshev nets".

In rough terms, the theory asks which (curved) surfaces are "dressable" with a piece of flat fabric. The first result in this direction which was stated by Chebyshev is that every real-analytic surface is locally dressable. It seems that the first rigorous proof is due to Bianchi (1902). Chebyshev also stated that a hemisphere is dressable, and he gave a sketch of a proof for that; it seems that a rigorous proof of it was done by Bakelman (1935). In the paper under review, the author obtains a more powerful result, namely, that it is possible to dress the whole sphere, except for a "seam". More precisely, he shows that the open set obtained from the sphere by deleting from it two orthogonal arcs of great circles meeting at their midpoints and with appropriate length is dressable. Chebyshev nets appeared later in relation with surfaces of negative curvature, and with the sine-Gordon equation.

Some other famous mathematicians were interested in garment cutting. The paper under review mentions Édouard Lucas (1842–1891), but it is also appropriate to mention Bill Thurston's work in high fashion. One cannot help but remember Thurston saying that "mathematics and design are both expressions of the human creative spirit".

The paper under review is beautifully written and contains many good ideas. Relations with works of Gauss, Hilbert and other mathematicians are presented in the paper.

Reviewed by *Athanase Papadopoulos*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.