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Ghys, É. (F-LILL); Tsuboi, T. [Tsuboi, Takashi] (J-TOKYS)
Différentiabilité des conjugaisons entre systèmes dynamiques de dimension 1. (French. English summary) [Differentiability of conjugations between dynamical systems of dimension 1]

This is a neat treatment of the following very natural problem: under what conditions is a $C^1$ conjugacy between two $C^r$ dynamical systems of dimension 1 automatically of class $C^r$?

In the first half the authors consider codimension 1 $C^r$ ($2 \leq r \leq \omega$) foliated compact manifolds $(M_i, \mathcal{F}_i)$. The result is: if the holonomy of $\mathcal{F}_1$ is nontrivial and if there exists a $C^1$ diffeomorphism $\varphi: M_1 \to M_2$ such that $\varphi^* \mathcal{F}_1 = \mathcal{F}_2$, then $\varphi$ is transversely class $C^r$ on the open subset of all the noncompact leaves of $\mathcal{F}_1$. This yields a rather natural new proof of the $C^1$ invariance theorem of G. Rabby of the Godbillon-Vey class.

The latter half of the paper is devoted to the study of $C^\omega$ endomorphisms $f_i$ of $S^1$ (possibly with critical points). Suppose that $f_1$ has periodic points, that $f_1$ is not constant and that neither iterate of $f_1$ is the identity. Then a $C^1$ diffeomorphism of $S^1$ conjugating $f_1$ with $f_2$ is shown to be $C^\omega$ except on finite points. If further $|\deg f_1| \geq 2$, then it is $C^\omega$ on the whole $S^1$. These results are shown by examples to be the best possible. $C^\infty$ endomorphisms are also dealt with in a completely satisfactory manner.

The authors also obtain a similar result about rational functions on the Riemann sphere.

Reviewed by Shigenori Matsumoto

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