MR949007 (89i:58119) 58F18 (57R30 58F10)

Ghys, É. (F-LILL); **Tsuboi, T. [Tsuboi, Takashi]** (J-TOKYOS)

Différentiabilité des conjugaisons entre systèmes dynamiques de dimension 1. (French. English summary) [Differentiability of conjugations between dynamical systems of dimension 1]

Ann. Inst. Fourier (Grenoble) **38** (1988), *no. 1,* 215–244.

This is a neat treatment of the following very natural problem: under what conditions is a C^1 conjugacy between two C^r dynamical systems of dimension 1 automatically of class C^r?

In the first half the authors consider codimension 1 C^r $(2 \leq r \leq \omega)$ foliated compact manifolds $(M_i, \mathcal{F}_i)$. The result is: if the holonomy of $\mathcal{F}_1$ is nontrivial and if there exists a C^1 diffeomorphism $\varphi\colon M_1 \to M_2$ such that $\varphi^*\mathcal{F}_1 = \mathcal{F}_2$, then φ is transversely class C^r on the open subset of all the noncompact leaves of $\mathcal{F}_1$. This yields a rather natural new proof of the C^1 invariance theorem of G. Rabby of the Godbillon-Vey class.

The latter half of the paper is devoted to the study of C^ω endomorphisms f_i of S^1 (possibly with critical points). Suppose that f_1 has periodic points, that f_1 is not constant and that neither iterate of f_1 is the identity. Then a C^1 diffeomorphism of S^1 conjugating f_1 with f_2 is shown to be C^ω except on finite points. If further $|\deg f_1| \geq 2$, then it is C^ω on the whole S^1. These results are shown by examples to be the best possible. C^∞ endomorphisms are also dealt with in a completely satisfactory manner.

The authors also obtain a similar result about rational functions on the Riemann sphere.

Reviewed by *Shigenori Matsumoto*