MR1452855 (98f:57050) 57R25 (57R15 57R30 58F18)
Gambaudo, Jean-Marc (F-NICE-NL2); **Ghys, Étienne** (F-ENSLY-PM)
Enlacements asymptotiques. (French) [Asymptotic links]
Topology **36** (1997), *no. 6,* 1355–1379.

Three types of numerical invariants of smooth vector fields and diffeomorphisms which preserve the volume form on a two-dimensional disc or on a homology three-sphere are considered. Those are the asymptotic Hopf invariant, introduced by Arnolʹd (it measures the mean asymptotic value of the linking number of two nearby orbits of a volume-preserving vector field on the three-sphere), the Calabi invariant of a symplectomorphism of the two-disc (it is equal to the integral of the generating function of the symplectomorphism), and the invariant of Ruelle (this is the mean value of the rotation number of the differentials of the flow diffeomorphisms).

The properties (some of them new) of the latter two invariants are discussed in detail in a clear geometric way. Their invariance with respect to conjugation by a homeomorphism is proven. However, these invariants cannot be extended to the case of homeomorphisms. Certain properties are shown to remain valid for a general measure form on the disc.

For a non-singular vector field on a homology three-sphere the Ruelle invariant is constructed provided that the normal bundle to the field is trivial. In this case its value does not depend on the trivialisation and it is proven to be a topological invariant.

To a symplectomorphism of the disc (equal to the identity near the boundary) one associates a volume-preserving vector field on the solid torus, whose Poincaré section mapping is the initial symplectomorphism. The asymptotic Hopf invariant of this field happens to coincide with the Calabi invariant of the symplectomorphism.

The results and techniques might be interesting to specialists in symplectic topology and hydrodynamics.

Reviewed by *V. M. Zakalyukin*

References

1. V. I. Arnold: The asymptotic Hopf invariant and its applications, *Sel. Math. Sov.* **5** (1986), 327–345. MR0891881 (89m:58053)
2. A. Banyaga: On the group of diffeomorphisms preserving an exact symplectic form, in *Differential topology*, Varenna (1976), pp. 5–9. MR0660656 (83g:58006)
3. E. Calabi: On the group of automorphisms of a symplectic manifold, in *Problems in analysis*, Symposium in honour of S. Bochner, R. C. Gunning, Ed., Princeton Univ. Press, Princeton (1970), pp. 1–26. MR0350776 (50 #3268)
4. G. Contreras and R. Iturriaga: Average linking numbers, preprint (1995). cf. MR 2001c:37022
5. A. Fathi: Transformations et homéomorphismes préservant la mesure. Systèmes dynamiques minimaux., Thèse Orsay (1980).
6. J. M. Gambaudo and E. Pécou: A topological invariant for volume preserving diffeomorphisms,

Ergod. Theory Dyn. Systems **15** (1995), 535–541. MR1336704 (96f:58137)

7. V. A. Naishul: Topological invariant of analytic and area-preserving mapping and their application to analytic differential equations in $\mathbf{C}^2$ and $\mathbf{CP}^2$, *Trans. Moscow Math. Soc.* **2** (1982), 239–242. MR0656288 (84f:58092)
8. D. Rolfsen: *Knots and links*, Math. Lect. Series **7**, Publish or perish, Berkeley, CA (1976). MR0515288 (58 #24236)
9. D. Ruelle: Rotation numbers for diffeomorphisms and flows, *Ann. Inst. Henri Poincaré, Physique Théorique* **42** (1985), 109–115. MR0794367 (86k:58075)
10. S. Schwartzmann: Asymptotic cycles, *Ann. Math.* **66** (1957), 210–215. MR0088720 (19,568i)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.