MR896095 (90c:57035) 57S30 (57R30 57R32 57R50 58D05 58F18)
Ghys, Étienne (F-LILL) ; **Sergiescu, Vlad** (F-LILL)
Sur un groupe remarquable de difféomorphismes du cercle. (French) [On a remarkable group of diffeomorphisms of the circle]
Comment. Math. Helv. **62** (1987), *no. 2,* 185–239.

The authors study the group G of piecewise linear homeomorphisms of the circle $\mathbf{R}/\mathbf{Z}$ which send $\mathbf{Q}_2/\mathbf{Z}$ onto itself and have singular points only on $\mathbf{Q}_2/\mathbf{Z}$, where $\mathbf{Q}_2$ is the ring of dyadic numbers. For any element g of G, there exists a positive integer N such that g restricted to the intervals $[k \cdot 2^{-N}, (k+1) \cdot 2^{-N}]$, $k \in \mathbf{Z}$, is of the form $x \longmapsto 2^n \cdot x + p \cdot 2^q$, $n, p, q \in \mathbf{Z}$.

The authors show the existence of a homeomorphism h such that hGh^{-1} is contained in the group of C^∞ diffeomorphisms of S^1; hence there is a representation of G in $\mathrm{Diff}^\infty_+(S^1)$. As for the uniqueness, they show that any representation $\varphi \colon G \longrightarrow \mathrm{Diff}^2_+(S^1)$, is semiconjugate to the inclusion $G \subset \mathrm{Homeo}(S^1)$.

The authors determine the rational cohomology ring of BG (as well as those of other related classifying spaces) as follows: $H^*(BG; \mathbf{Q}) = \mathbf{Q}[\alpha, \chi]/\alpha \cdot \chi = 0$, $|\alpha| = |\chi| = 2$, where α corresponds to the Euler class and χ is explicitly written down as a 2-cocycle (the discrete Godbillon-Vey cocycle). They note that, though the discrete Godbillon-Vey cocycle is similar to the usual Godbillon-Vey cocycle, the usual Godbillon-Vey class pulled back by any homomorphism of G to $\mathrm{Diff}^2_+(S^1)$ is zero.

They show that the representations $\varphi \colon G \longrightarrow \mathrm{Diff}^\infty_+(S^1)$ have several interesting dynamical properties. For example, there are the ones which have exceptional minimal sets. They also give an explicit example of C^∞-foliated S^1-bundles over a surface with an exceptional minimal set and with nontrivial Euler class.

Reviewed by *Takashi Tsuboi*