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Groupes d’holonomie des feuilletages de Lie. (French) [Holonomy groups of Lie foliations]

Following A. Haefliger’s point of view [see J. Differential Geom. 15 (1980), no. 2, 269–284; MR0614370 (82j:57027)], to a foliation $\mathcal{F}$ on a manifold $M$ is associated an equivalence class $[\mathcal{F}]$ of pseudogroups (a representative of which is the pseudogroup $\mathcal{F}$ induced by holonomy on a complete transversal submanifold). A natural problem is to study what sort of pseudogroup is this holonomy pseudogroup if $M$ is a closed manifold.

In this paper, the author studies the case where $(M, \mathcal{F})$ is a $\mathfrak{g}$-foliation on a closed manifold [E. Fedida, “Feuilletages du plan; feuilletages de Lie”, Thèse, Univ. Louis Pasteur, Strasbourg, 1973; BullSocMathFrance 110 (1974):2599; see Differential topology, foliations and Gelfand-Fuks cohomology (Rio de Janeiro, 1976), 183–195, Lecture Notes in Math., 652, Springer, Berlin, 1978; see MR 80a: 57012]; if $G$ is the simply connected Lie group with $\mathfrak{g}$ as Lie algebra, then $\mathcal{F}$ is equivalent to a subgroup $\Gamma$ of $G$. This “holonomy group” of $(M, \mathcal{F})$ is a quotient of the fundamental group $\pi_1(M)$. Hence $\Gamma$ is finitely generated. Moreover, as $M$ fibers over $G/\Gamma$, this homogeneous space is compact.

The author obtains a new property of such a subgroup $\Gamma$, involving the notion of “real cohomological dimension” of a CW-complex. If $\text{rcd}(X)$ denotes this dimension, and if $\Gamma$ is a group, then by definition, $\text{rcd}(\Gamma) = \text{rcd}(K(\Gamma, 1))$. The principal result of the author is the inequality $\text{rcd}(\Gamma) \geq \dim G - \dim K$, where $K$ is a maximal compact subgroup of $G$. Using this property, the author shows that, if $G$ is the affine group of $\mathbb{R}$, and $\Gamma$ is generated by elements $(a_1 b_1, \cdots, a_l b_l)$, a necessary condition in order that $\Gamma$ may be the holonomy group of a $\mathfrak{g}$-Lie foliation is that $a_1, b_1, \cdots, a_l, b_l$ be algebraically dependent over $\mathbb{Q}$.

Reviewed by P. Molino

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