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Groupes d'homéomorphismes du cercle et cohomologie bornée. (French. English summary)
[Homeomorphism groups of the circle and bounded cohomology]

The Lefschetz centennial conference, Part III (*Mexico City*, 1984), 81–106, *Contemp. Math.*, 58, *III*, *Amer. Math. Soc.*, *Providence*, *RI*, 1987.

The author associates to a group of homeomorphisms of the circle an element in bounded cohomology which generalizes the Euler class and the rotation number. If $\varphi\colon \Gamma \to \mathrm{Homeo}^+(S^1)$, $\varphi^*(e) \in H^2_b(\Gamma;\mathbf{Z})$, the bounded cohomology defined by M. L. Gromov [Inst. Hautes Études Sci. Publ. Math. No. 56 (1982), 5–99; MR0686042 (84h:53053)]. This class is the pull-back of a universal class $e \in H^2_b(\mathrm{Homeo}^+(S^1),\mathbf{Z})$; its image in the usual cohomology $H^2(\Gamma,\mathbf{Z})$ is the Euler class of the corresponding S^1-bundle over $B\Gamma$. In case $\Gamma = \mathbf{Z}$, $H^2_b(\mathbf{Z};\mathbf{Z}) = \mathbf{R}/\mathbf{Z}$ and $\varphi^*(e)$ is the rotation number of $\varphi(1)$.

The author defines a weak notion of semiconjugacy of two actions φ_1 and φ_2: for all $\gamma \in \Gamma$, $\varphi_1(\gamma) \circ h = h \circ \varphi_2(\gamma)$ for some not necessarily continuous function $h\colon S^1 \to S^1$ whose lift to the line is a nondecreasing function $\overline{h}$ with $\overline{h}(x+1) = \overline{h}(x) + 1$. With this definition, semiconjugacy is an equivalence relation and two representations φ_1 and φ_2 are semiconjugate if and only if the classes $\varphi_1^*(e)$ and $\varphi_2^*(e)$ are equal.

Reviewed by *John W. Wood*