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**★Sur les groupes hyperboliques d'après Mikhael Gromov. (French) [Hyperbolic groups in the theory of Mikhael Gromov]**

Papers from the Swiss Seminar on Hyperbolic Groups held in Bern, 1988.
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This book grew out of the "Swiss Seminar" organized by the Troisième Cycle Romand de Mathématiques held at the University of Bern in the summer of 1988 on the theory of hyperbolic groups following a fundamental paper by Gromov [in *Essays in group theory*, 75–263, Springer, New York, 1987; MR0919829 (89e:20070)].

Gromov studies groups from a geometric viewpoint: Let $\Gamma$ be a group admitting a finite set $S$ of generators; then $S$ defines in a natural way a distance function $d_S$ on $\Gamma$ (word-length distance). This distance can be used to introduce geometric concepts such as geodesics, triangles, volume growth, etc. Roughly speaking, hyperbolic groups $\Gamma$ are finitely generated groups such that the metric space $(\Gamma, d_S)$ shares a lot of properties with Riemannian manifolds of negative sectional curvature. Thus many geometric ideas coming from hyperbolic geometry can be used to study these groups.

This viewpoint turned out to be extremely useful in group theory as well as in geometry.

In the book under review the authors give a detailed introduction to parts of the theory with many examples and applications. A highlight is the geometric construction of infinite torsion groups. In the reviewer's opinion the authors have done an excellent job and the book is extremely useful to mathematicians interested in this field.

Most of the chapters are written by editors Ghys (É.G.) and de la Harpe (P.H.). Additional contributions are by Troyanov (M.T.), Salem (É.S.), Ballmann (W.B.), Haefliger (A.H.) and Strebel (R.S.).

The following are short outlines of the chapters. Chapter 1, Panorama (É.G. and P.H.): Here the fundamental concepts and results of the theory are discussed in a more intuitive way without detailed proofs. Chapter 2, Espaces métriques hyperboliques (É.G. and P.H.): Different definitions and characterizations of metric hyperbolic spaces are given. To state only one, a geodesic space $X$ is hyperbolic if there is a constant $\delta > 0$ such that for any geodesic triangle $\Delta$ in $X$ and any point $p$ on one of the edges of $\Delta$ the distance to the union of the two other edges is bounded by $\delta$. Thus large triangles look $\delta$-close to triangles in a tree. The relation between hyperbolic spaces and infinite trees is discussed. Chapter 3, Espaces à courbure négative et groupes hyperboliques (M.T.): There is a short introduction to triangle comparison statements. It is furthermore proved that the fundamental group of a compact Riemannian manifold with negative sectional curvature is hyperbolic.

Chapter 4, Premières propriétés des groupes hyperboliques (É.S.): The Rips complex of the hyperbolic group is constructed and as a consequence it is proved that a hyperbolic group $\Gamma$ has the properties: (1) $\Gamma$ has a finite presentation; (2) there are only finitely many conjugate classes of torsion elements; (3) the cohomology groups $H^k(\Gamma, \mathbf{Q})$ are trivial for $k$ large enough. Chapter 5, Quasi-isométries et quasi-géodésiques (É.G. and P.H.): The metric $d_S$ on $\Gamma$ depends on the set $S$. For a different set $S^*$ of generators the corresponding distance $d_{S^*}$ is quasi-isometric to $d_S$. Therefore all relevant geometric concepts have to be invariant under quasi-isometries in order to be an invariant of the group $\Gamma$. Chapter 6, Le bord d'un arbre (É.G. and P.H.): Now the authors begin to study the boundary at infinity of a hyperbolic space. In this chapter they consider the special case that $X$ is an infinite tree.

Chapter 7, Le bord d'un espace hyperbolique (É.G. and P.H.): Here the boundary of a general hyperbolic space is investigated. In particular, the quasi-conformal structure on the boundary at infinity is discussed. Chapter 8, L'action au bord des isométries (É.G. and P.H.): Similar to the situation of simply connected Riemannian manifolds of negative curvature one can define three classes of isometries: elliptic, hyperbolic and parabolic. Chapter 9, La propriété de Markov pour les groupes hyperboliques (É.G. and P.H.): It is proved that the hyperbolic groups are strongly Markov. Roughly speaking, this means that for any given set $S$ of generators all elements $\gamma$ of the group $\Gamma$ and their $d_S$-distance from the neutral element can be described by a finite Markov grammar. There are strong relations to the theory of automatic groups.

Chapter 10, Singular spaces of nonpositive curvature (W.B.): Singular spaces (in particular polyhedrons) of nonpositive curvature are constructed. Chapter 11, Orbi-espaces (A.H.): An orbi-espace is a space $X$ which can be locally described as $V_i/\Gamma_i$ where $V_i$ is locally compact and $\Gamma_i$ is a finite group of homeomorphisms. $X$ is said to be developable if $X$ can be globally written as $\widetilde{X}/\Gamma$. It is shown that a compact orbi-espace with curvature $\leq 0$ is developable. Chapter 12, Groupes de torsion (É.G. and A.H.): Using the results of Chapters 10 and 11, Gromov's construction of infinite torsion groups is discussed. Appendix, Small cancellation groups (R.S.): This is a quite detailed introduction to the theory of small cancellation groups.

Reviewed by *Viktor Schroeder*