On the measurable dynamics of $z \mapsto e^z$.


Measure-theoretic properties of the complex exponential map $E(z) = e^z$ are studied. In particular, the exponential map is shown to be recurrent with respect to the full orbit equivalence relation generated by $E$, and to have no nontrivial topologically conjugate deformations. A trivial deformation is a map in the 3-complex-parameter family $z \mapsto e^{az+b} + c$ ($a, b, c \in \mathbb{C}$). The recurrence derives from this by the same type of argument (but easier) as in Sullivan’s proof of the nonexistence of wandering domains for rational maps [Ann. of Math. (2) 122 (1985), no. 3, 401–418]: a wandering set of positive measure would give a nontrivial invariant line field, whence nontrivial quasiconformal deformations. Recurrence is then used to show that the fibres of the map from $\mathbb{C}$ to the symbolic dynamics space, constructed for the exponential map by R. L. Devaney and M. Krych [Ergodic Theory Dynamical Systems 4 (1984), no. 1, 33–52; MR0758892 (86b:58069)], have measure 0. A different argument has to be used for fibres over periodic sequences.

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