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**On the measurable dynamics of  $z \mapsto e^z$ .**

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Measure-theoretic properties of the complex exponential map  $E(z) = e^z$  are studied. In particular, the exponential map is shown to be recurrent with respect to the full orbit equivalence relation generated by  $E$ , and to have no nontrivial topologically conjugate deformations. A trivial deformation is a map in the 3-complex-parameter family  $z \mapsto e^{az+b} + c$  ( $a, b, c \in \mathbf{C}$ ). The recurrence derives from this by the same type of argument (but easier) as in Sullivan's proof of the nonexistence of wandering domains for rational maps [Ann. of Math. (2) **122** (1985), no. 3, 401–418]: a wandering set of positive measure would give a nontrivial invariant line field, whence nontrivial quasiconformal deformations. Recurrence is then used to show that the fibres of the map from  $\mathbf{C}$  to the symbolic dynamics space, constructed for the exponential map by R. L. Devaney and M. Krych [Ergodic Theory Dynamical Systems **4** (1984), no. 1, 33–52; [MR0758892 \(86b:58069\)](#)], have measure 0. A different argument has to be used for fibres over periodic sequences.

Reviewed by *M. Rees*

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