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Ghys, Étienne (F-LILL); **Carrière, Yves** (F-LILL)
Relations d'équivalence moyennables sur les groupes de Lie. (French. English summary)
[Amenable equivalence relations on Lie groups]
C. R. Acad. Sci. Paris Sér. I Math. **300** (1985), *no. 19,* 677–680.

Let X be a standard measure space with a possibly infinite measure μ. Suppose the free group on 2 generators acts essentially freely on X, leaving μ quasi-invariant, and furthermore the automorphisms corresponding to the generators are conservative. Then the equivalence relation generated by the action is not amenable. As a consequence, the equivalence relation generated by a countable subgroup Γ on a Lie group G is not amenable, provided Γ contains a nondiscrete free subgroup on 2 generators. In particular, this applies to any nondiscrete nonsolvable subgroup of $\mathrm{SL}(2, \mathbf{R})$. A foliation induces a discrete standard equivalence relation on any total transversal manifold. Suppose the manifold is compact, the foliation admits a transverse invariant measure, and almost all leaves are without holonomy; then the foliation is amenable if and only if almost all leaves satisfy the Følner condition: There exists a sequence of compact submanifolds K_n with boundary ∂K_n increasing to the leaf and such that $\mathrm{vol}\,\partial K_n/\mathrm{vol}\,K_n$ tends to 0. An ergodic Riemannian foliation is amenable if and only if each leaf satisfies the Følner condition.

Reviewed by *Bruce L. Reinhart*