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Right-handed vector fields & the Lorenz attractor. (English summary)

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Inspired by an open question raised by J. S. Birman and R. F. Williams [Topology **22** (1983), no. 1, 47–82; MR0682059 (84k:58138)] on the Lorenz attractor, namely whether there is some natural meaning to the fibrations associated to Lorenz links, the author introduces a concept of right-handed vector fields. A vector field X on the 3-sphere is right-handed if the quadratic linking form is positive on the convex set of invariant probability measures. This is equivalent to the fact that there is some Gauss linking form $\overline{\Omega}$ which is pointwise positive on X. Grosso modo, such a vector field is characterized by the property that any two orbits link positively. The author shows that for a right-handed vector field in the 3-sphere, any finite collection of periodic orbits is a fibered link. The positive Gauss linking form $\overline{\Omega}$ seems to be the global object which incarnates the collection of the fibrations of all the finite links of periodic orbits. Naturally among the provided examples, the Lorenz flow is given and is shown to be "almost" right-handed.

Reviewed by *Quach thi Câm Vân*

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