Let $\Gamma_g$ be the fundamental group of a closed oriented surface of genus $g \geq 2$. In the paper under review, the author considers representations $\Phi: \Gamma_g \to \text{Diff}^r_+(S^1)$ into the orientation-preserving $C^r$-diffeomorphisms of the circle. For the usual presentation of $\Gamma_g$ with generators $a_1, b_1, \ldots, b_g$, one chooses lifts $\tilde{A}_i$ and $\tilde{B}_i$ of $\Phi(a_i)$ and $\Phi(b_i)$ to diffeomorphisms of $\mathbb{R}$, and defines the Euler number $\text{eu}(\Phi)$ to be the integer $\tilde{A}_1 \tilde{B}_1 \tilde{A}_1^{-1} \cdots \tilde{B}_g^{-1}$. J. Milnor and J. Wood proved that $\text{eu}(\Phi)$ is at most $2g - 2$, and $\text{eu}(\Phi) = 2g - 2$ if $\Phi$ is an imbedding with image a discrete subgroup of $\text{PSL}(2, \mathbb{R})$. Up to conjugacy, such imbeddings parameterize the Teichmüller space of the surface.

The author's main theorem says that if $r \geq 3$ and $\text{eu}(\Phi) = 2g - 2$, then there is a diffeomorphism of the circle, of class $C^r$, which conjugates $f$ to an imbedding onto a discrete cocompact subgroup of $\text{PSL}(2, \mathbb{R})$. The existence of a homeomorphism conjugating $\Phi$ to have values in $\text{PSL}(2, \mathbb{R})$ is a result of S. Matsumoto [Invent. Math. 90 (1987), no. 2, 343–358; MR0910205 (88k:58016)]. The author also notes that any two imbeddings as discrete cocompact subgroups of $\text{PSL}(2, \mathbb{R})$ are conjugate by a homeomorphism of $S^1$, but are conjugate by a $C^1$-diffeomorphism only when they are conjugate in $\text{PSL}(2, \mathbb{R})$, and hence represent the same point of Teichmüller space.

The proof of the main theorem involves examination of Anosov diffeomorphisms of the torus and Anosov flows on 3-manifolds. In particular, the author proves that an Anosov flow of class $C^r$ ($r \geq 2$) on a closed 3-manifold whose stable and weakly unstable leaves are of class $C^{1,1}$ must be $C^r$-equivalent to either a quasi-Fuchsian flow or to the suspension of a diffeomorphism of the torus. In the final section the author gives an application of the main theorem to holomorphic deformations of Fuchsian groups.

Reviewed by Darryl McCullough

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