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Totally geodesic foliations on 4-manifolds.
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The basic fact about a geodesic foliation $\mathcal{F}$ on a Riemannian manifold M is that the transport of a leaf to another along the orthogonal distribution $\mathcal{F}^{\perp}$ respects the Riemannian structure along the leaves. Following R. A. Blumenthal and J. J. Hebda [Quart. J. Math. Oxford Ser. (2) **35** (1984), no. 140, 383–392; MR0767769 (86e:53021)] and Cairns [C.R. Acad. Sci. Paris Sér. I Math. **297** (1983), no. 9, 525–527; MR0735491 (85g:53040)], by lifting to the bundle of orthogonal frames along the leaves, one obtains a foliation endowed with a tangential parallelism invariant by the lifted distribution $\hat{\mathcal{F}}^{\perp}$. It is something like the "dual" of the procedure of lifting a Riemannian foliation to the bundle of transverse orthogonal frames. Of course, $\hat{\mathcal{F}}$ is generally not integrable, but it defines a "singular" Riemannian foliation. Several invariants appear in this way, as explained in the work of Cairns.

In this paper, the authors use elegant arguments to study the case where (M,g) is a compact 4-manifold, $\mathcal{F}$ a codimension 2 foliation. They show that for another metric g' (such that $\mathcal{F}$ is always geodesic), all the leaves have the same constant curvature, which is $+1$ (elliptic case), 0 (parabolic case), or -1 (hyperbolic case). They give precise results in each case. They show, for example, that in the elliptic case, $\mathcal{F}$ is defined by a fibration, with S^2 as typical fiber. If M is 1-connected, $\mathcal{F}$ is defined by a fibration, with S^2 as fiber. If the Euler characteristic of M is negative, M admits a fibration by spheres S^2 which is either everywhere tangent to or everywhere transverse to $\mathcal{F}$.

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