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Transformations holomorphes au voisinage d’une courbe de Jordan. (French. English summary) [Holomorphic transformations around a Jordan curve]

Denote by $U$ an open neighbourhood of the Jordan curve $J$ in the plane and by $F: U \to \mathbb{C}$ a univalent map. Suppose further that $J$ is invariant under $F$, which preserves the orientation of $J$, and denote the restriction of $F$ to $J$ by $f$. By combining conformal mapping with Denjoy’s theorem on homeomorphisms of the circle, the author shows that either $f$ has a periodic point or every orbit of $f$ is dense in $J$.

The real number $\alpha$ is said to satisfy a Diophantine condition if there exist $\beta \geq 0$, $c > 0$ such that for every rational $p/q$ one has $|\alpha - (p/q)| \geq cq^{-(2+\beta)}$. If $\alpha$ is the rotation number of $f$ and if $\alpha$ satisfies a Diophantine condition it is shown that $J$ must in fact be an analytic curve and in the neighbourhood of $J$ the map $F$ is analytically conjugate to the rotation $z \to e^{2\pi i \alpha} z$ in the neighbourhood of the unit circle.

If $g(z) = e^{2\pi i \alpha} z + \cdots$, $\alpha$ irrational, is analytic in the neighbourhood $W$ of 0, define $S$ to be a maximal connected open set such that $0 \in S \subset W$ and such that $S$ is invariant under $g$ and $g$ is analytically conjugate to a rotation in $S$. If $\alpha$ satisfies a Diophantine condition then such an $S$ exists and, moreover, if the boundary of $S$ is a Jordan curve $J \subset W$, then $J$ contains a critical point of $g$.

Reviewed by I. N. Baker

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