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Transformations holomorphes au voisinage d'une courbe de Jordan. (French. English summary) [Holomorphic transformations around a Jordan curve]

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Denote by U an open neighbourhood of the Jordan curve J in the plane and by $F\colon U \to \mathbf{C}$ a univalent map. Suppose further that J is invariant under F, which preserves the orientation of J, and denote the restriction of F to J by f. By combining conformal mapping with Denjoy's theorem on homeomorphisms of the circle, the author shows that either f has a periodic point or every orbit of f is dense in J.

The real number α is said to satisfy a Diophantine condition if there exist $\beta \geq 0$, $c > 0$ such that for every rational p/q one has $|\alpha - (p/q)| \geq cq^{-(2+\beta)}$. If α is the rotation number of f and if α satisfies a Diophantine condition it is shown that J must in fact be an analytic curve and in the neighbourhood of J the map F is analytically conjugate to the rotation $z \to e^{2\pi i\alpha}z$ in the neighbourhood of the unit circle.

If $g(z) = e^{2\pi i\alpha}z + \cdots$, α irrational, is analytic in the neighbourhood W of 0, define S to be a maximal connected open set such that $0 \in S \subset W$ and such that S is invariant under g and g is analytically conjugate to a rotation in S. If α satisfies a Diophantine condition then such an S exists and, moreover, if the boundary of S is a Jordan curve $J \subset W$, then J contains a critical point of g.

Reviewed by *I. N. Baker*