Comptage bijectif et profil des cartes simples

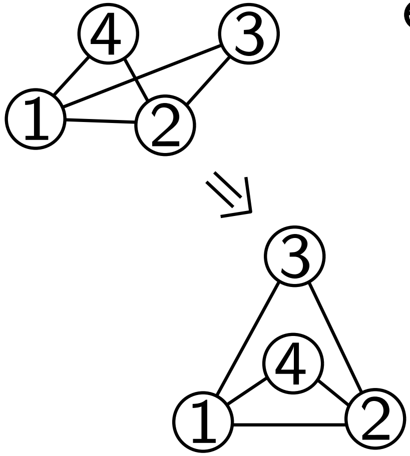
Éric Fusy (CNRS/LIX)
travail avec Olivier Bernardi et Gwendal Collet

+ convergence vers la carte brownienne
travail en cours avec Marie Albenque

Journées Cartes, Mars 2014, ENS Lyon

Planar graphs

planar graph = graph that can be embedded in the plane



Encoding: list of edges

$$\{1,2\},\ \{2,3\},\ \{1,3\}$$
$$\{1,4\}, \{2,4\}$$

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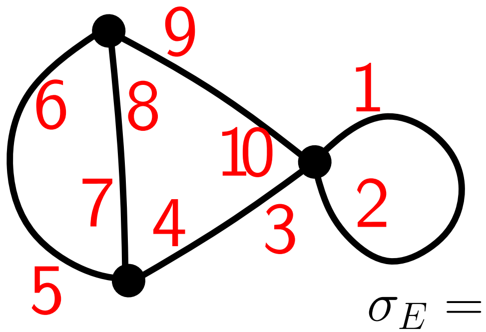

Encoding: two permutations

$$\sigma_E = (1,2)(3,4)(5,6)(7,8)(9,10)$$
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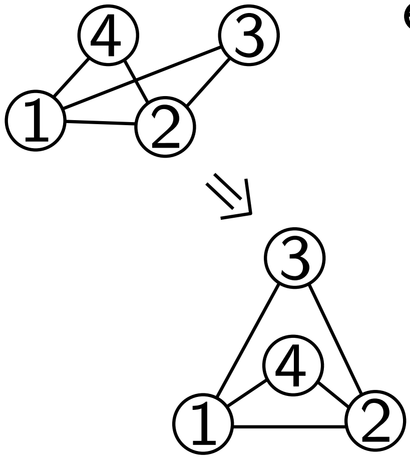

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typically: labelled at vertices
no loops nor multiple edges

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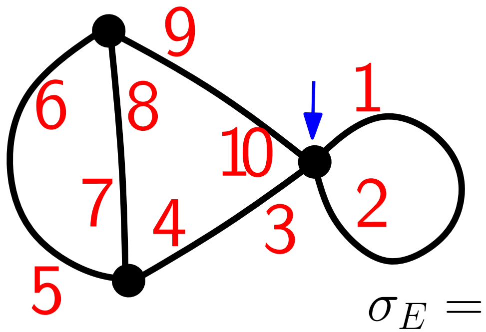

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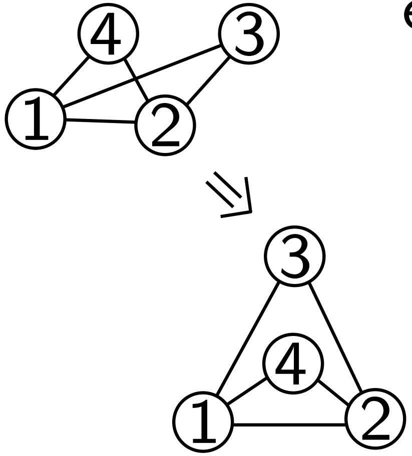

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Characterization of planarity:

- positive: existence of a planar embedding
- negative: $\exists\ K_5$ or $K_{3,3}$ minor (Kuratowski)

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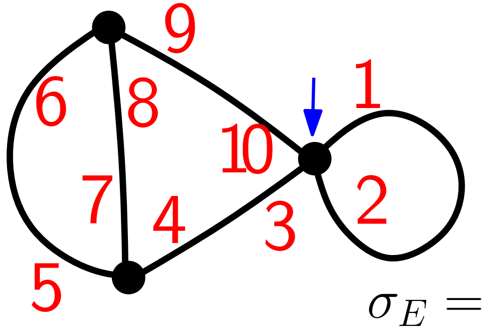

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$$\boxed{\ell(\sigma_V) + \ell(\sigma_V \circ \sigma_E) = \ell(\sigma_E) + 2}$$

Euler relation

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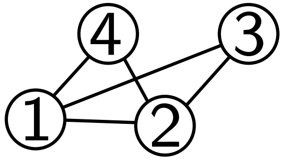

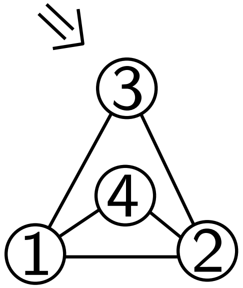

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Enumeration methods:

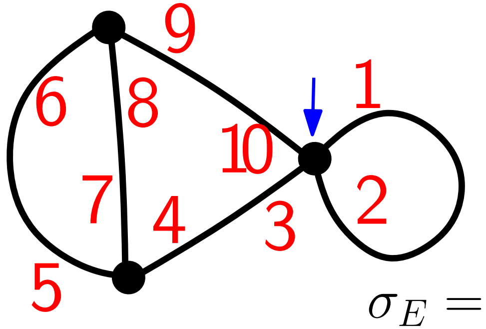
composition method (Tutte+Whitney)

planar graph = tree of 3-connected maps components

(asymptotics in [Giménez,Noy'05])

Planar maps

planar map = graph equipped with a planar embedding



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Enumeration methods:

- loop-equations [Tutte'60s]
- composition method [Tutte'60s]
- matrix integrals [Brezin et al]
- Bijections [Cori-Vauquelin,Schaeffer,...]

Simple planar maps

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Counting by composition (core-extraction) **from rooted maps**

For $i \in \{1, 2, 3\}$, let $M_i \equiv M_i(t_i) =$ GF rooted maps girth $\geq i$ (by edges)

$M_1(t_1)$: maps, $M_2(t_2)$: loopless maps, $M_3(t_3)$: simple maps

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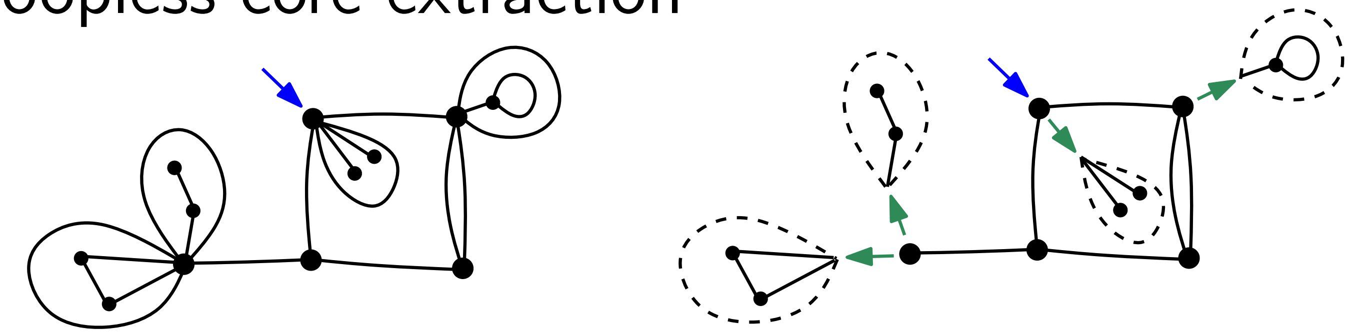

$$\Rightarrow \boxed{M_1(t_1) = t_1 M_1(t_1)^2 + M_2(t_2),}$$

with $t_2 = \frac{t_1}{(1 - t_1 M_1(t_1))^2}$

[Lehman-Walsh'75]

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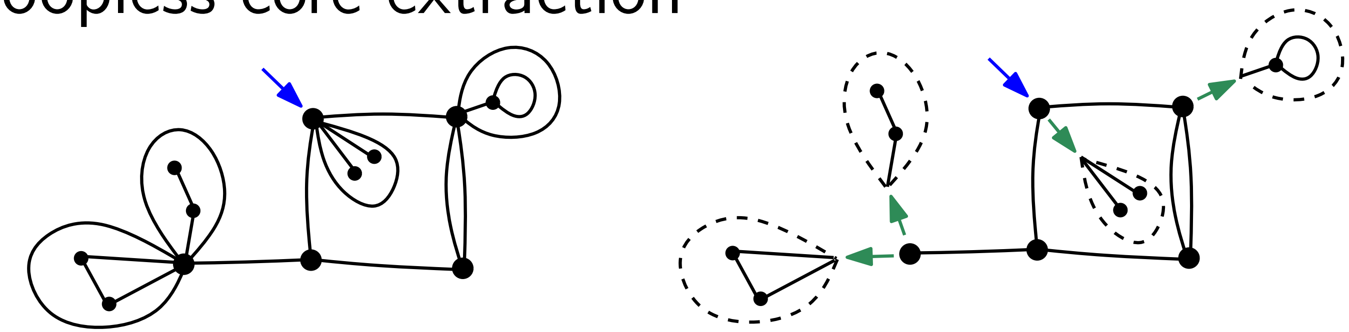

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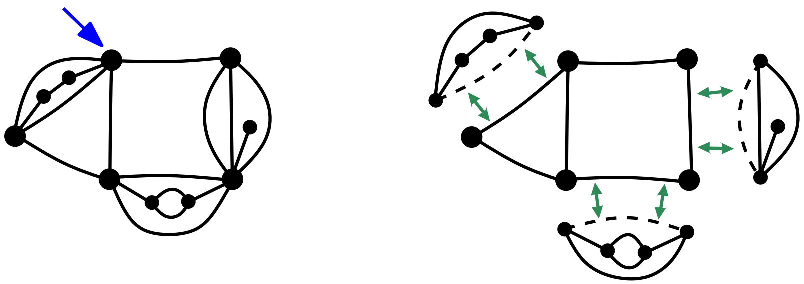

$$\Rightarrow \boxed{M_2(t_2) = M_3(t_3),}$$

with $t_3 = t_2 M_2(t_2)$

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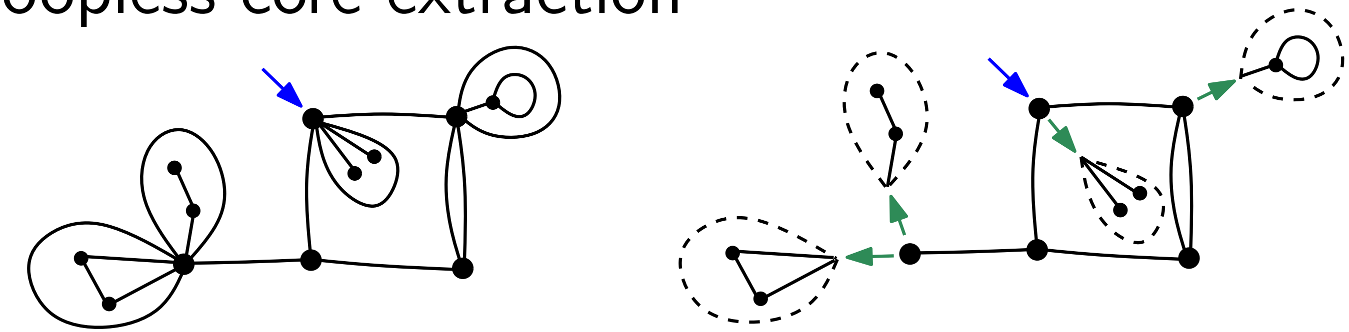

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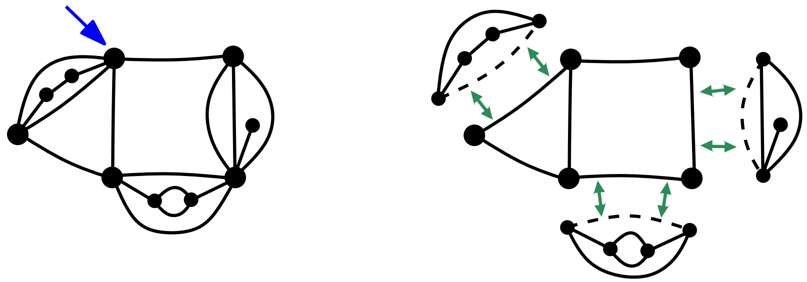

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$$\Rightarrow \boxed{M_3 = \frac{(1+2u)^2}{(1+u)^3}, \quad t_3 = \frac{u}{(1+2u)^2}}$$

[Banderier et al'03]
[Noy'12]

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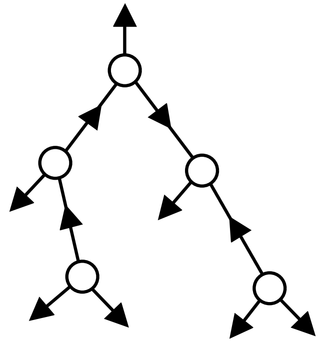
The series $M \equiv M(t)$ of rooted simple maps (by edges) is given by

$$M = \frac{(1+2u)^2}{(1+u)^3},$$

with $u = t(1+2u)^2$ the series of rooted oriented binary trees



$$M(t) = 1 + t + 2t^2 + 6t^3 + 23t^4 + 103t^5 + 512t^6 + 2740t^7 + 15485t^8 + \cdots$$

Rk: appears in Sloane, #1342-avoiding permutations of size n [Bona'97]

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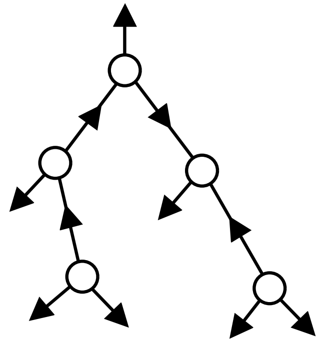

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Rk: the GF $B \equiv B(t)$ of rooted bipartite maps is expressed in terms of same u

$$B = 1 + u - u^2, \text{ with } u = t(1+2u)^2$$

(also $B = 1 + \sum_{n\geq 1} 3 \cdot 2^{n-1} \frac{(2n)!}{n!(n+2)!}$)

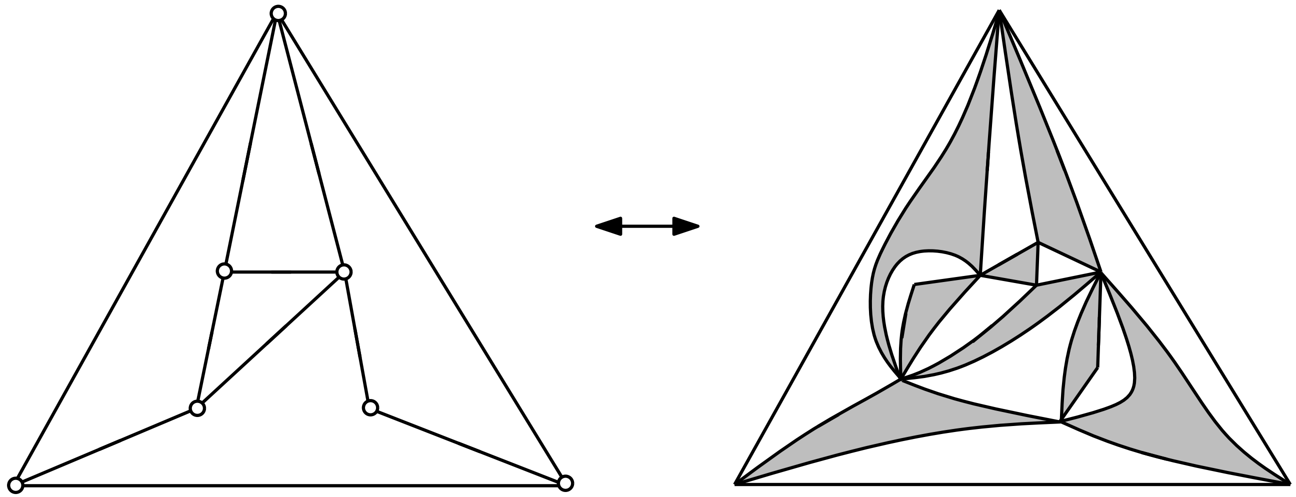
$$\Rightarrow \boxed{M(t) = \frac{1}{1 - tB(t)}}$$

Overview

- Bijective proof of the formula

$$M(t) = \frac{1}{1 - tB(t)}$$

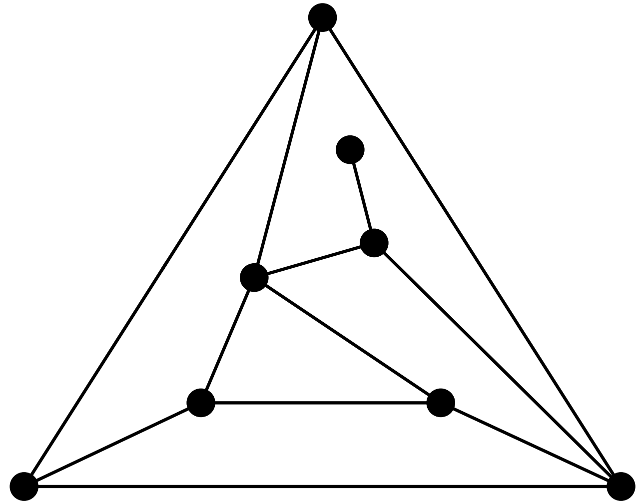
that links (the GFs of) simple maps and bipartite maps



- Applications
  - enumeration of simple maps
  - distance profile and convergence to the brownian map (sketch)

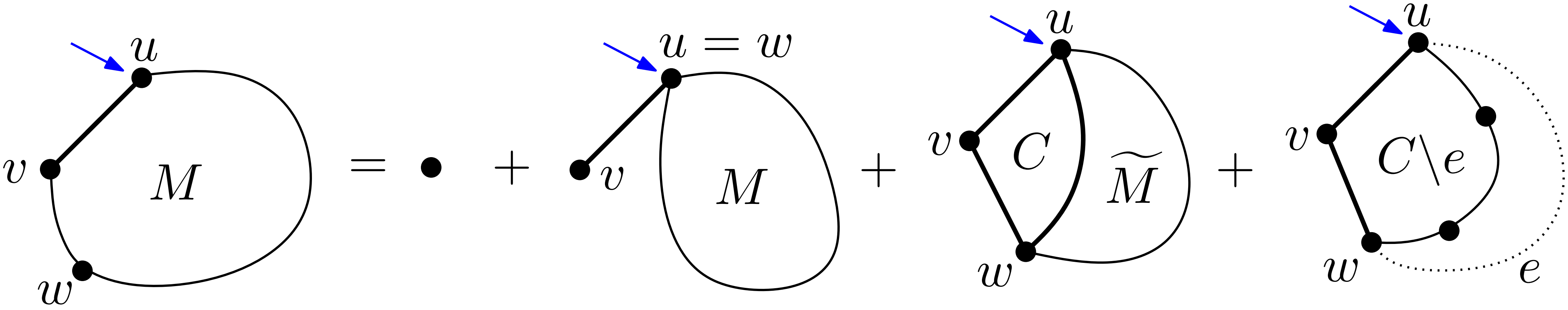
First observations (1)

Consider subfamily $\mathcal{C}$ of outertriangular simple maps



$C(z)$ generating series for rooted ones according to edges

- Decomposition of $\mathcal{M} = 1 + \widetilde{\mathcal{M}}$ in terms of $\mathcal{C}$



u, w adjacent $\quad u, w$ not adjacent

$$\Rightarrow M(t) = 1 + tM(t) + \frac{1}{t}C(t)(M(t)-1) + \frac{1}{t}C(t)$$

$$\Rightarrow \boxed{M(t) = \frac{1}{1 - t(1 + C(t)/t^2)}} \Rightarrow$$

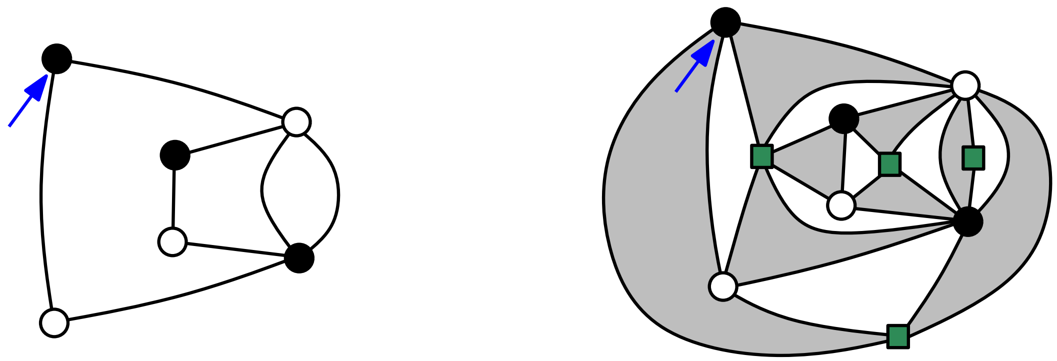
Have to prove bijectively that

$$B(t) = 1 + C(t)/t^2$$

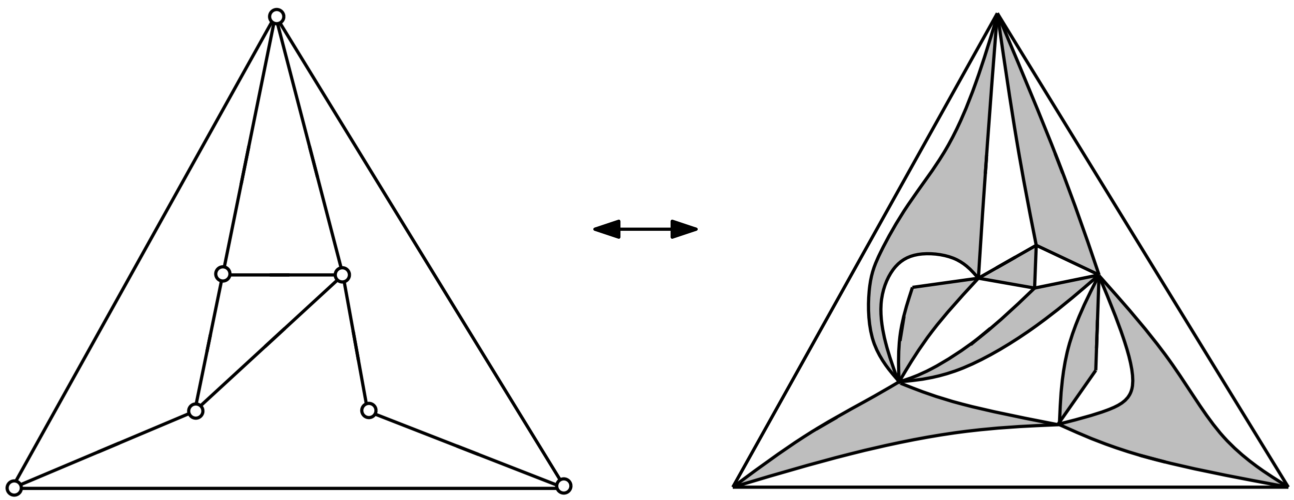
First observations (2)

Well known (Tutte's "trinity mapping"):

$B(t) - 1$ is the generating function of rooted eulerian triangulations where t marks the number of dark triangles



$\Rightarrow$ proving $M(t) = \frac{1}{1 - tB(t)}$ bijectively reduces to finding a bijection between outer-triangular simple maps with n inner edges and eulerian triangulations with n inner dark faces

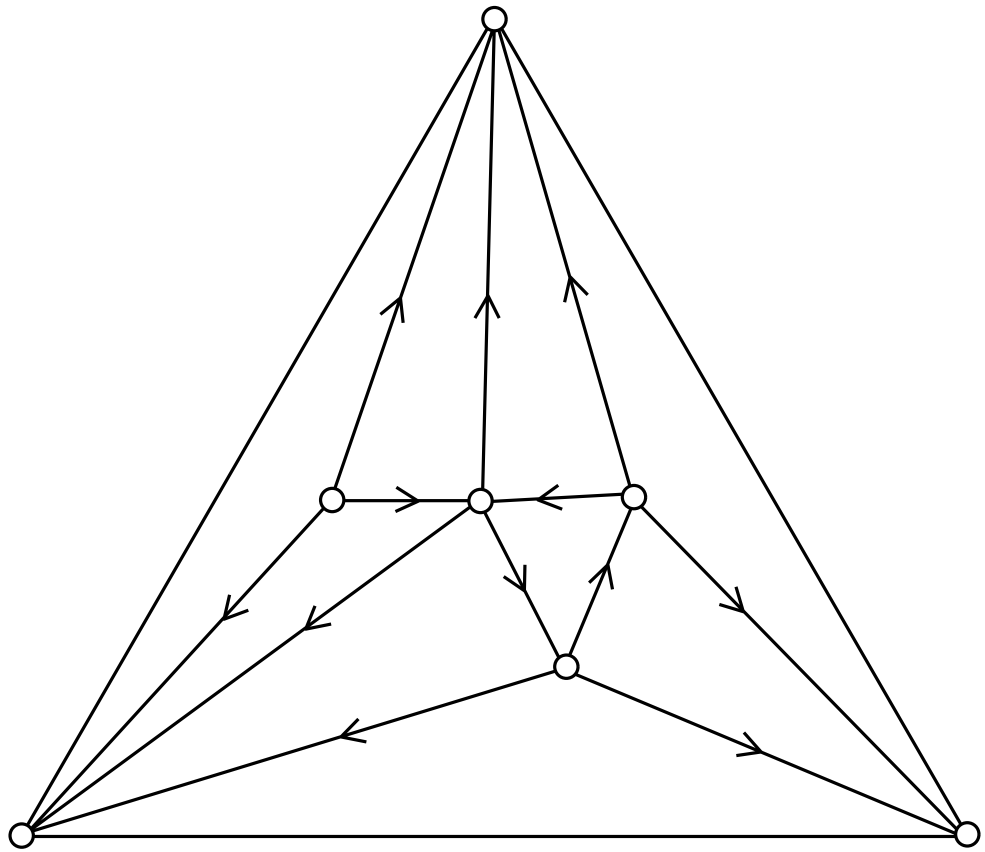


Canonical orientations for outer-triang. simple maps

Well-known: the (maximal) case of simple triangulations [Schnyder'89]

Each simple triangulation has a unique orientation such that

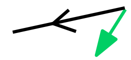
- Inner (resp. outer) vertices have outdegree 3 (resp. 0)
- no clockwise circuit

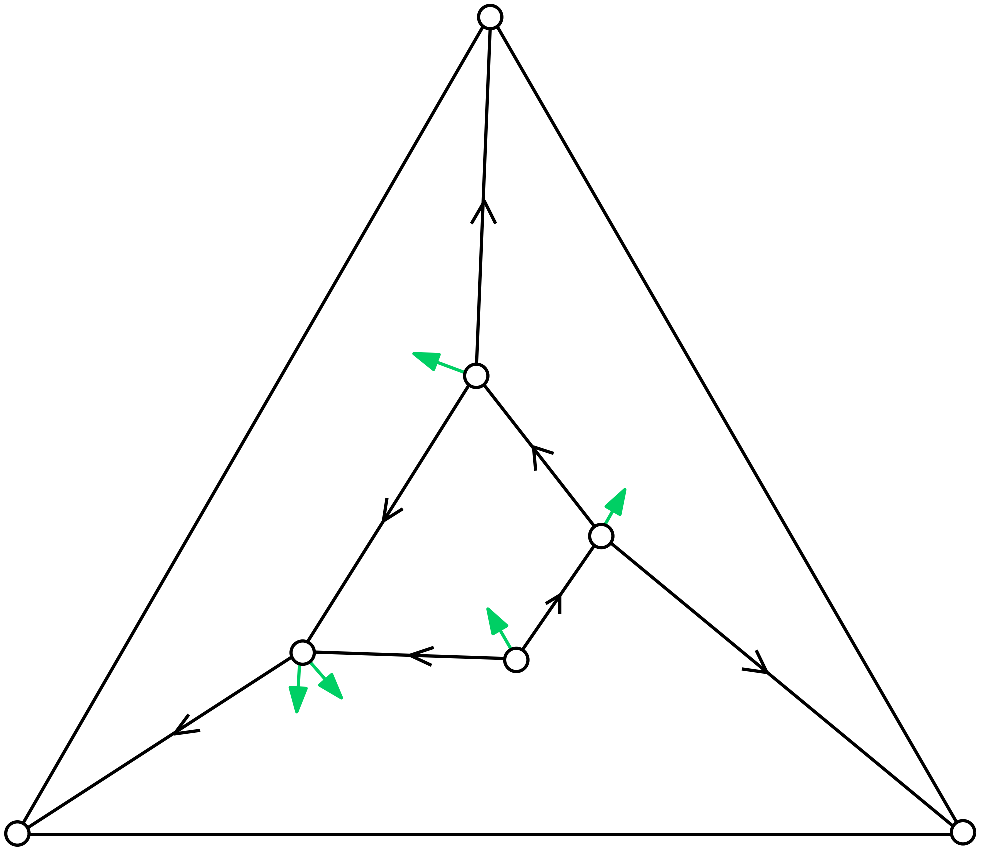


Rk: the outer face is accessible from every inner vertex

Canonical orientations for outer-triang. simple maps

General case: [Bernardi,F'11] Each outer-triangular simple map has a unique orientation "with buds" such that:

- Inner (resp. outer) vertices have outdegree 3 (resp. 0)
- each inner face of degree $3 + d$ has d buds
- no clockwise circuit
- Local property:  implies 



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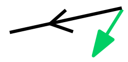
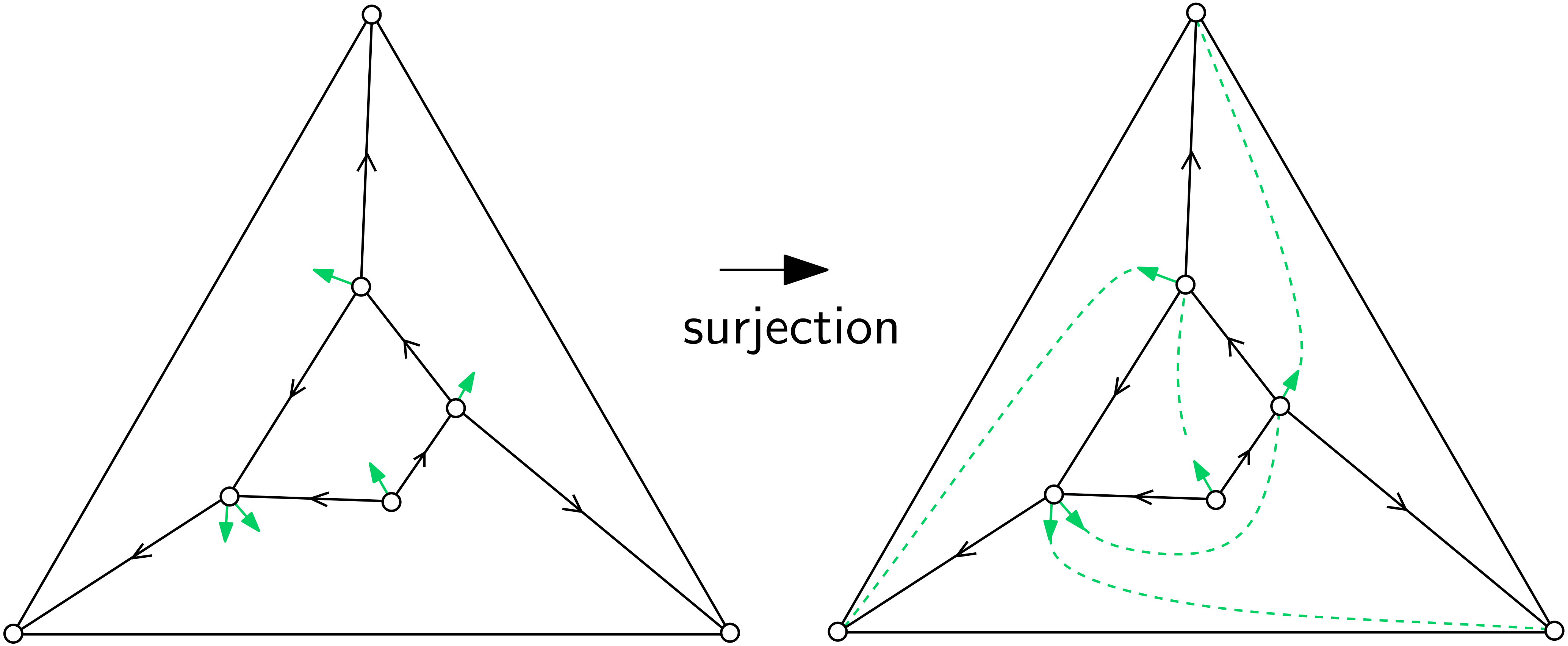
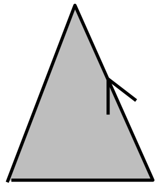
- Local property:  implies 

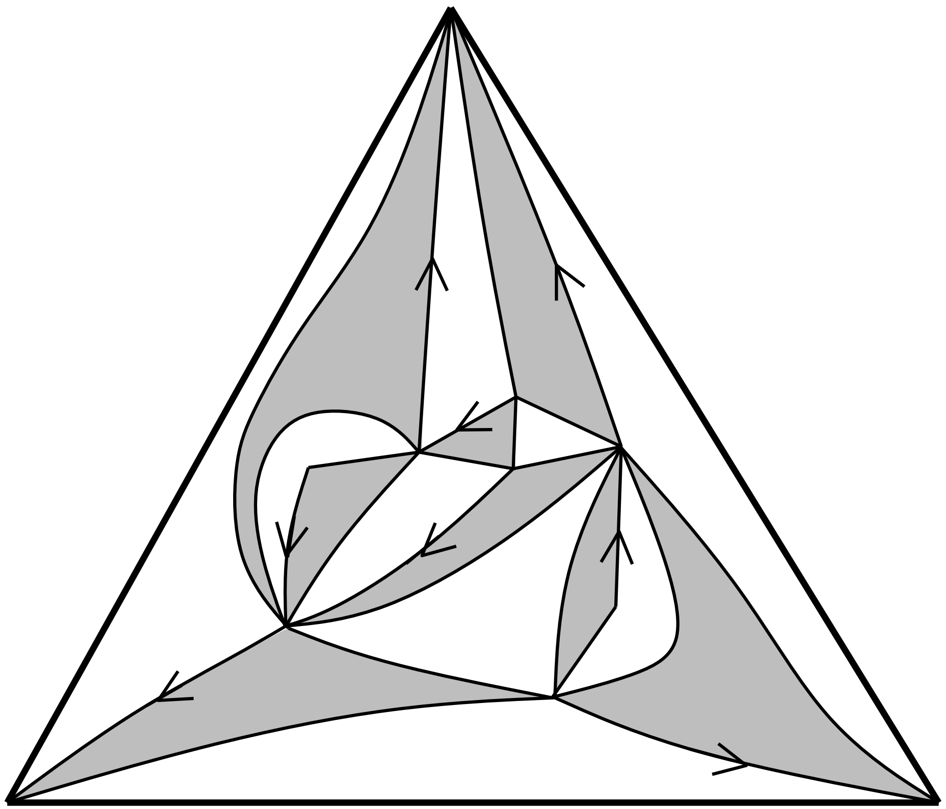


Rk: yields canonical way to triangulate an outer-triangular simple map

Canonical orientations for eulerian triangulations

[Bousquet-Mélou-Schaeffer'00]: each eulerian triangulation has a unique (partial) orientation such that:

- the oriented edges form a forest of 3 trees (one toward each outer vertex)
- each inner dark face is of the form 



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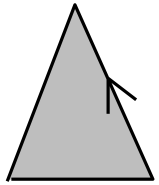
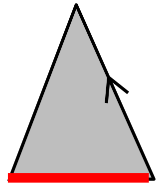
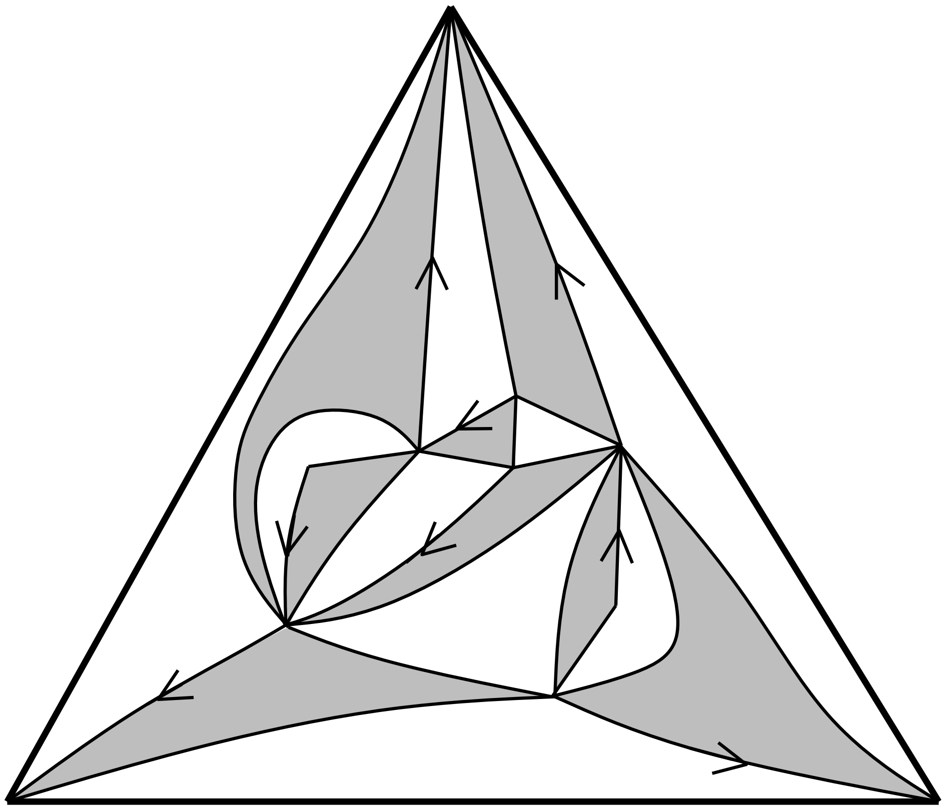
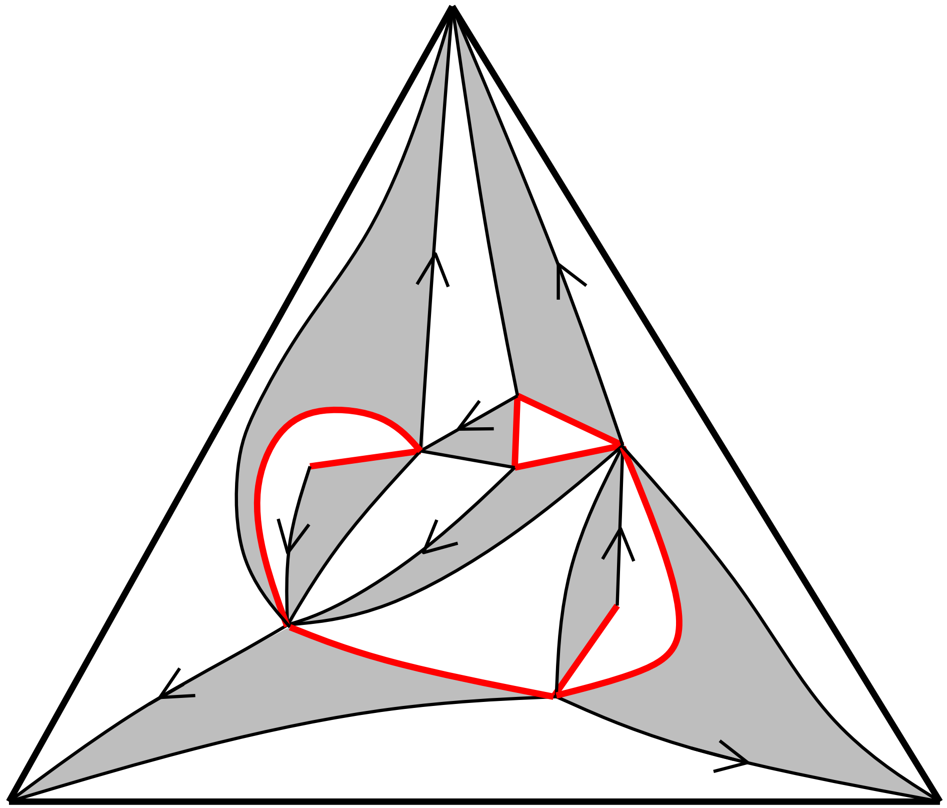


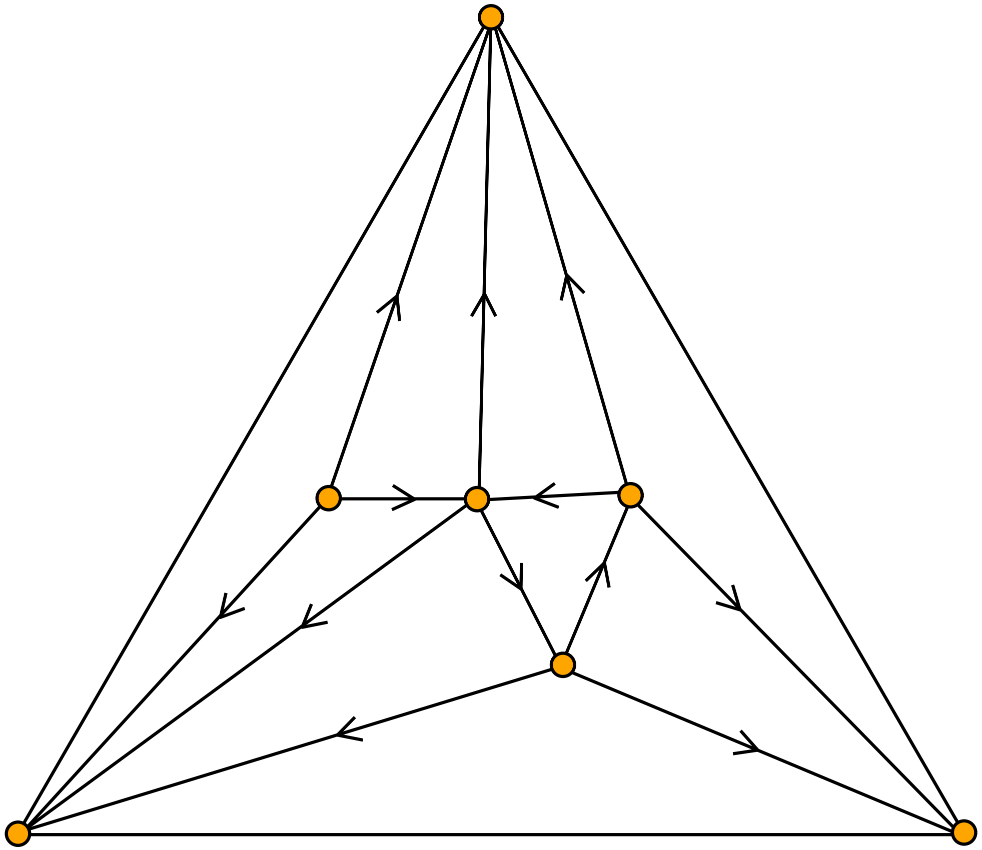


color red each "base-edge"





Simple triangulation to eulerian triangulation



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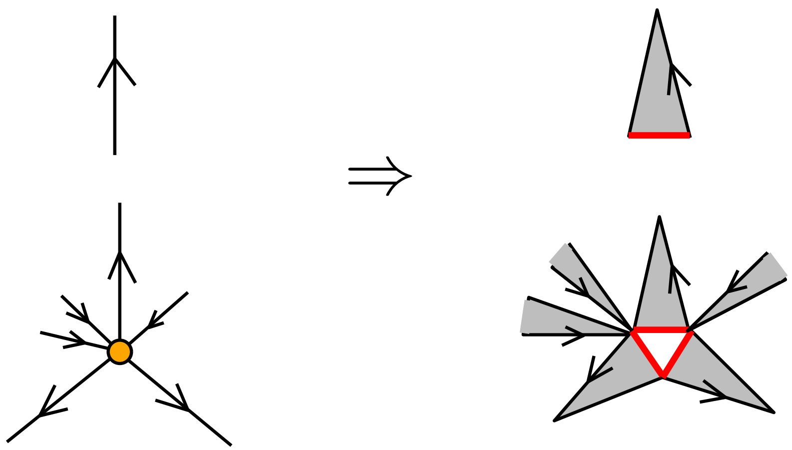

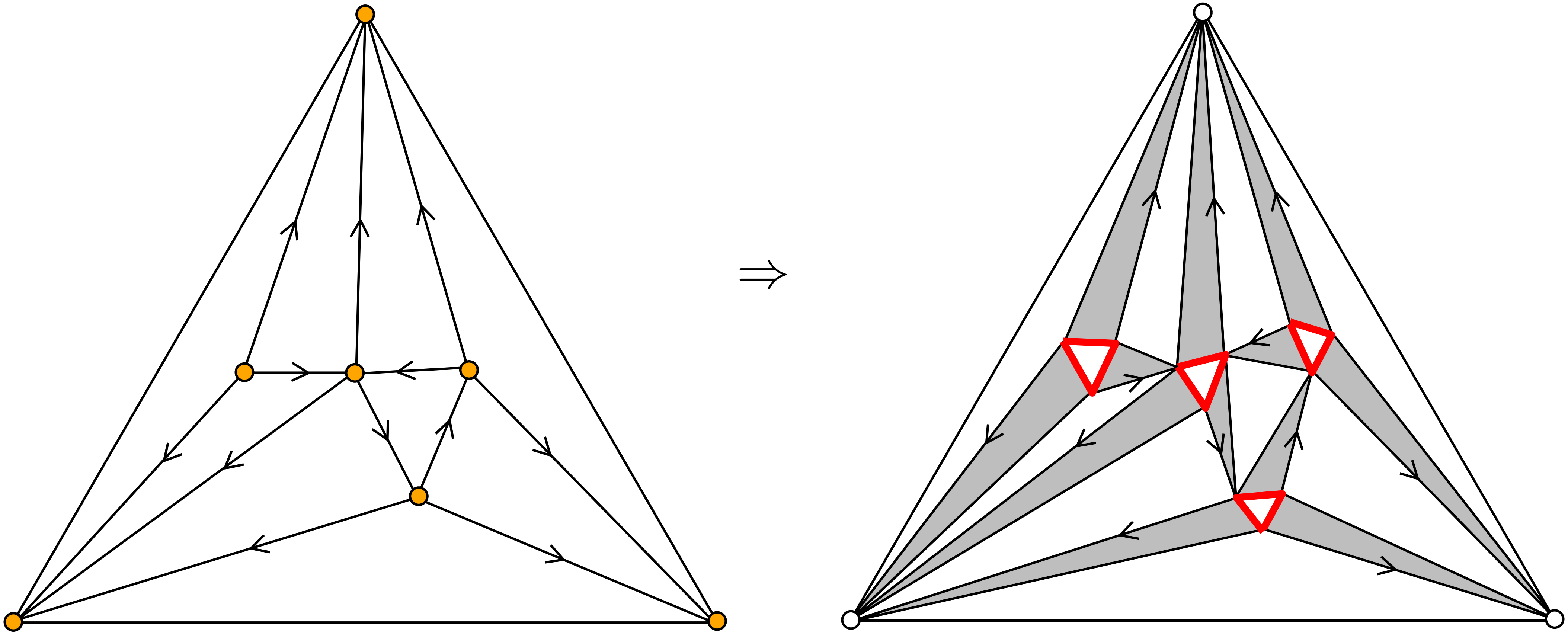

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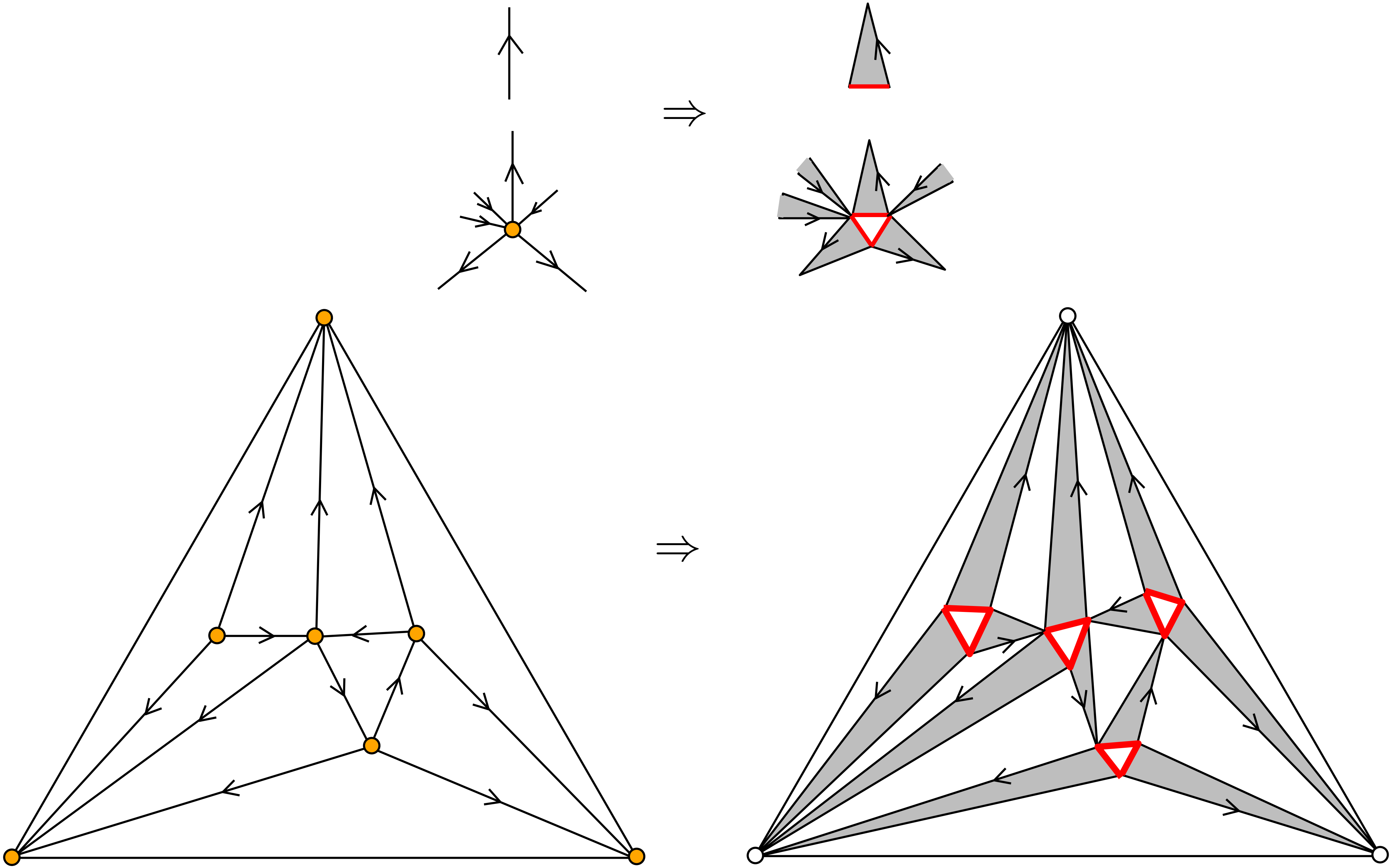
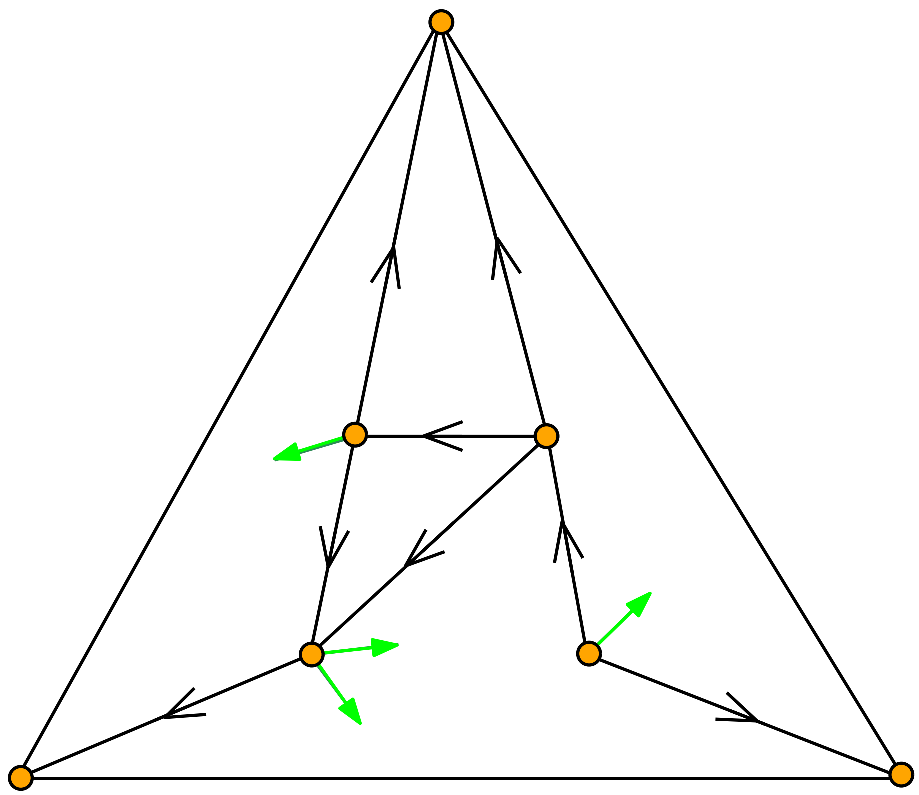


Not bijective ! (each white triangle has 0 or 3 red edges)

Simple outer-triang. map to eulerian triangulation



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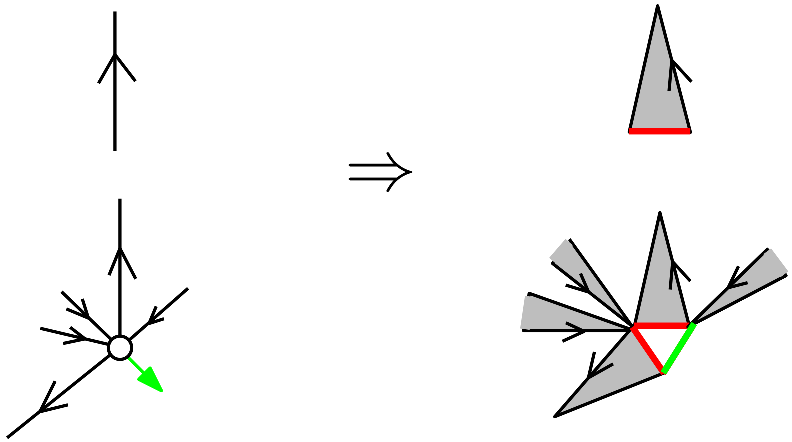
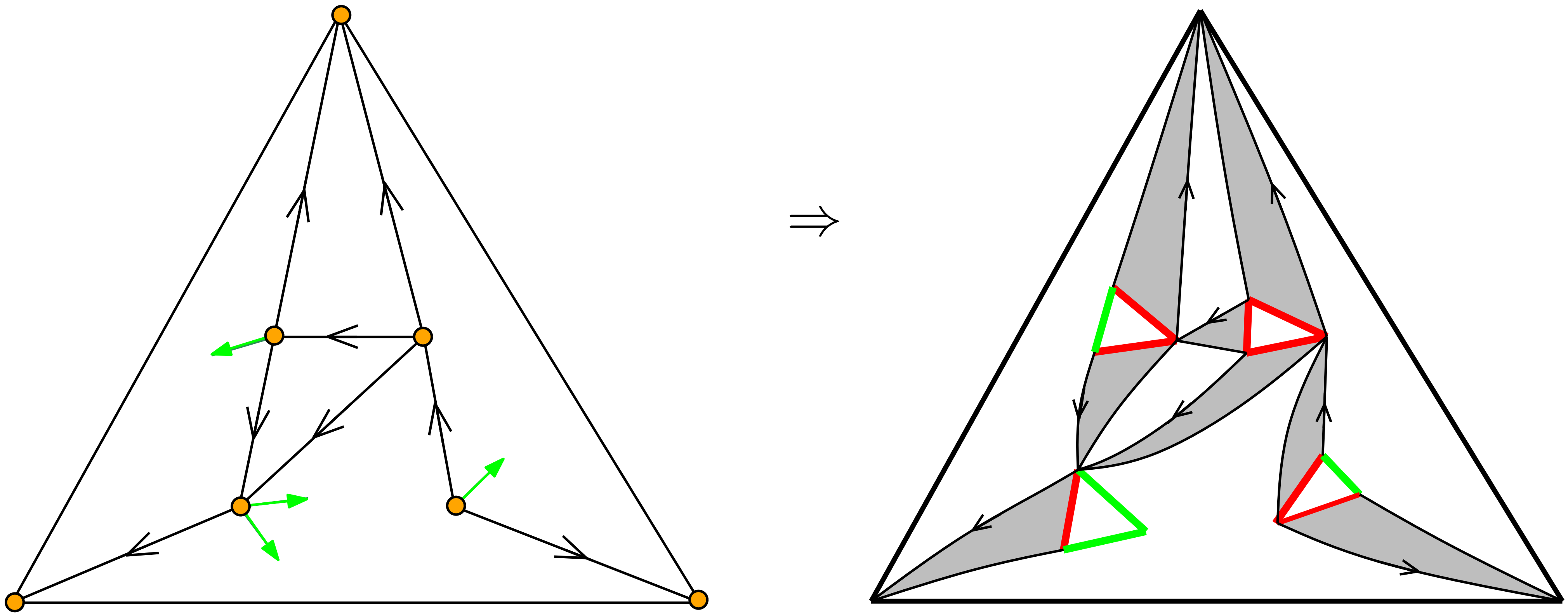
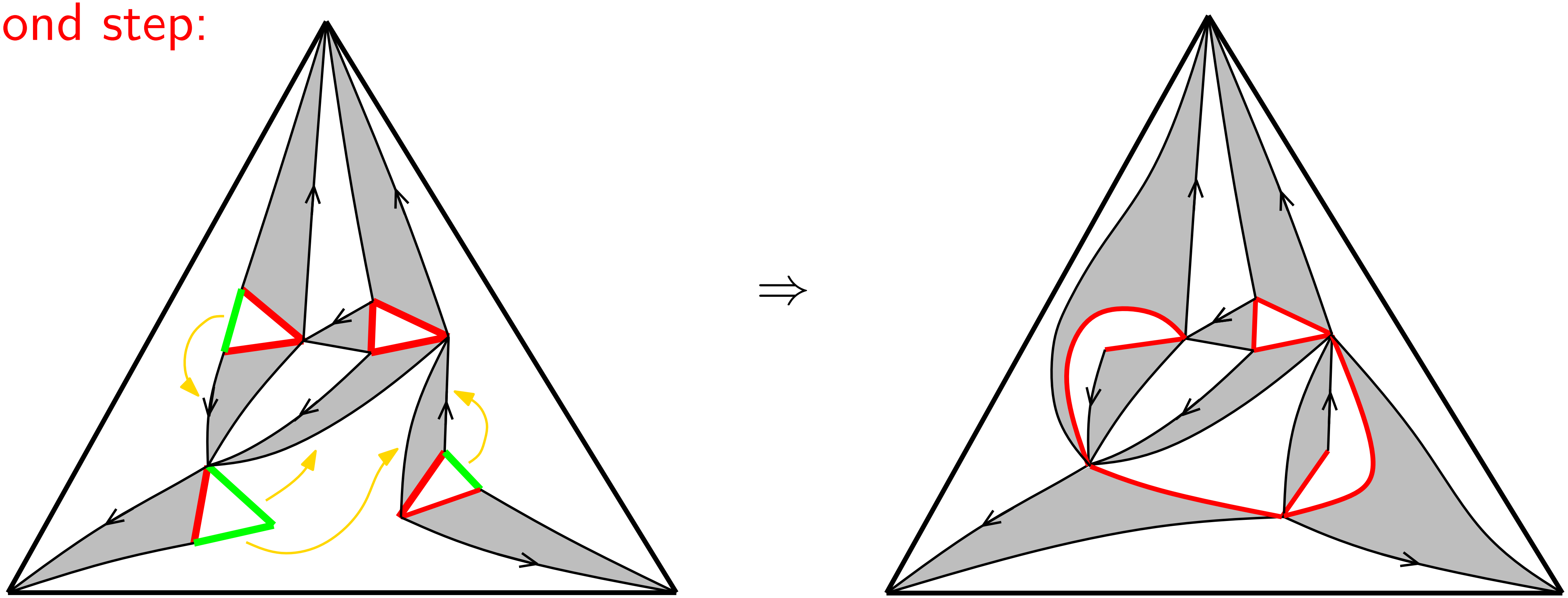
First step:



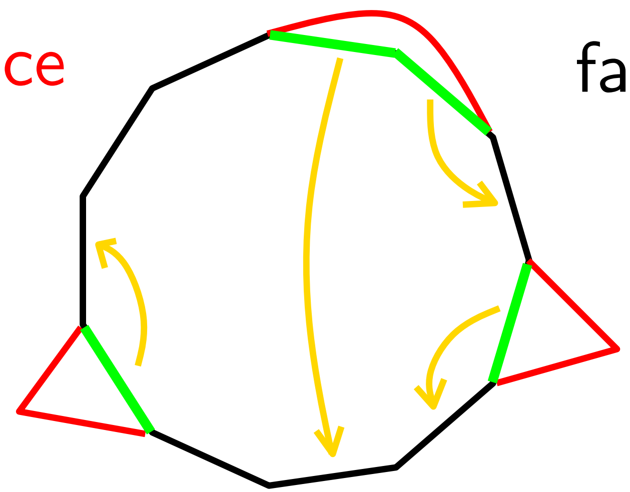


Simple outer-triang. map to eulerian triangulation

Second step:

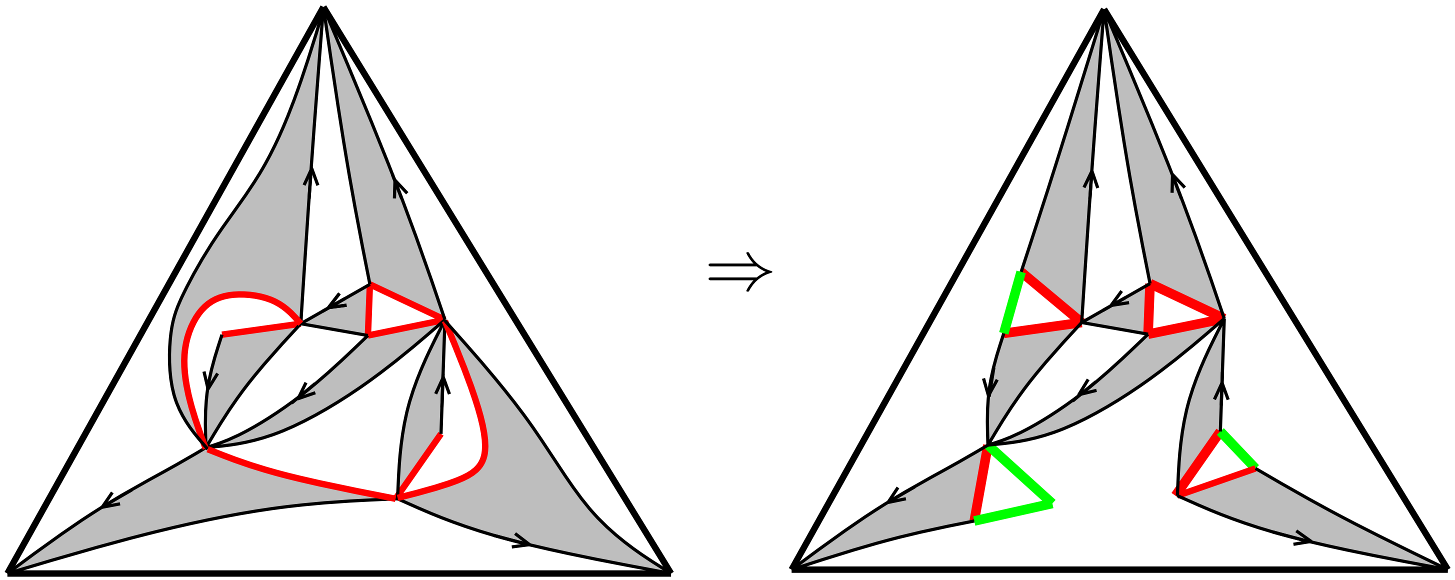


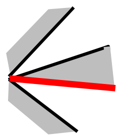
Generic situation in a face

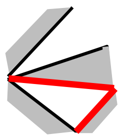
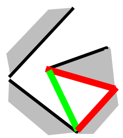


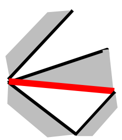
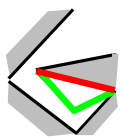
face becomes white triangle
with no red edge

Inverse mapping

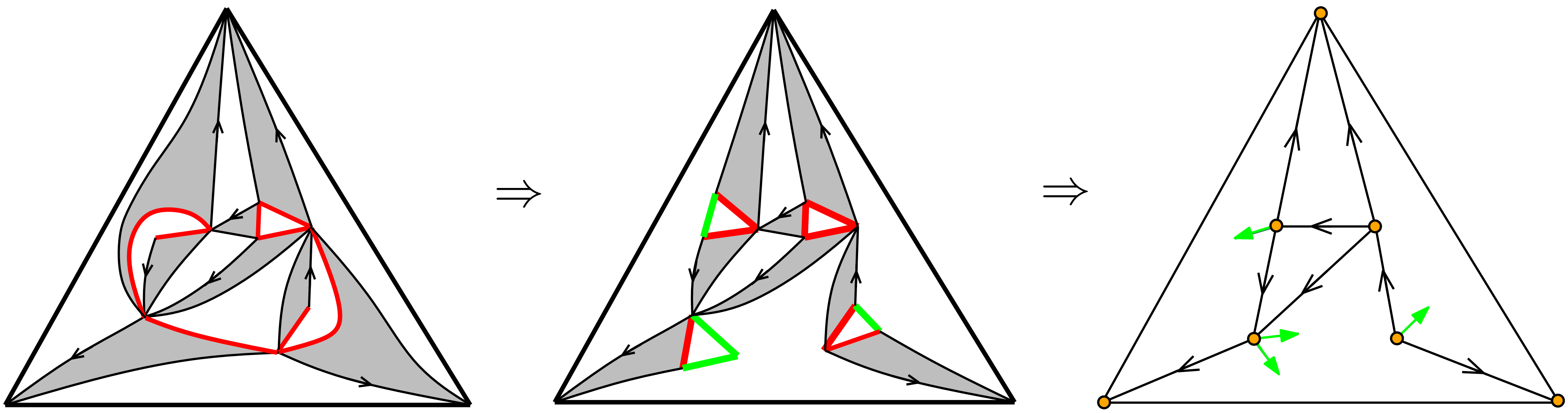


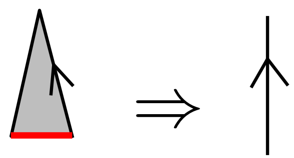
As long as there is a  do

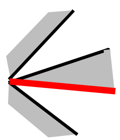
Case a  ⇒ 

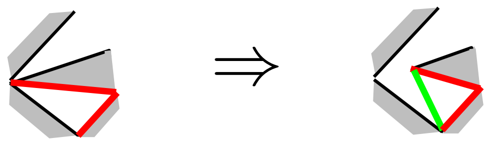
Case b  ⇒ 

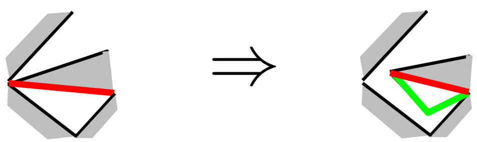
Inverse mapping





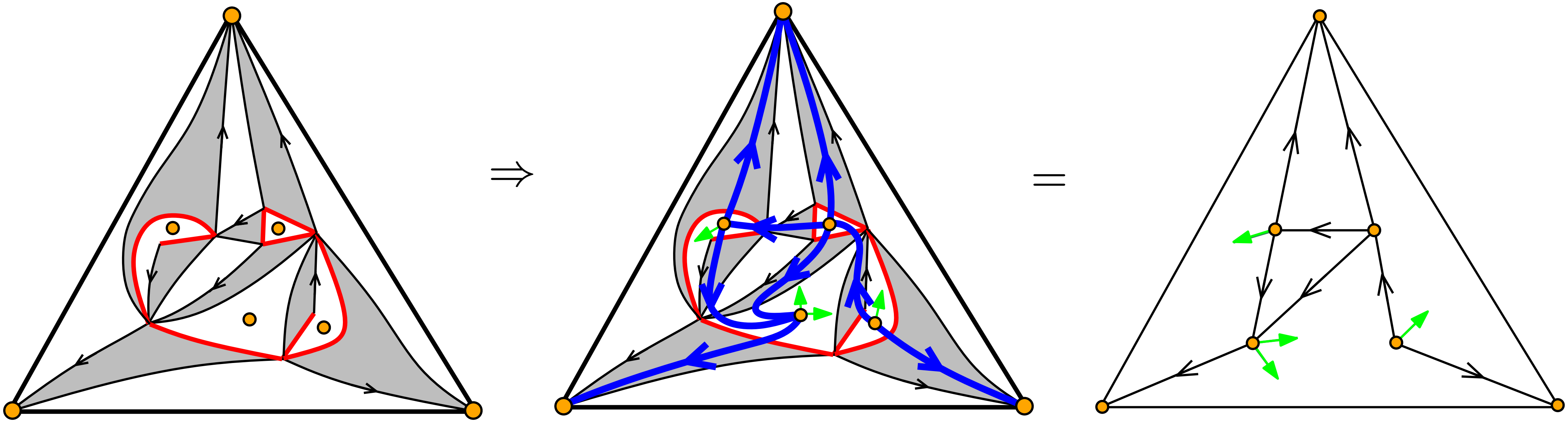
As long as there is a  do

Case a 

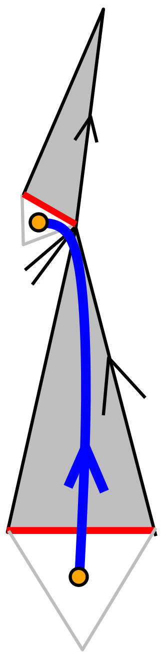
Case b 

Inverse mapping (more local formulation)

Rk: Inner vertices of the simple map are in white triangles with > 0 red edge

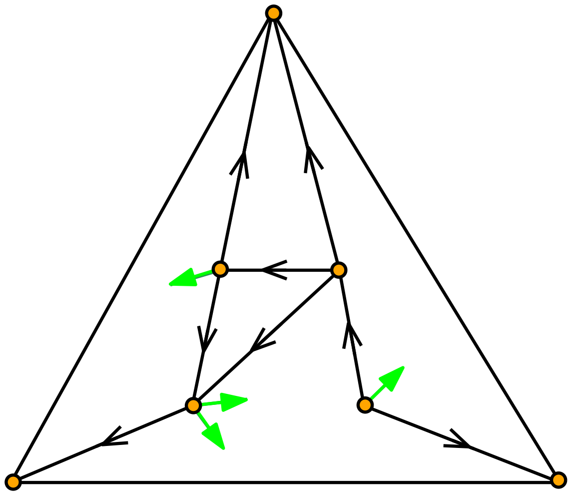


Each dark triangle
yields an edge

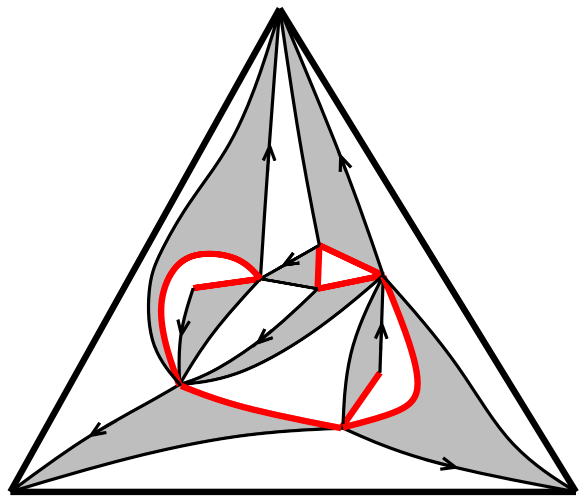


Summary

There is a bijection between outer-triangular simple maps with n inner edges and eulerian triangulations with n inner dark faces



$\leftrightarrow$



inner face $\leftrightarrow$ white face with no red edge

inner vertex with $i \in \{0, 1, 2\}$ buds $\leftrightarrow$ white face with $3 - i$ red edges

---

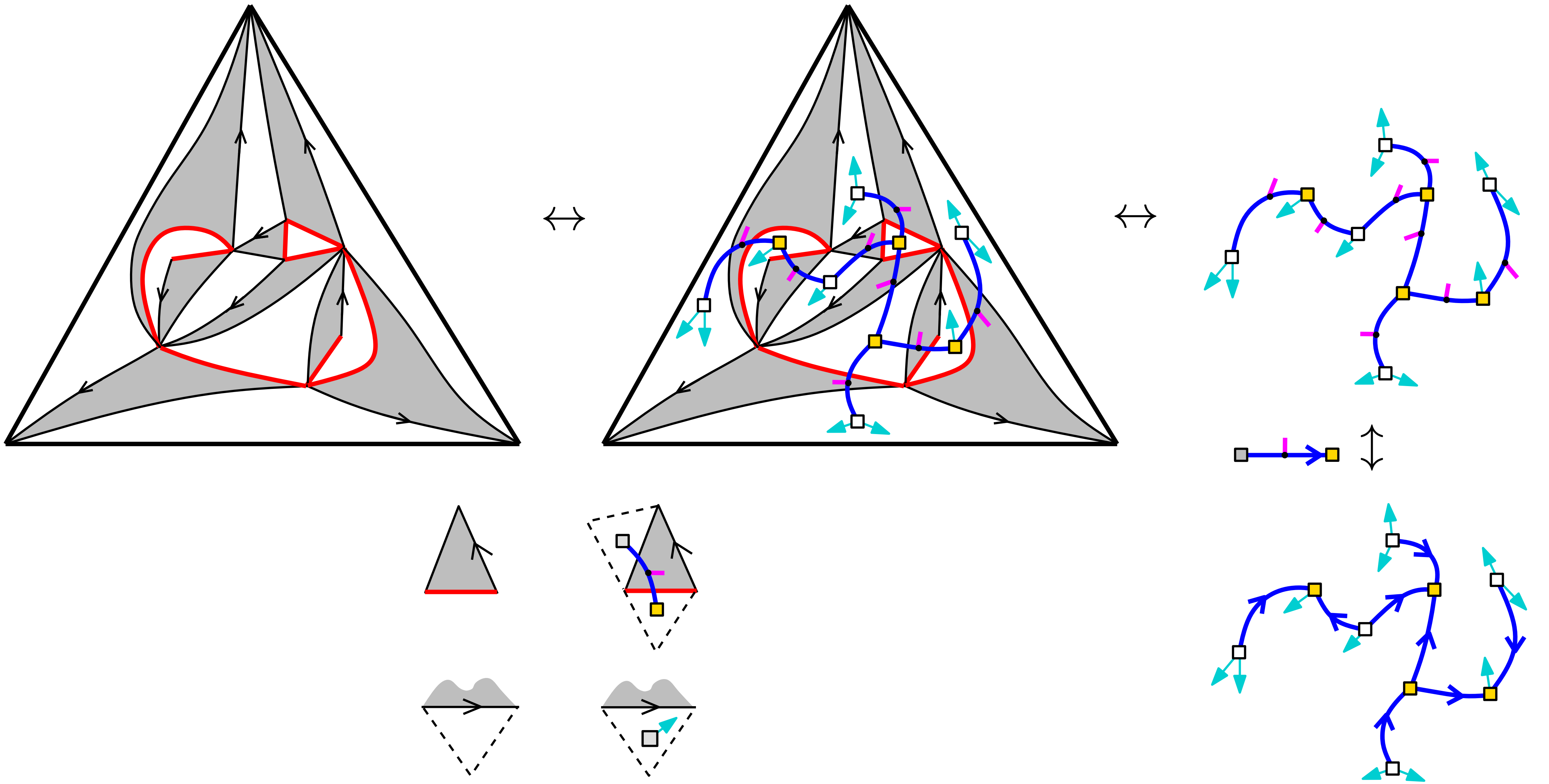
gives bijective proof of the formula

$$M(t) = \frac{1}{1 - tB(t)}$$

that links (the GFs of) rooted simple maps and bipartite maps

Eulerian triangulations ↔ oriented binary trees

[Bousquet-Mélou-Schaeffer'00] eulerian triangulations are in bijection with oriented binary trees



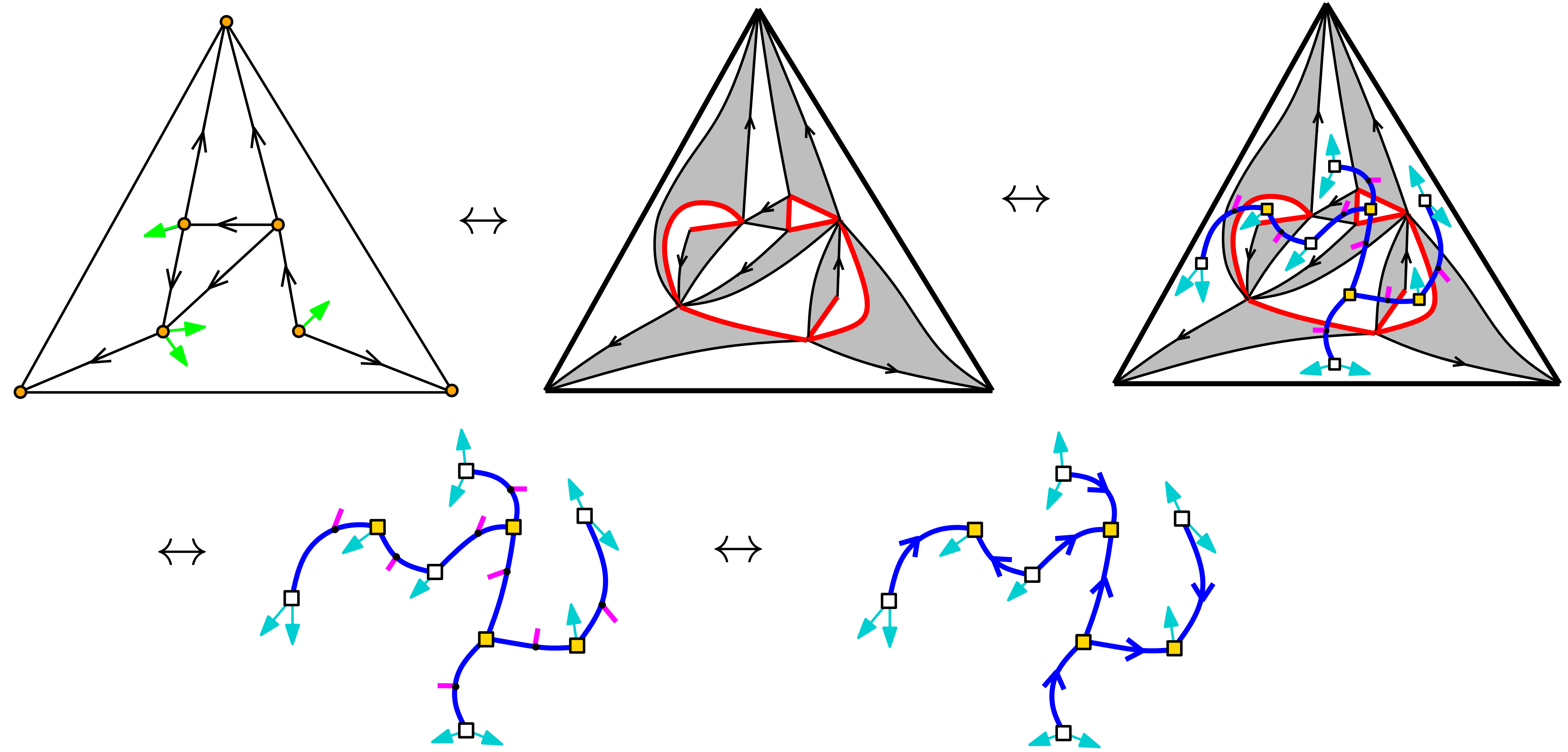
Outer-triang. simple maps ↔ oriented binary trees

Composing the bijections, we obtain a bijection:

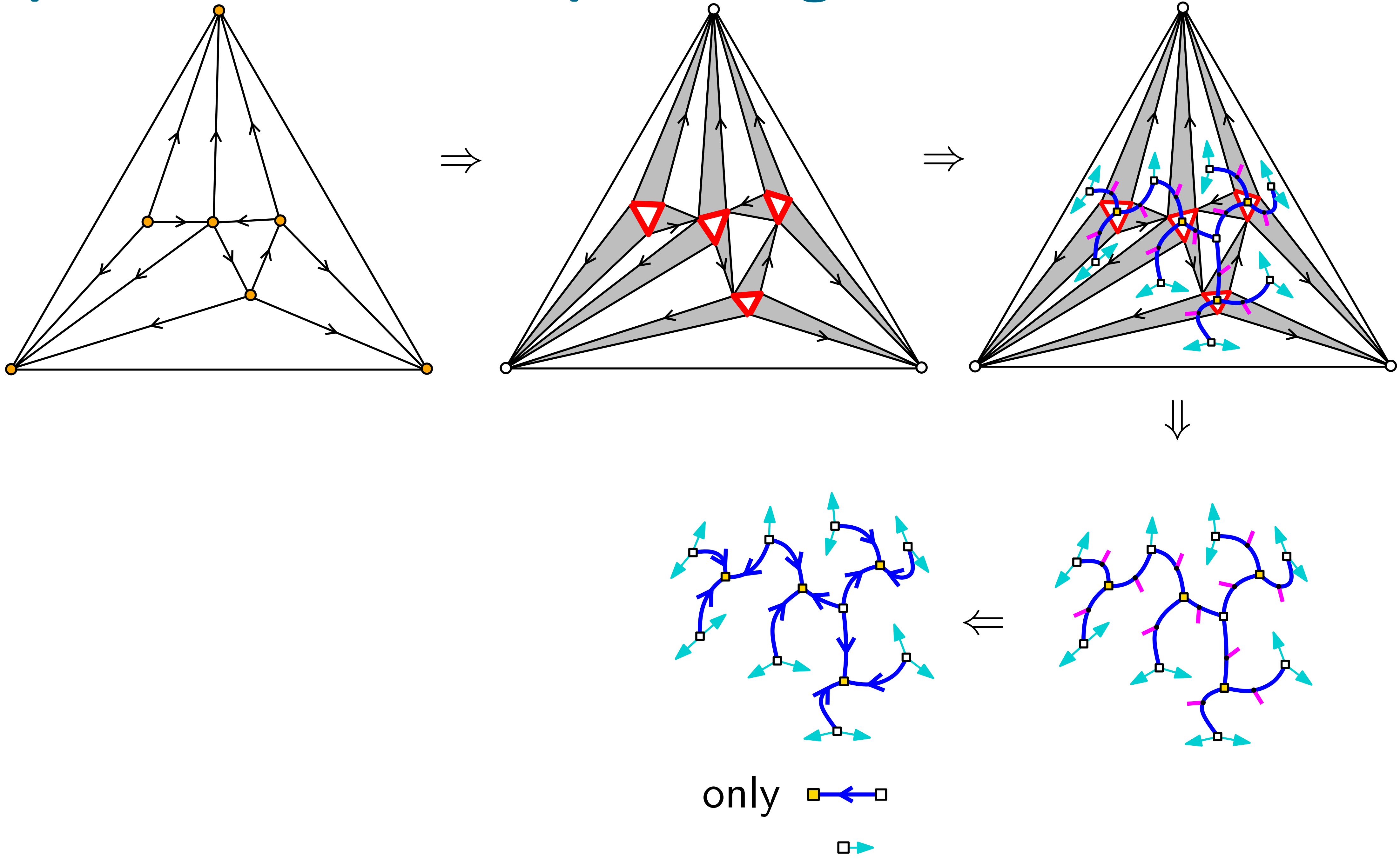
outer-triangular simple maps ↔ oriented binary trees

where

inner face ↔ source inner node
inner vertex ↔ non-source inner node



Specialization to simple triangulations



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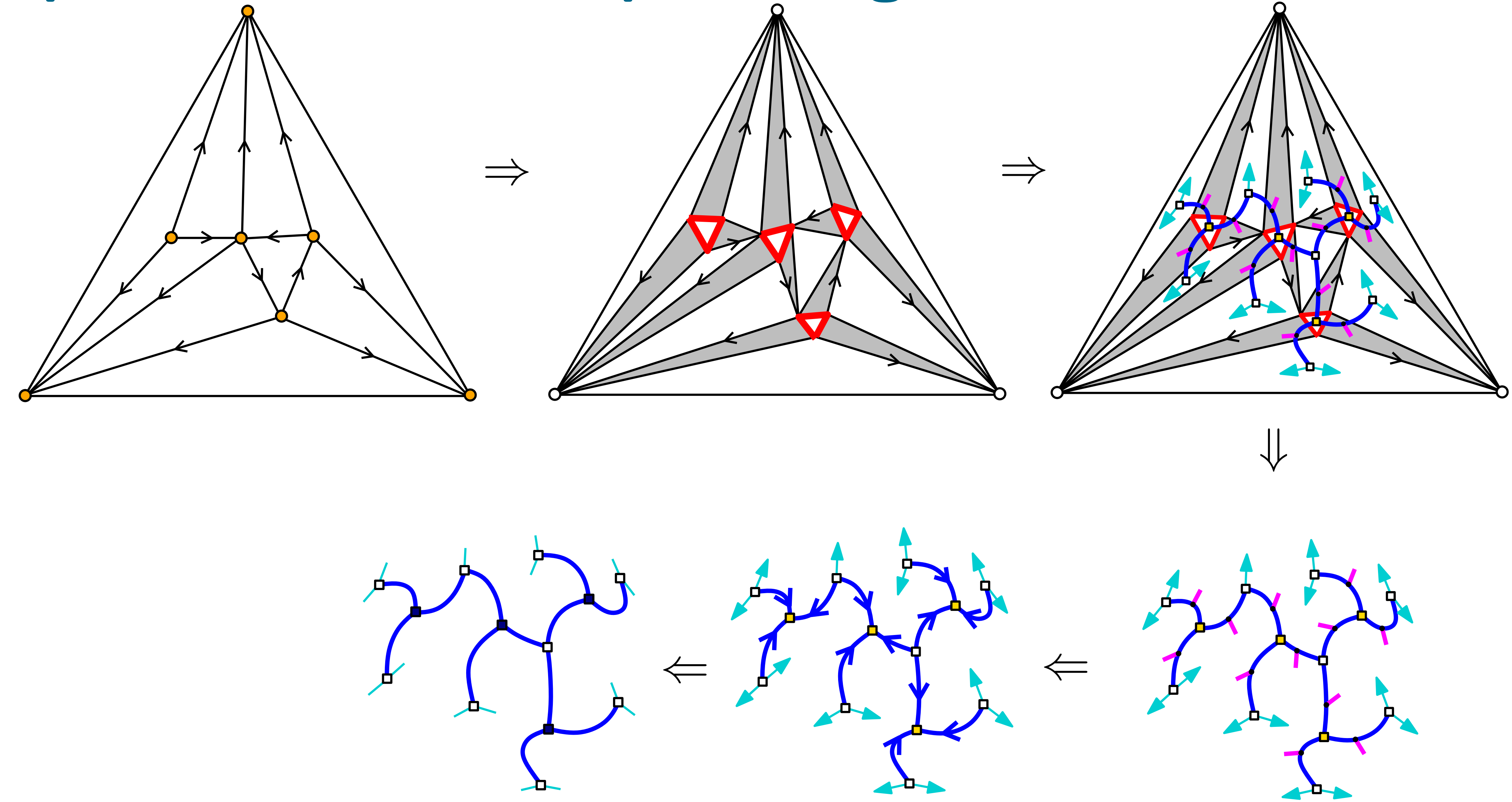
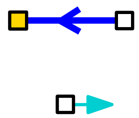
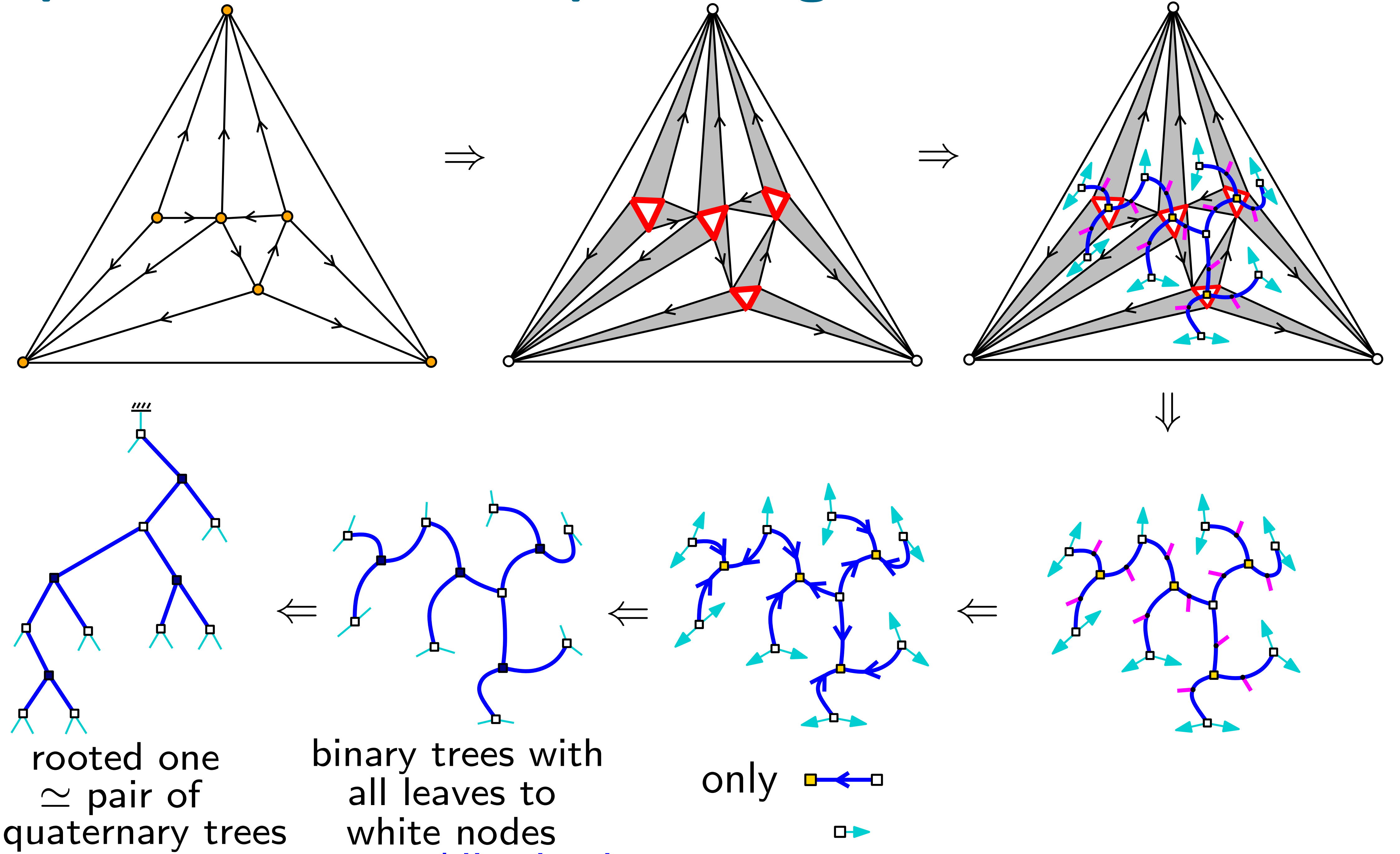


binary trees with
all leaves to
white nodes

only 

recover bijection in
[F, Poulalhon, Schaeffer'05]

Specialization to simple triangulations



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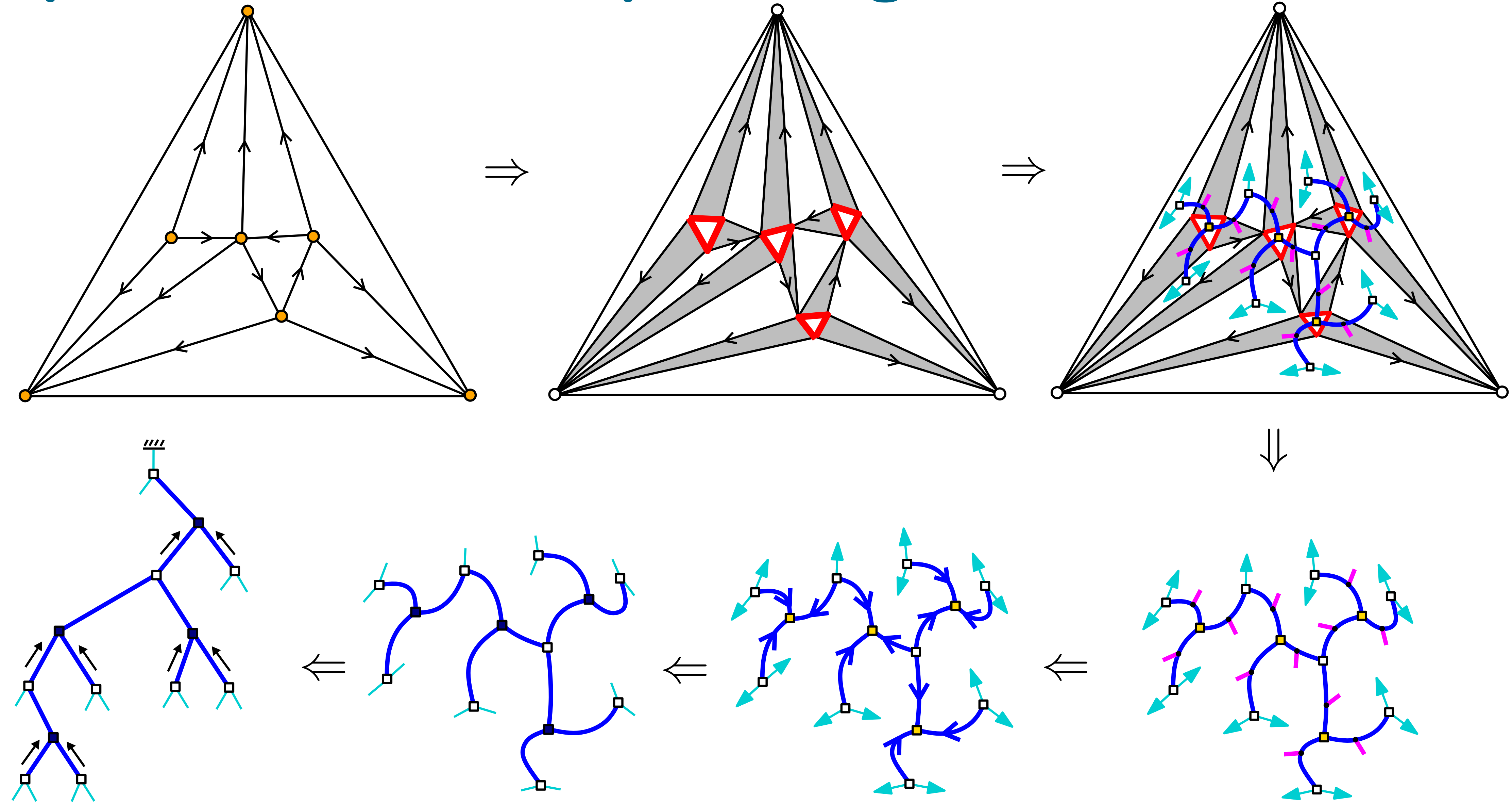
Specialization to simple triangulations



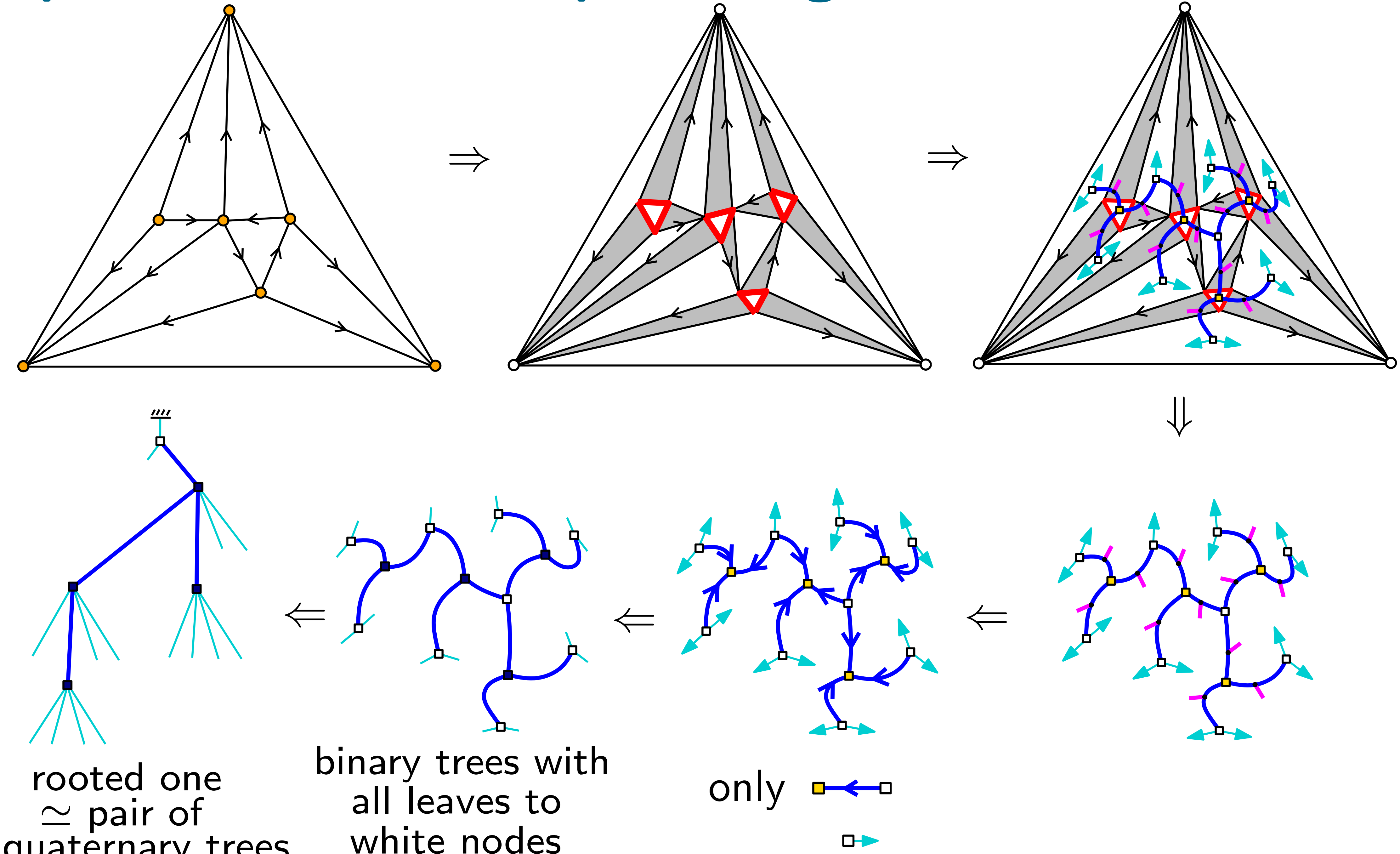
rooted one $\simeq$ pair of quaternary trees

binary trees with all leaves to white nodes

only

recover bijection in [F, Poulalhon, Schaeffer'05]

Specialization to simple triangulations



recover bijection in
[F, Poulalhon, Schaeffer'05]

Counting results

- Exact bivariate enumeration,

The series $M(t,x)$ of rooted simple maps by edges & vertices satisfies

$$M = \frac{x^2 t + x^3 U \cdot (1 - V/t)}{1 - xt - xU \cdot (1 - V/t)}$$

where $\begin{cases} U = (t+V)^2 + 2xU(t+V)^2 + xU^2 \\ V = x(t+U+V)^2 \end{cases}$

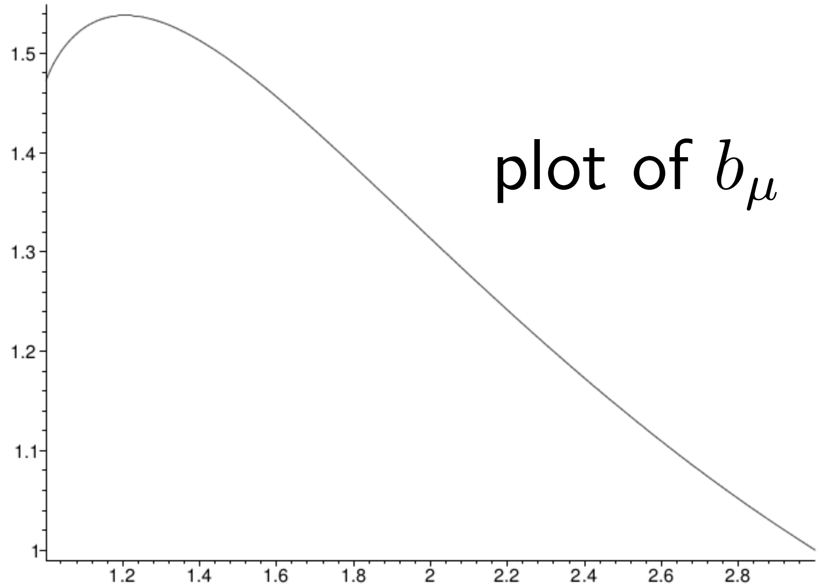
- Asymptotic expected number of planar embeddings

Let $e_{n,m}$ = number of embeddings in a random (connected unembedded) planar graph with n vertices and m edges

Then, for fixed $\mu \in (1,3)$
as $n \to \infty$ and $m/n \to \mu$

$$E(e_{n,m}) \sim c_\mu b_\mu^n$$

with c_μ, b_μ explicit

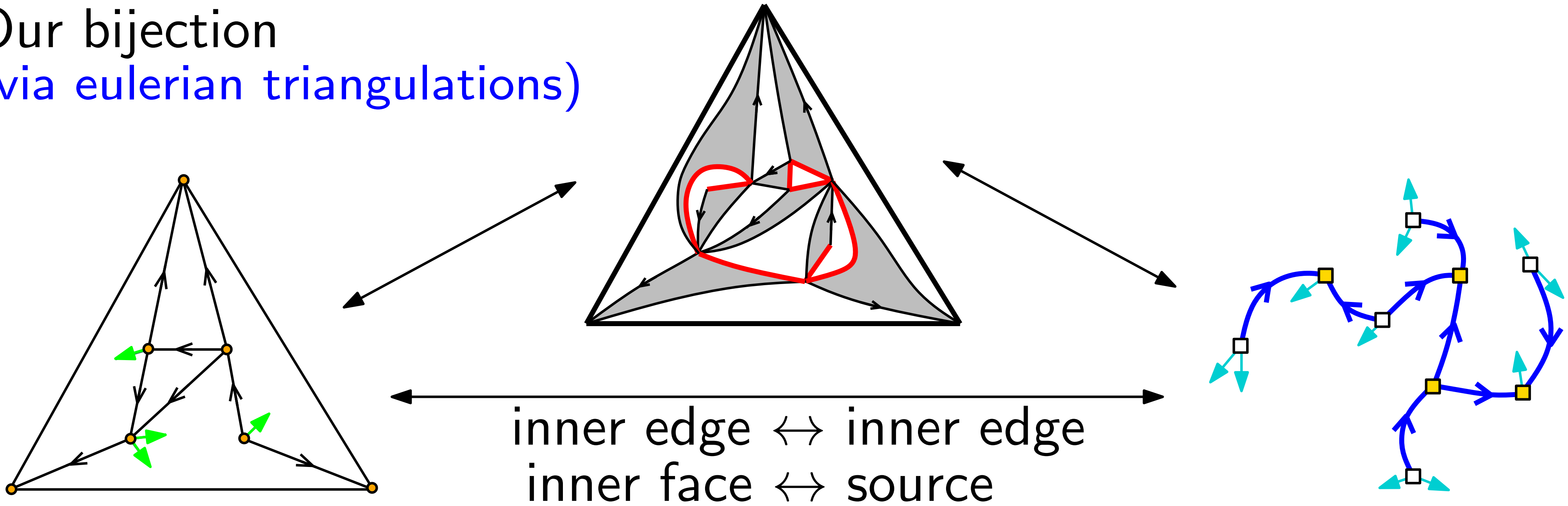


plot of b_μ

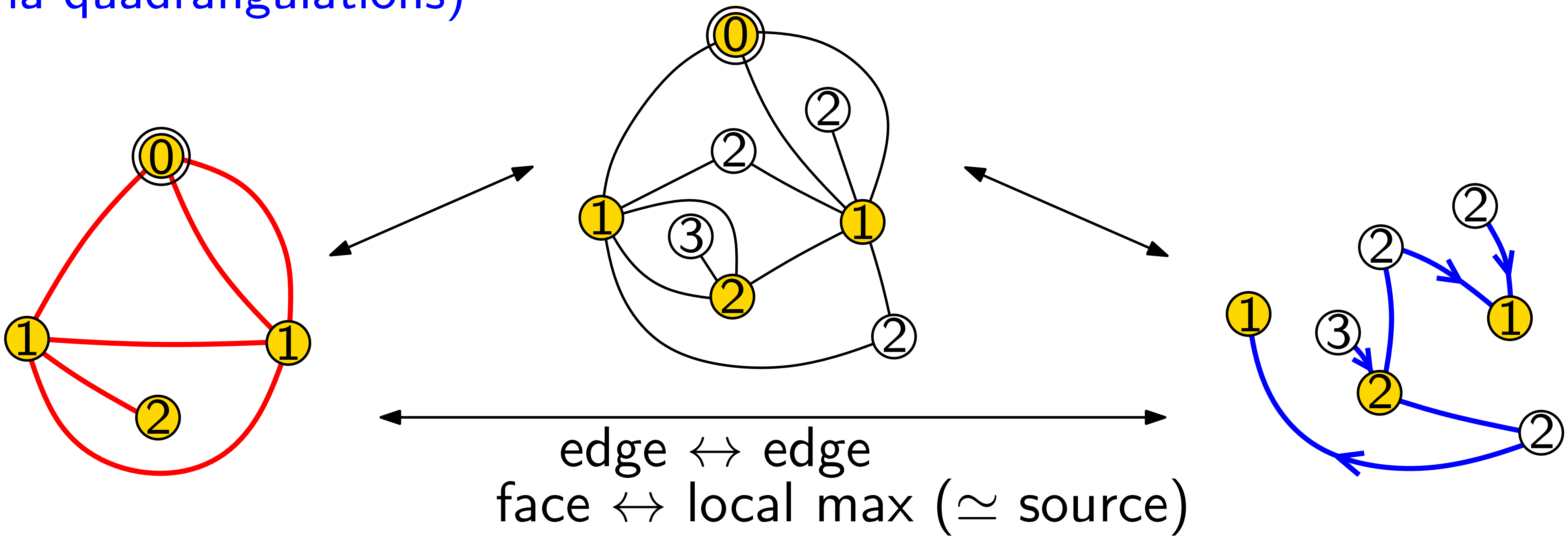
Conjecture: $\log(e_{n,\lfloor \mu n \rfloor})/n$ is concentrated around $a_\mu \leq \log(b_\mu)$

Similarities with the Ambjørn-Budd bijection

- Our bijection
  (via eulerian triangulations)



- The Ambjørn-Budd bijection
  (via quadrangulations)

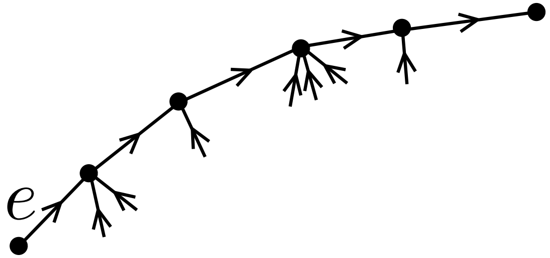


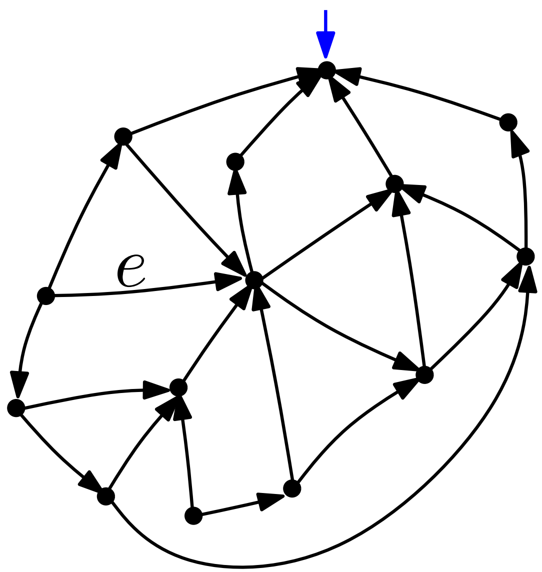
Typical distances, scaling limit

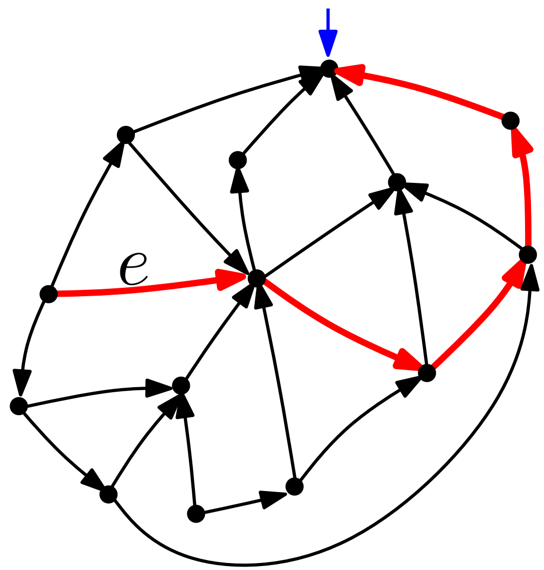
Rightmost paths

Let O be an orientation of a rooted planar map with no cw circuit and such that the root is accessible from every vertex

For e an edge of O, the **rightmost path** from e is the unique directed path $P(e)$ starting at e that turns right "as much as possible"







[Bernardi'06] the rightmost path ends at the root (does not loop)

Rightmost paths can be considered on canonical 3-orientations of simple triangulations (more generally of outer-triangular simple maps)

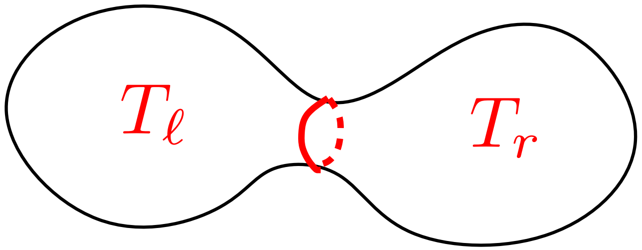
Rightmost paths are quasi-geodesic in 3-orientations

[Addario-Berry&Albenque'2013]

Lemma: Let T a simple triangulation with n vertices, e an edge of T
If there is another path Q from e to the root such that

$$|Q| \leq |P(e)| - \epsilon n^{1/4}$$

then one can extract from $P(e) \cup Q$ a cycle C of length $O(1/\epsilon)$
such that both parts T_ℓ, T_r after cutting along C have diameter $\Omega(\epsilon n^{1/4})$



Proposition: Let $A_{n,\epsilon}$ the event that a random simple triangulation with n vertices has an edge e such that $\mathrm{dist}(e, \mathrm{root}) \leq |P(e)| - \epsilon n^{1/4}$.

Then $P(A_{n,\epsilon}) \to 0$ as $n \to \infty$

Rightmost paths are quasi-geodesic in 3-orientations

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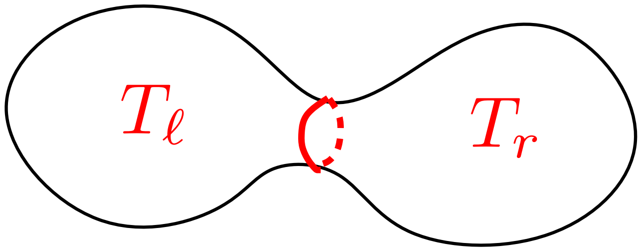

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Then $P(A_{n,\epsilon}) \to 0$ as $n \to \infty$

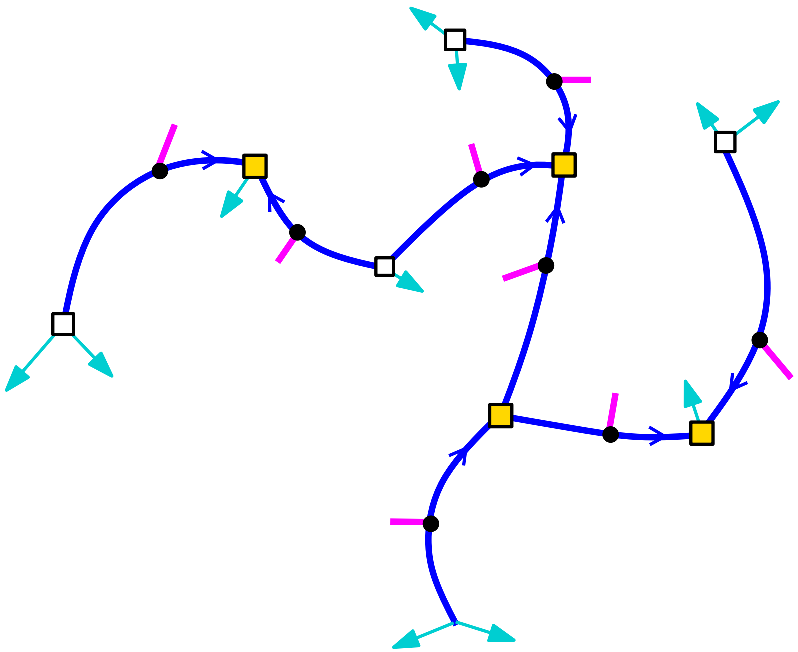
From the same lemma, we can prove the analogue proposition for random simple outer-triangular maps with n edges

The bijection starting from oriented binary trees

[Bousquet-Mélou&Schaeffer'00]: Turning ccw around the tree, consider

as opening parentheses

as closing parentheses



oriented binary tree

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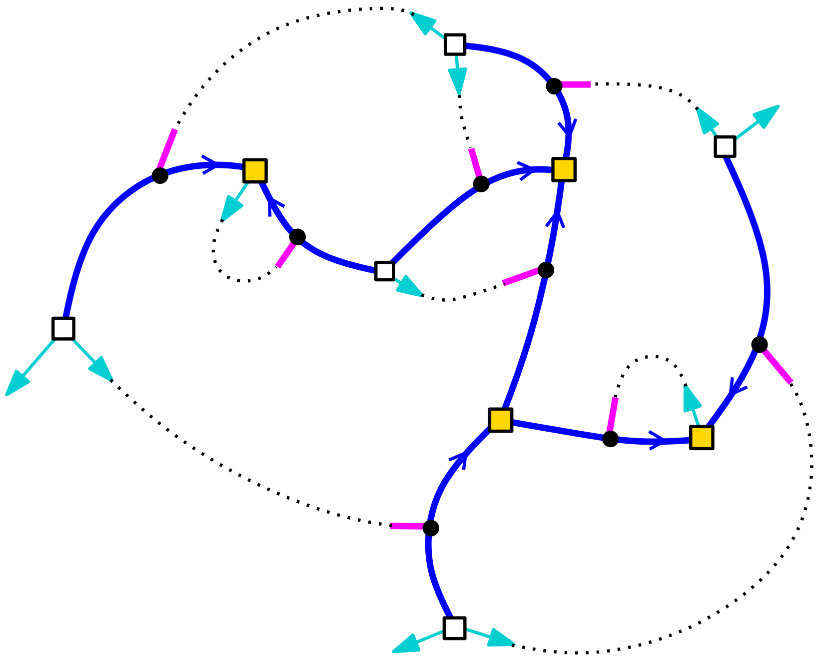

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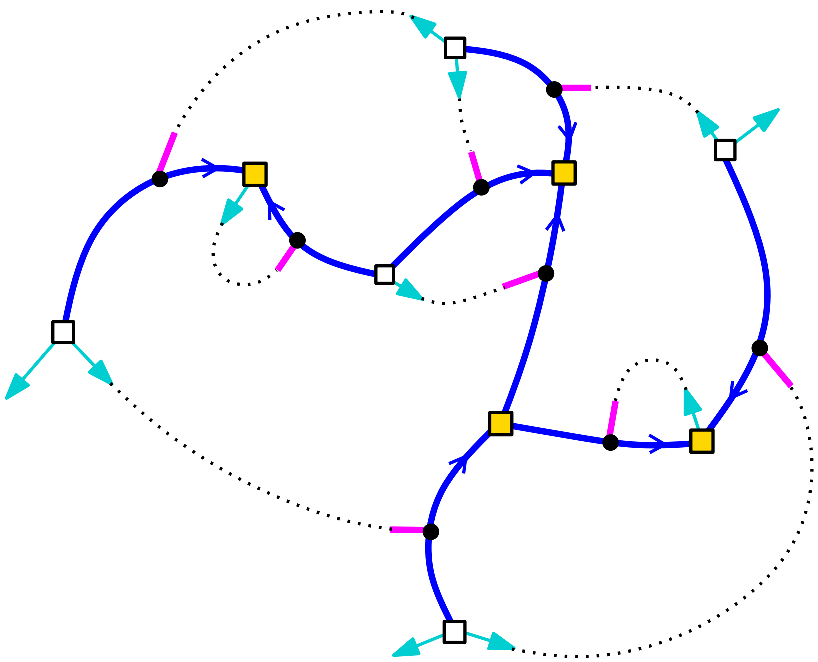
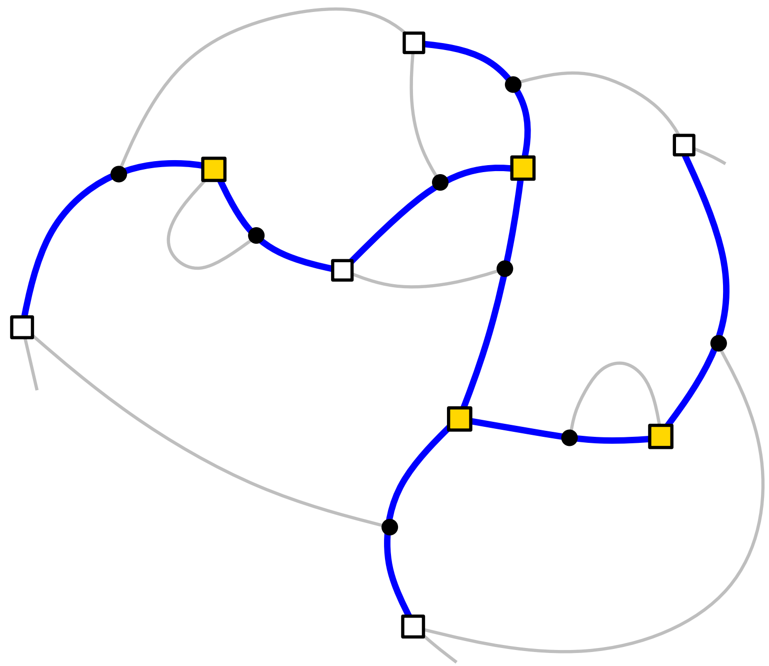


oriented binary tree



The bijection starting from oriented binary trees

1st step: oriented binary tree $\rightarrow$ vertex-pointed bipartite cubic map

[Bousquet-Mélou&Schaeffer'00]: Turning ccw around the tree, consider

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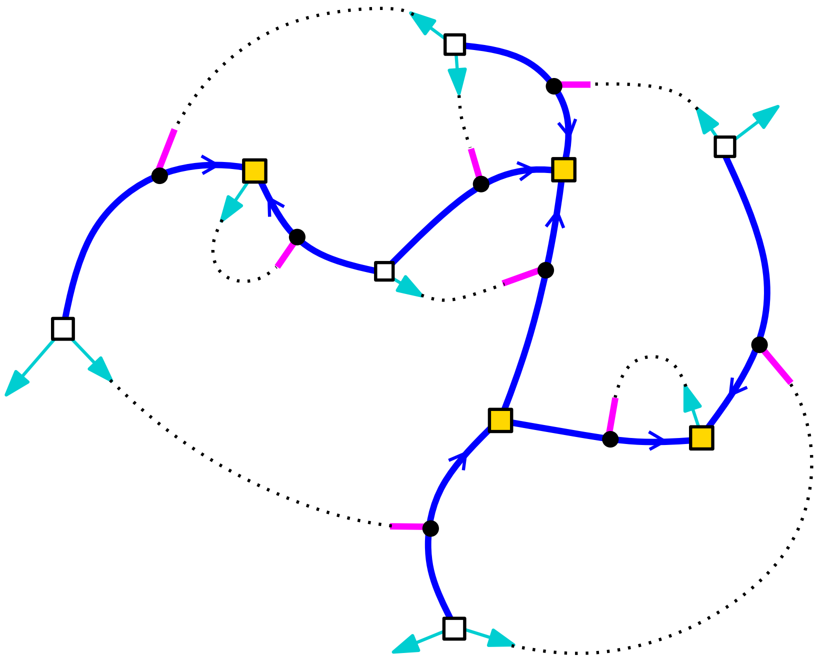
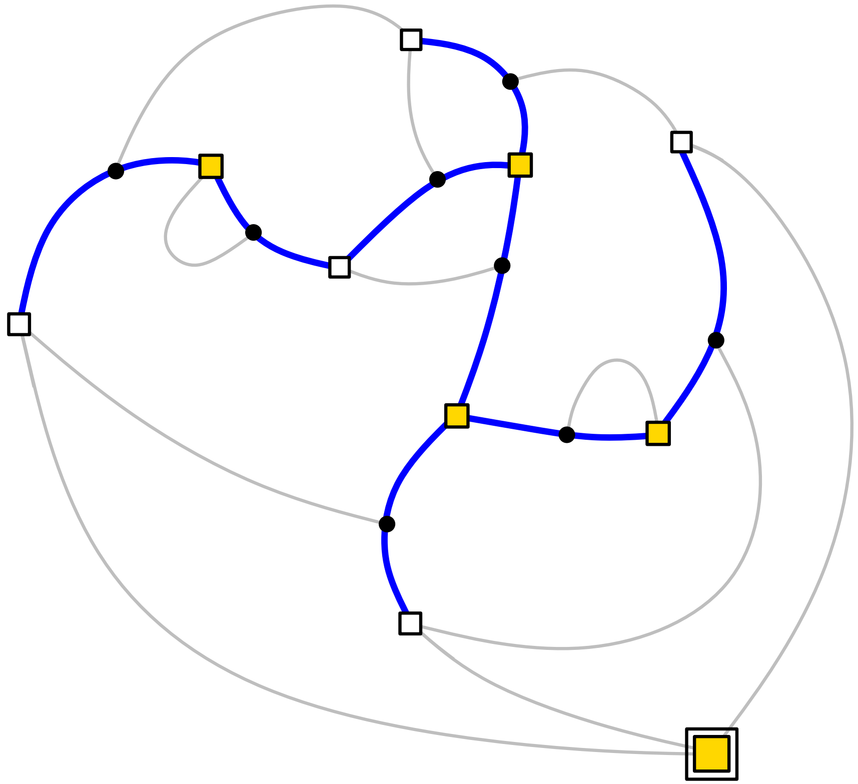


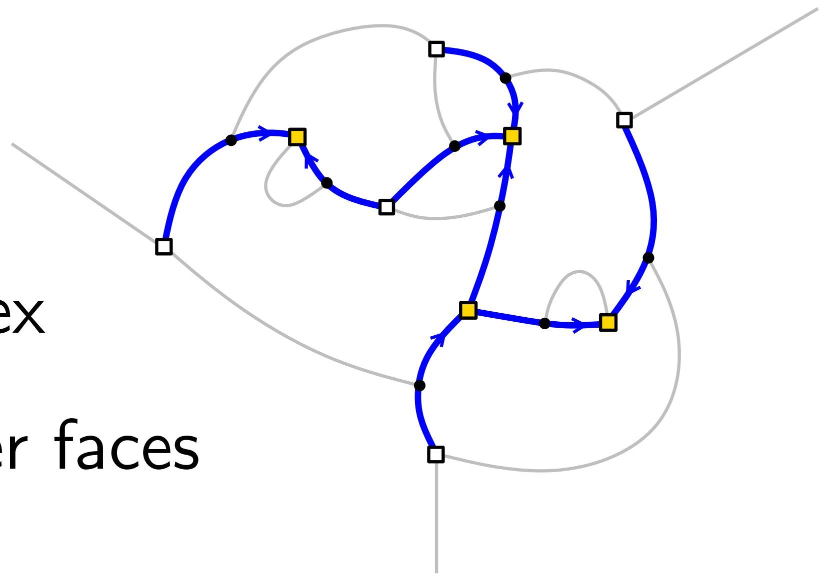
oriented binary tree



vertex-pointed bipartite cubic map (dual to eulerian triangulation)

The bijection starting from oriented binary trees

2nd step: pointed bicubic map $\rightarrow$ outer-triangular simple map



pointed vertex
at infinity
hence 3 outer faces

The bijection starting from oriented binary trees

2nd step: pointed bicubic map $\to$ outer-triangular simple map

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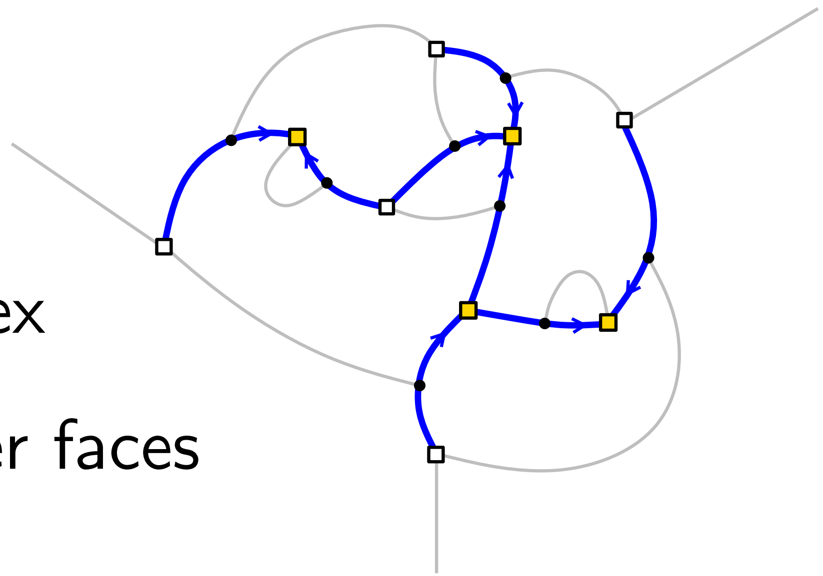
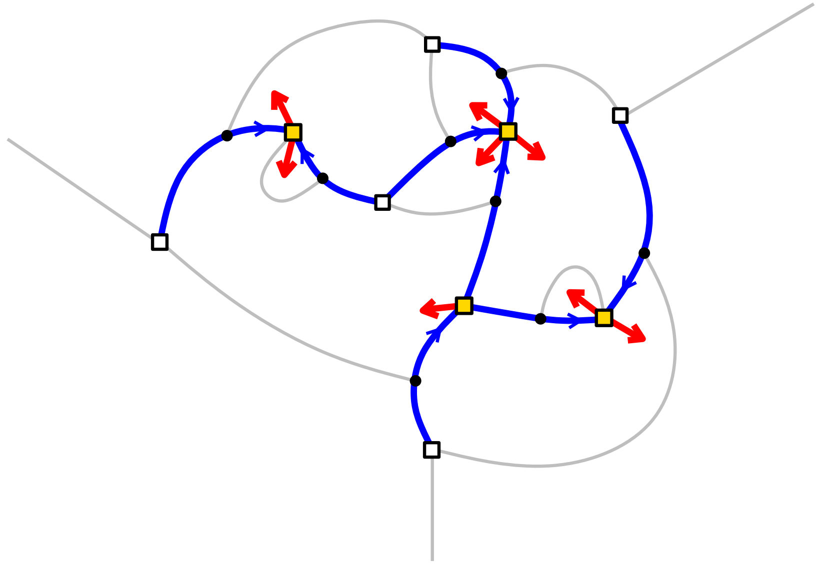
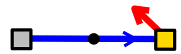




local rule:

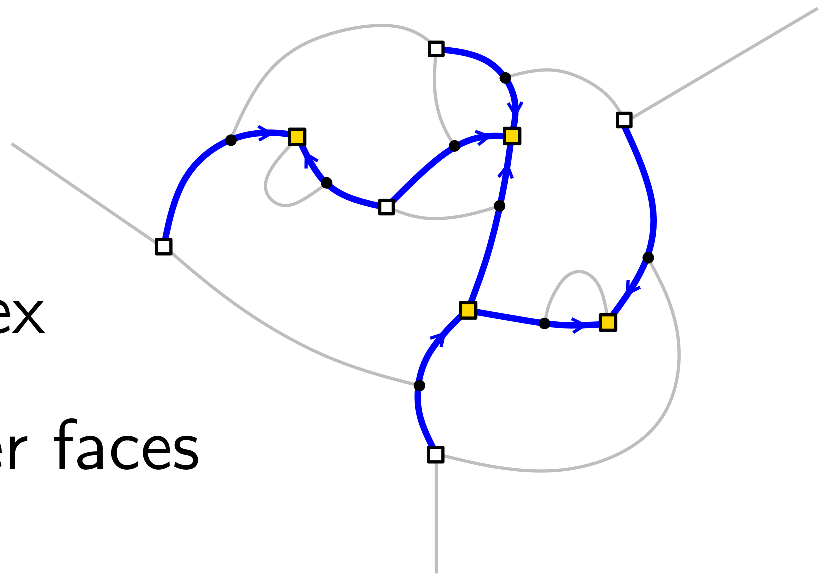


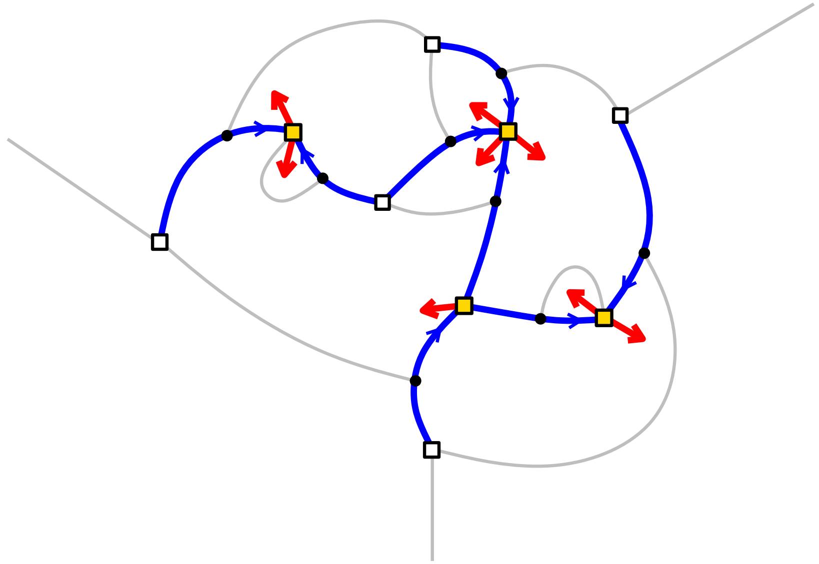
one ↑ in each inner face f
call v_f the incident vertex

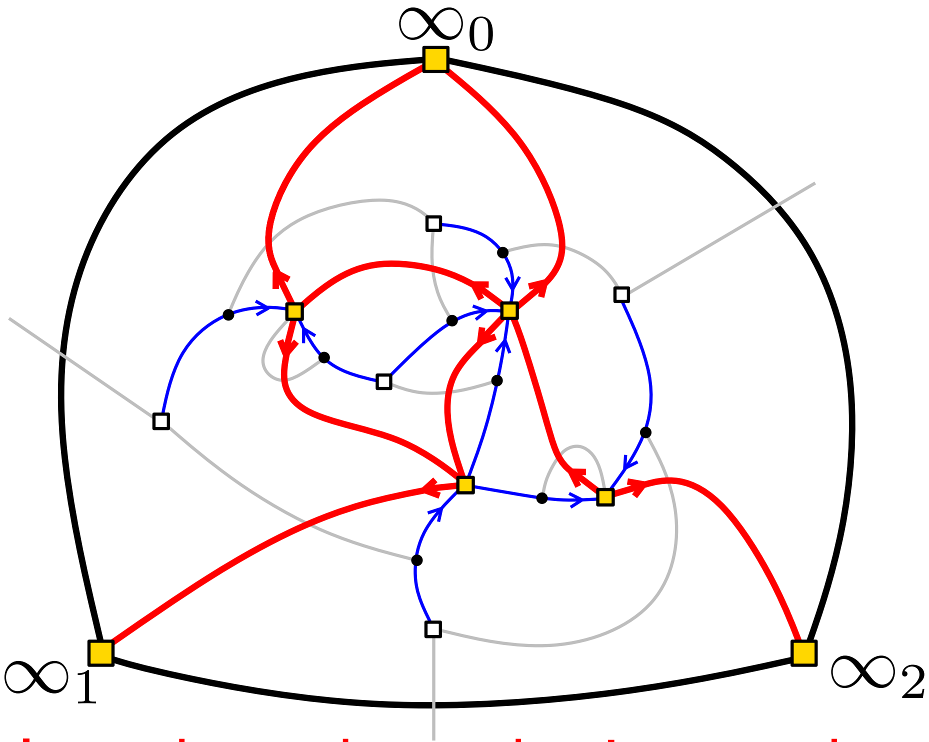
The bijection starting from oriented binary trees

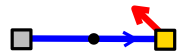
2nd step: pointed bicubic map → outer-triangular simple map

pointed vertex
at infinity
hence 3 outer faces







local rule: 

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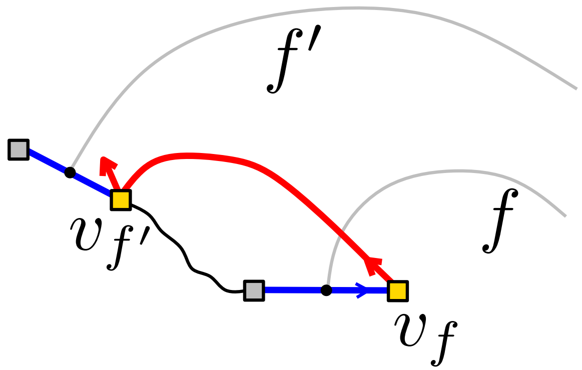
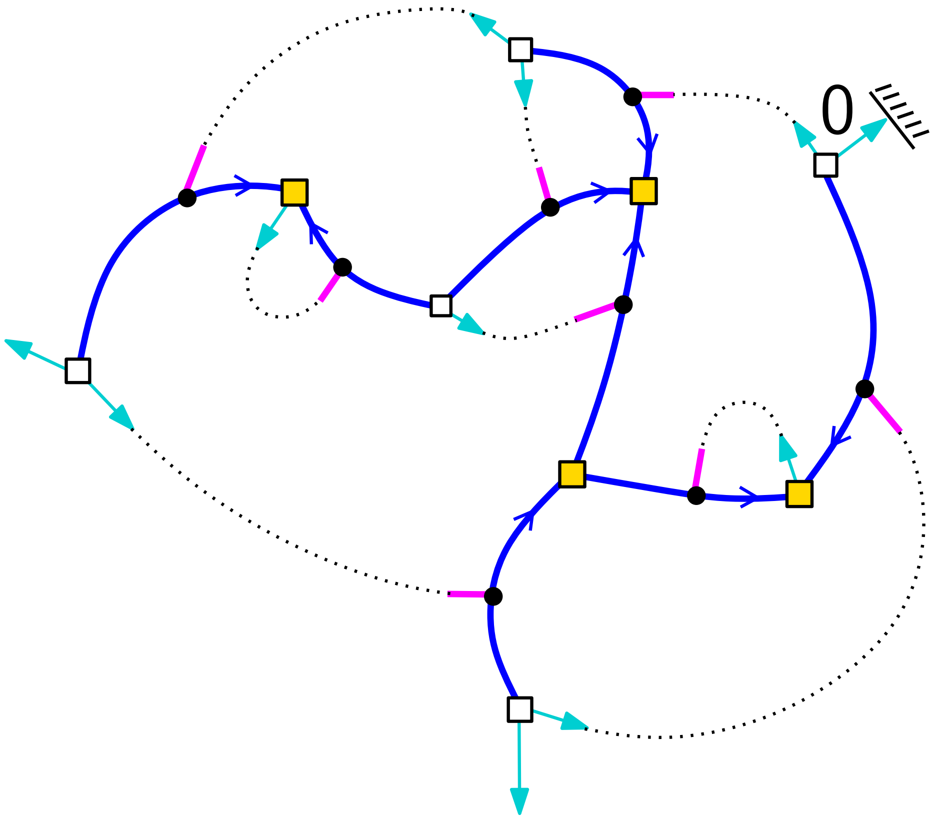


generic situation to
draw a red edge

In red we draw the inner edges of the outer-triangular simple map

Canonical labelling of a rooted oriented binary tree

cf [Bouttier,Di Francesco, Guitter'03], [Chassaing&Schaeffer'04]:
label corners such that



Canonical labelling of a rooted oriented binary tree

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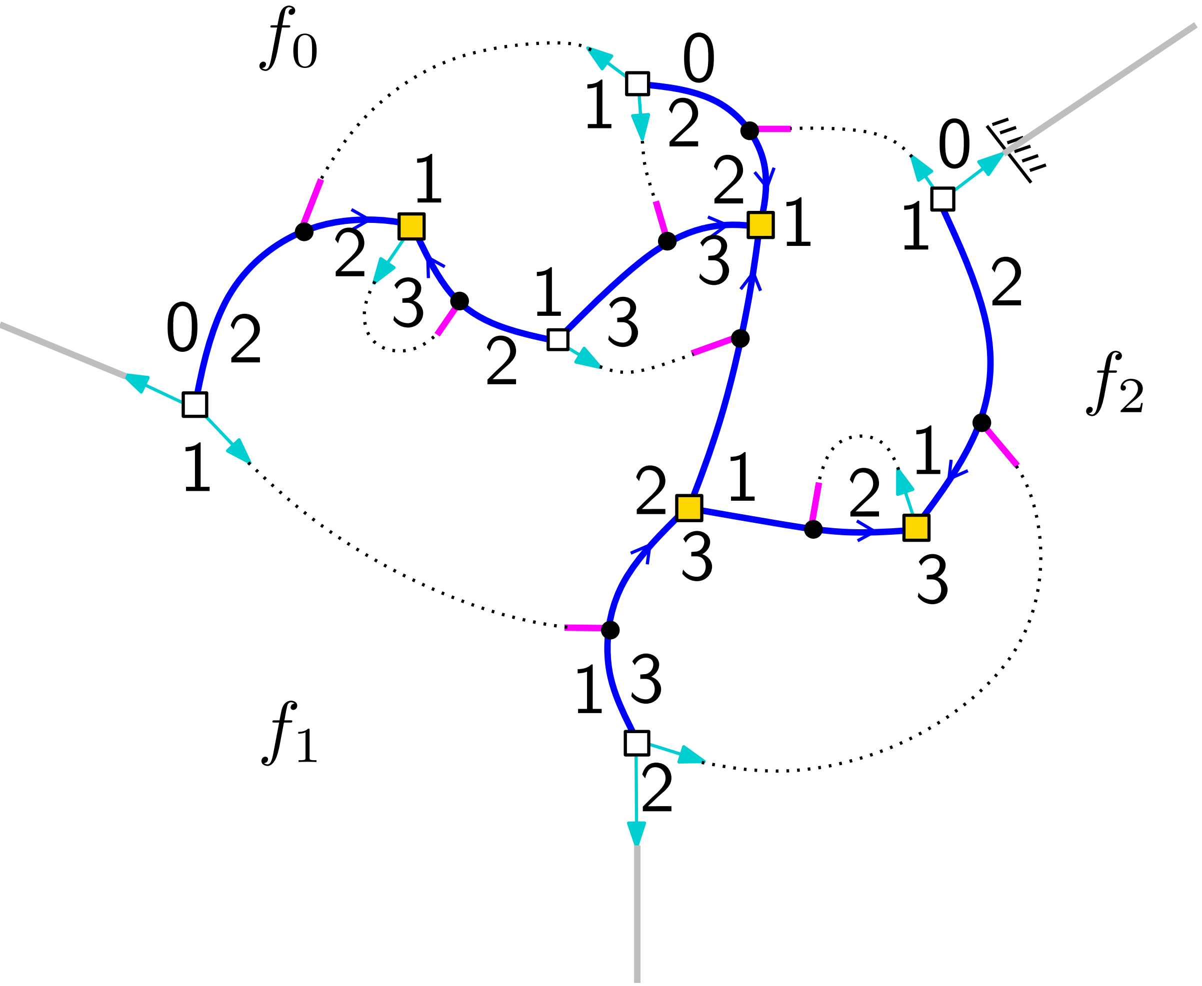
label gives "depth" of each face



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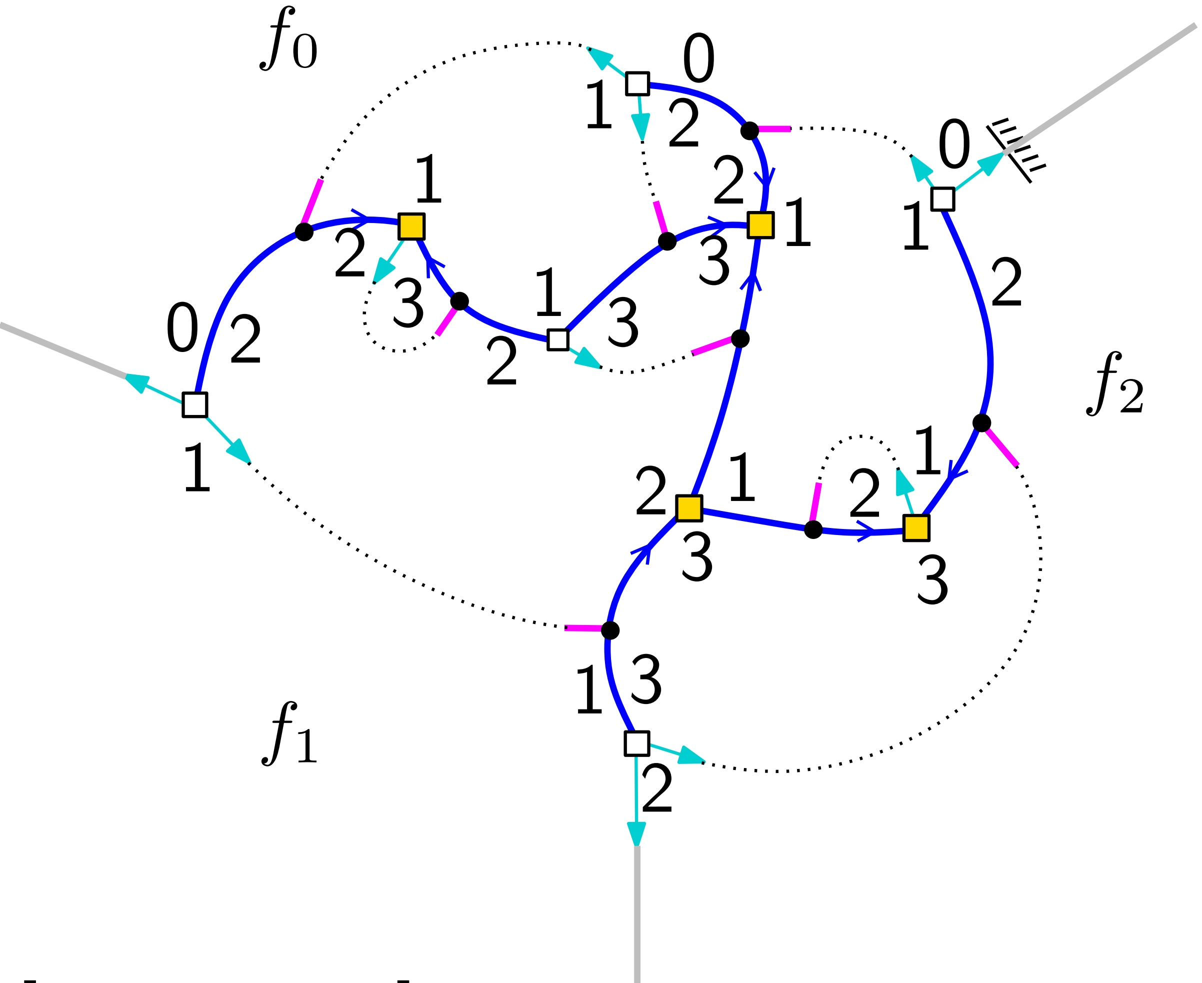
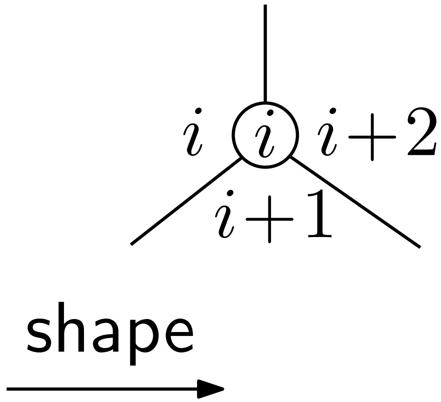
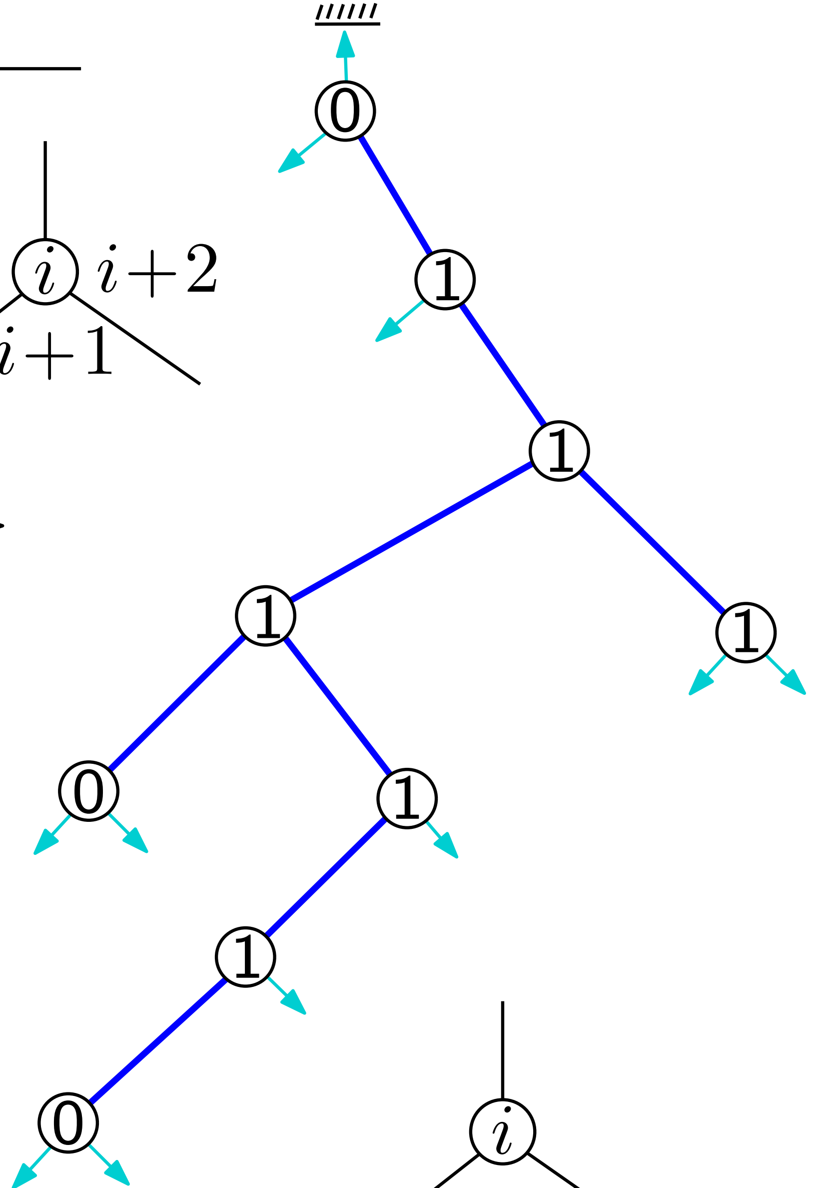
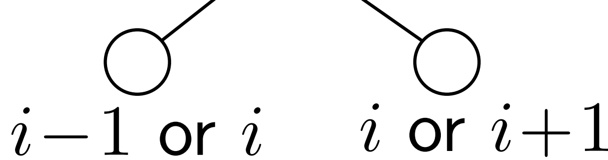






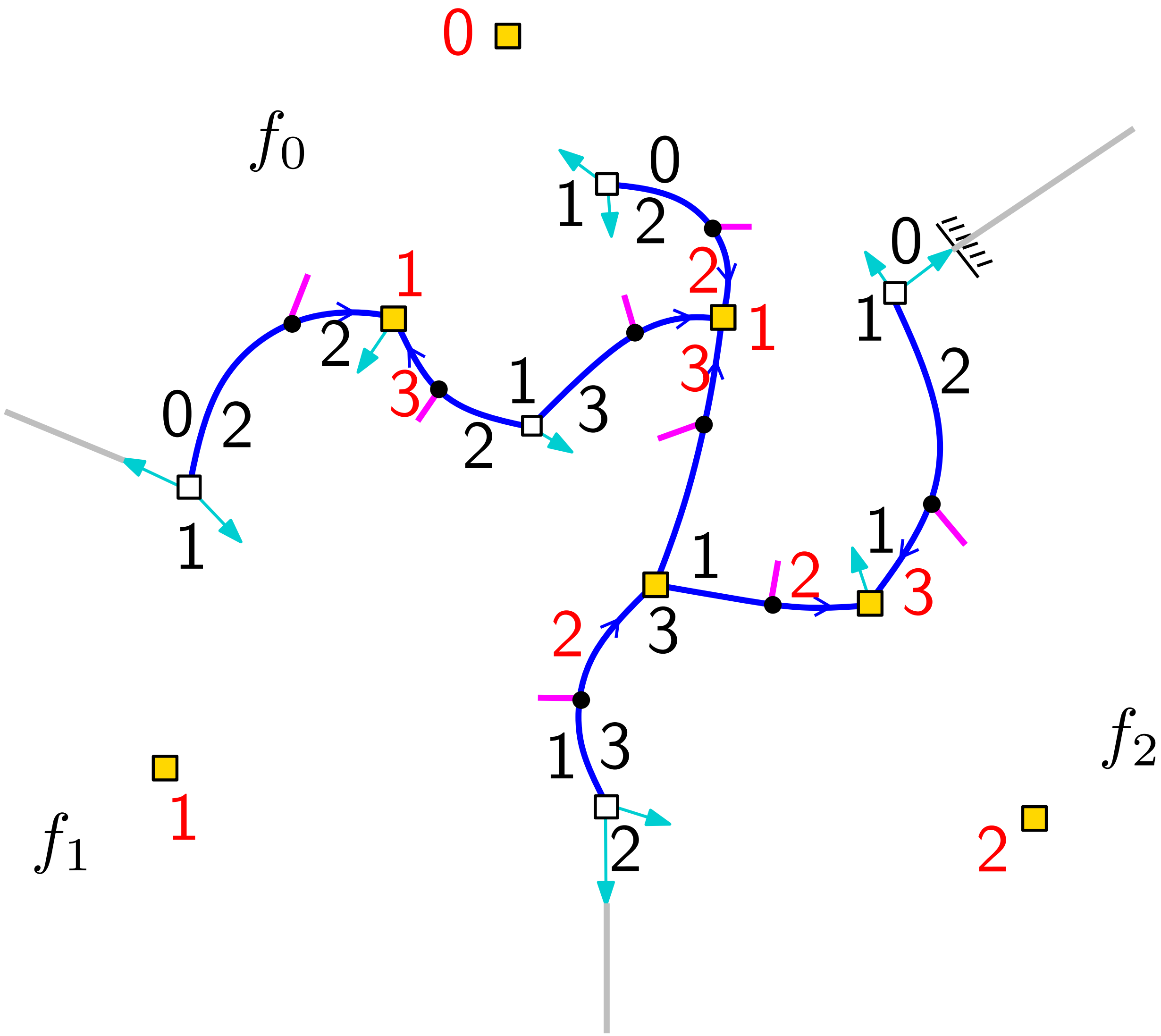


[Marckert'08]: convergence to the Brownian snake with the labels rescaled by $(2n)^{1/4}$

Shortcutting bicubic maps in the bijection

Endow the rooted oriented binary tree with its canonical corner-labelling

Color red the corners that are just before a (descending) leg



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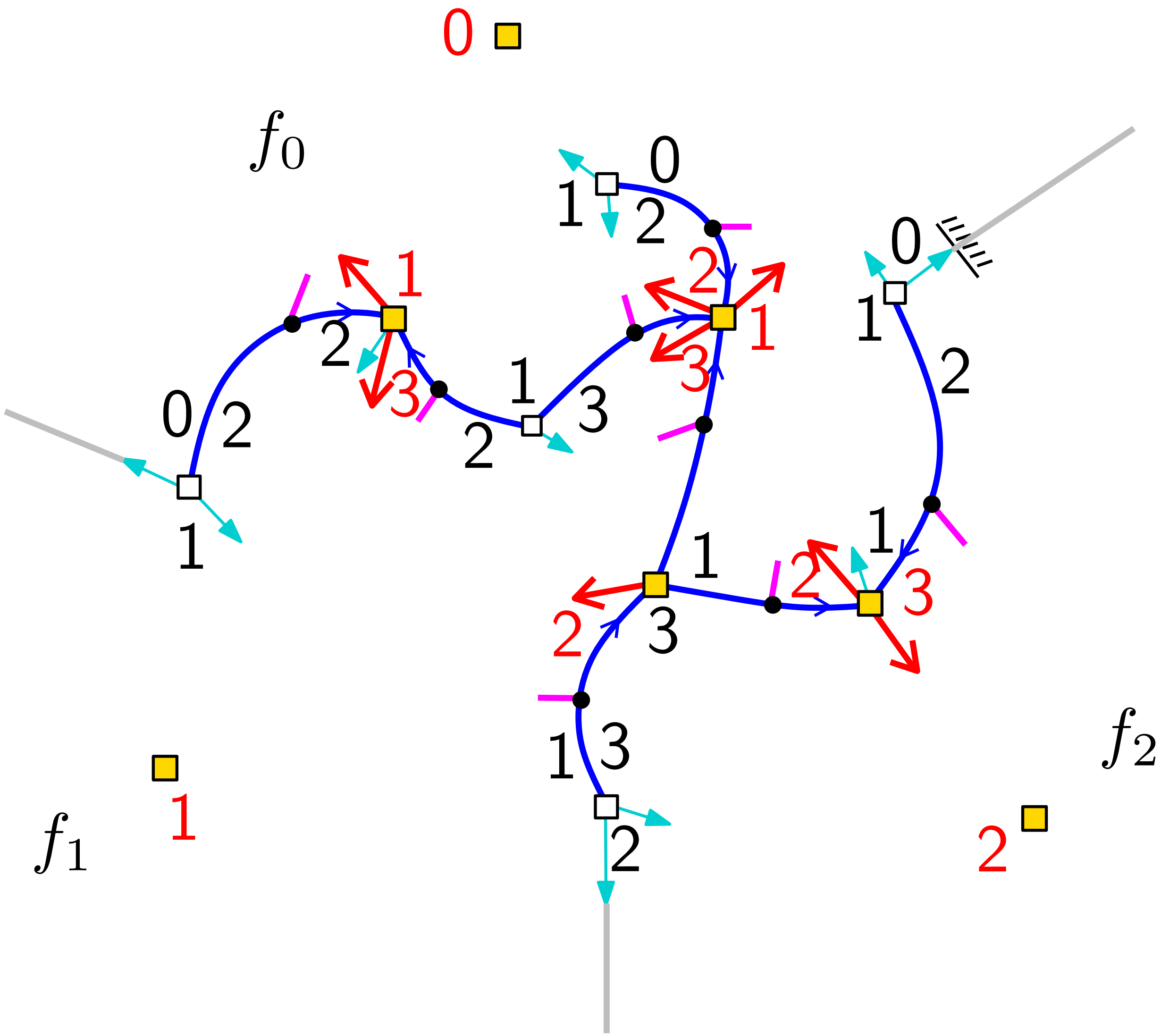

draw a red arrow
from each red corner

Shortcutting bicubic maps in the bijection

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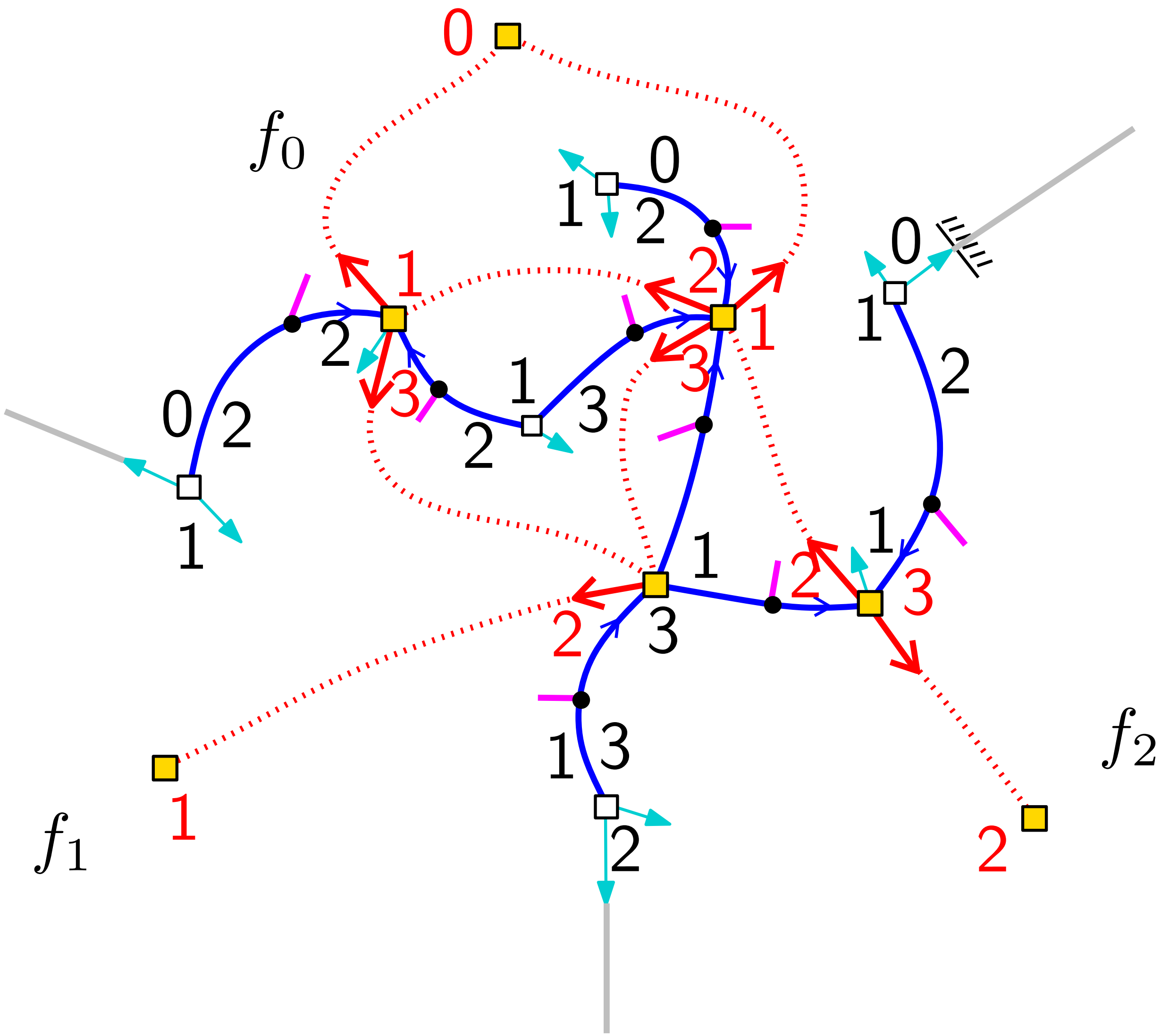

draw a red arrow from each red corner

connect each red arrow of label i to the next red corner of label $i-1$ in ccw order around the tree

Shortcutting bicubic maps in the bijection

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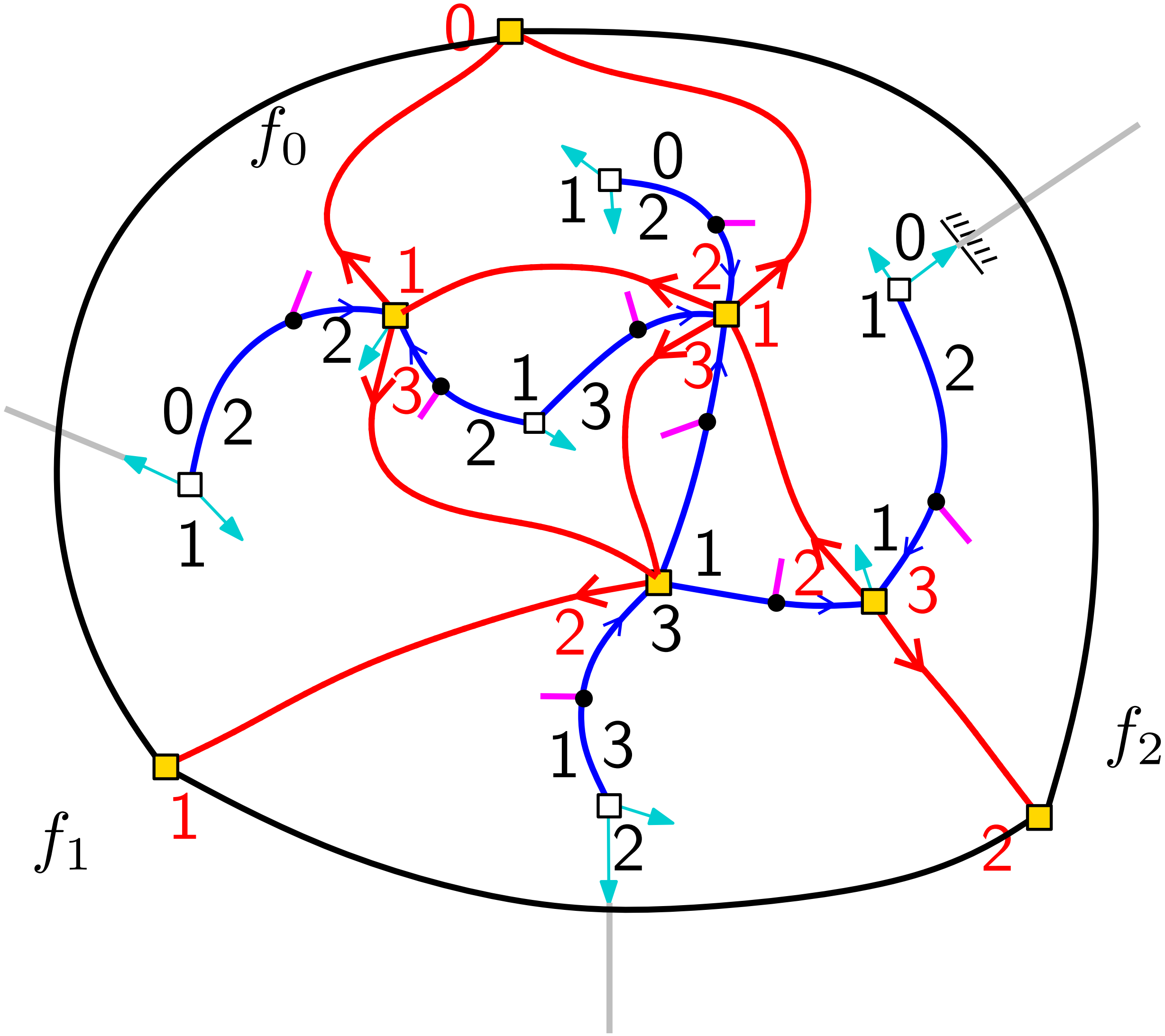

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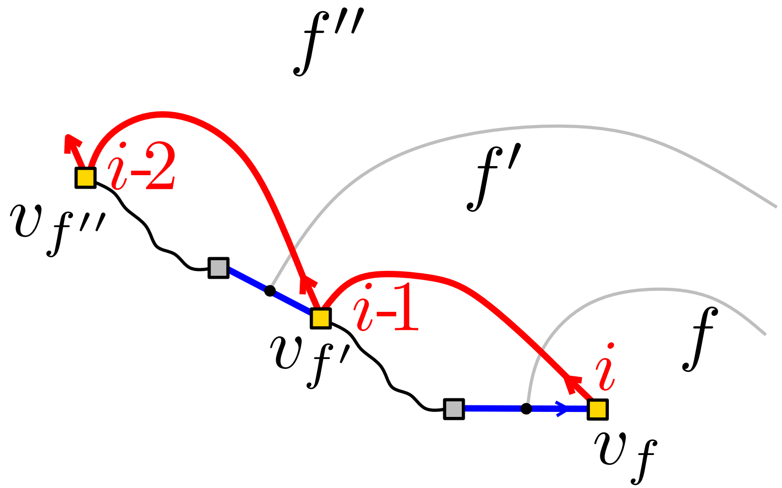
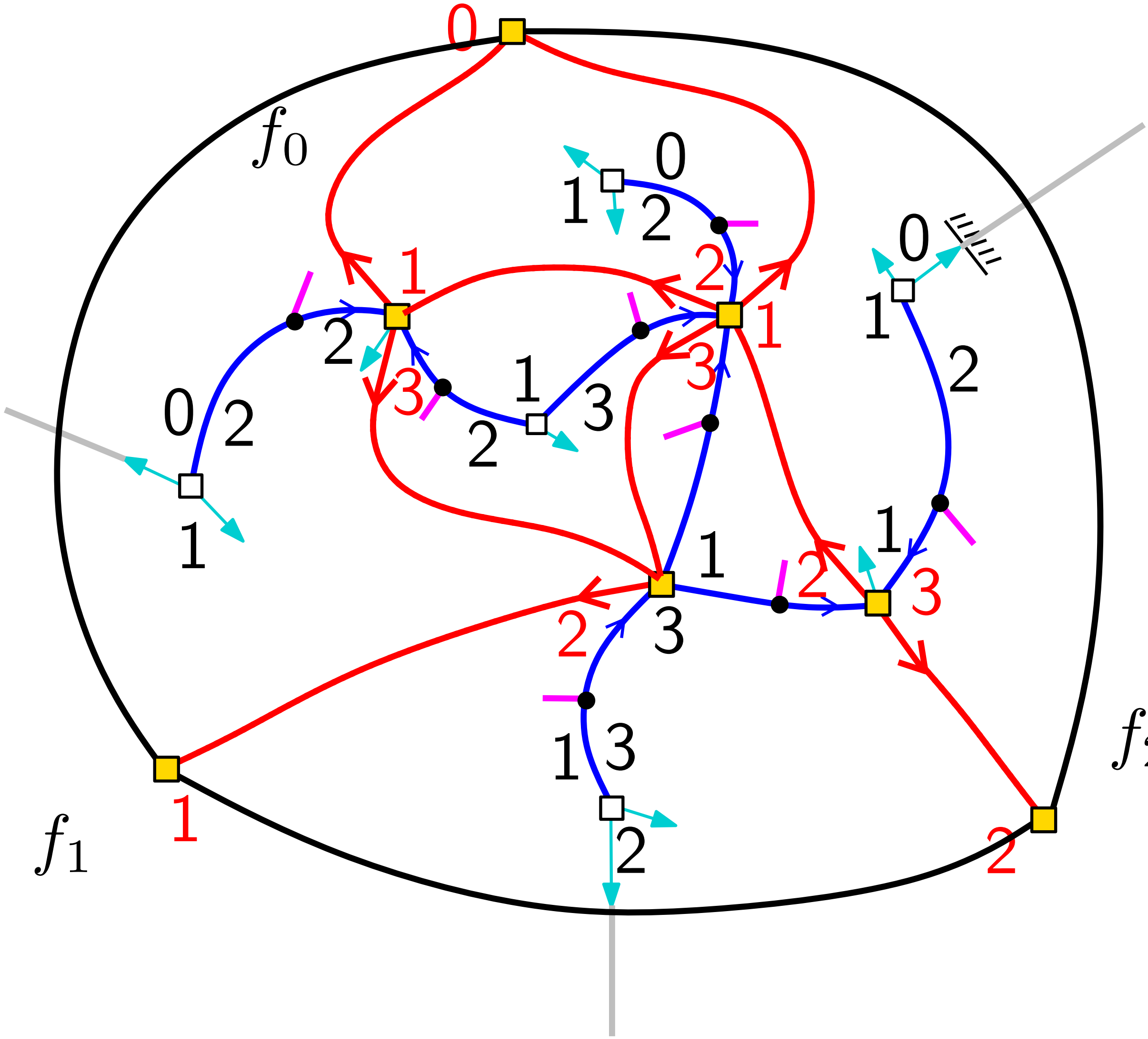


Note that the label i gives
length of the rightmost path

Shortcutting bicubic maps in the bijection

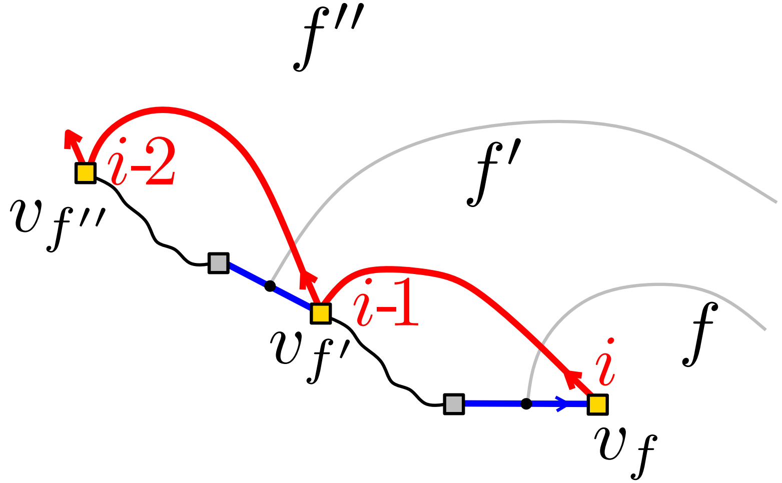
Endow the rooted oriented binary tree with its canonical corner-labelling

Color red the corners that are just before a (descending) leg



draw a red arrow
from each red corner

connect each red arrow
of label i to the next
red corner of label $i-1$
in ccw order around the tree



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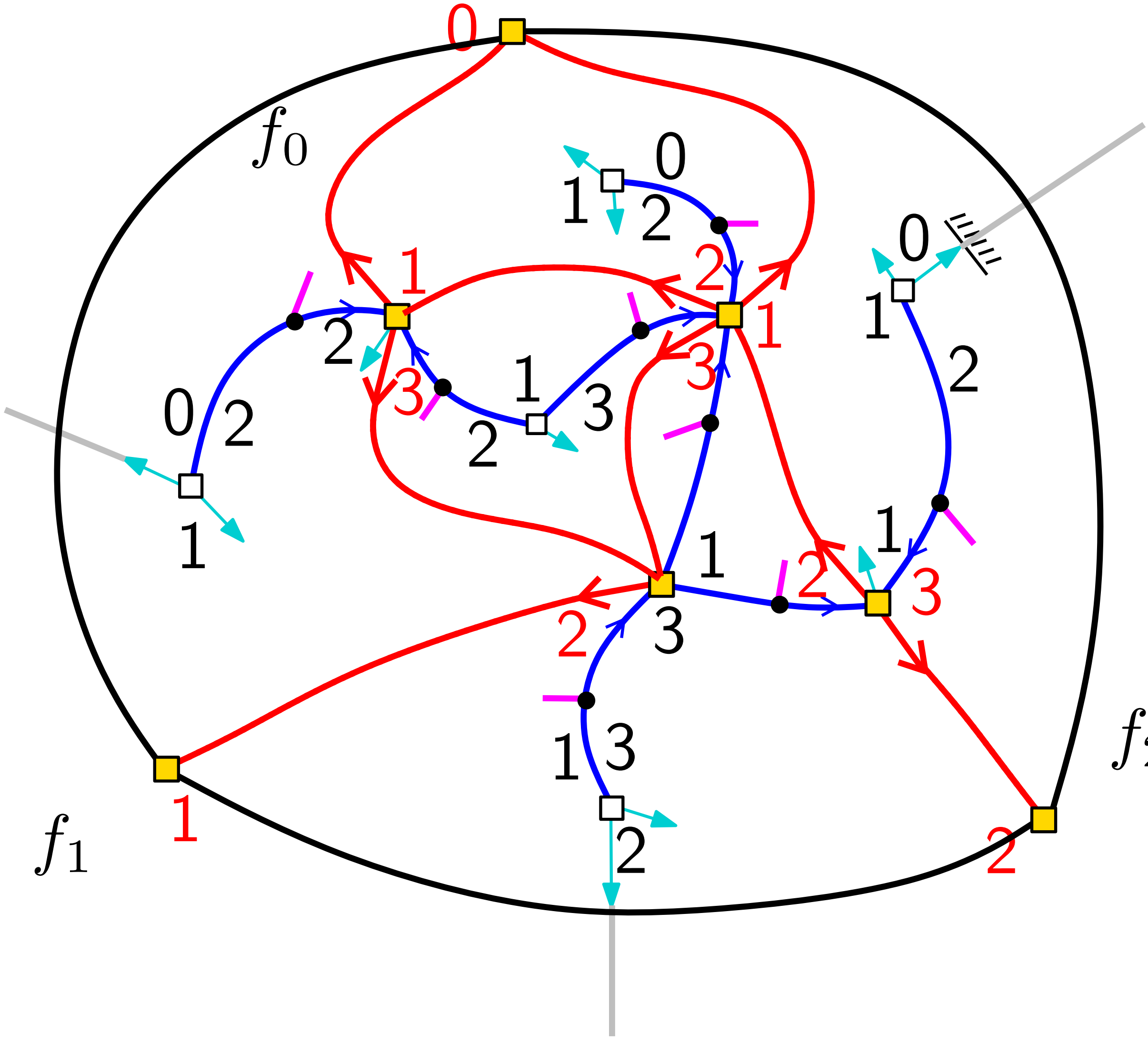

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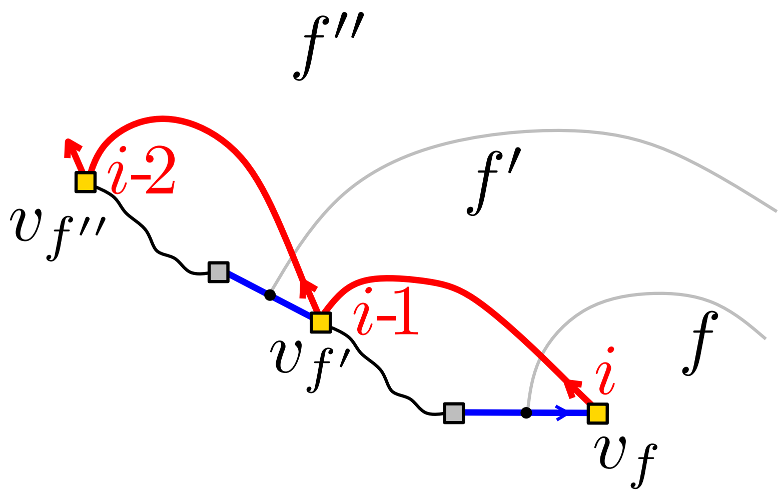

This is typically a bijection “à la Schaeffer”

$\Rightarrow$ makes it possible to prove convergence
using general criteria in [Le Gall’13]

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Conclusion and perspectives

- We have a bijection

  outer-triang. simple map $\leftrightarrow$ eulerian triang. $\leftrightarrow$ oriented binary trees

- We can shortcut eulerian triangulations to obtain a "Schaeffer" bijection from oriented binary trees to outer-triangular simple maps

- This bijection is well suited to prove convergence of the random rooted simple map with n edges (rescaled by $(2n)^{1/4}$) to the Brownian map (using [Addario-Berry&Albenque'13] and [Le Gall'13])

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- The random simple triangulation with n vertices

using Proposition in [Addario-Berry&Albenque'13] (rightmost paths) & since our bijection specializes well (canonically labelled quaternary trees) would recover [Addario-Berry&Albenque'13] for simple triang.