Introduction to Exceptional Field Theory I

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Corfu workshop on “Dualities and Generalized Geometries” 09/2018

Lab PHYS
ENS de LYON

plan

exceptional field theories

- dimensional reduction and duality symmetries
- exceptional geometry & tensor hierarchy
- invariant action functionals
- embedding of supergravity

applications

- generalized Scherk-Schwarz reductions
- examples of consistent truncations: $AdS_5 \times S^5$
- other developments

based on work with Olaf Hohm, Arnaud Baguet,
Hadi Godazgar, Mahdi Godazgar, Hermann Nicolai, Edvard Musaev,
Gianluca Inverso, Marc Magro, Emanuel Malek, Mario Trigiante

exceptional field theory (ExFT)

manifestly duality covariant formulation of maximal supergravity

- upon toroidal reduction on T^d, eleven-dimensional supergravity exhibits the global exceptional symmetry group $E_{d(d)}$

  after proper dualisation/reorganisation of the fields

$$
\begin{tikzcd}
\boxed{\text{D=11 supergravity}} \arrow[d, "T^6"'] & \boxed{\text{IIB supergravity}} \arrow[dl, "T^5"] \\
\boxed{\text{D=5 supergravity}} &
\end{tikzcd}
$$

[Cremmer, Julia]

maximal supersymmetry, global $E_{6(6)}$

- ExFT : reformulate D=11 supergravity such that $E_{d(d)}$ (or its remnants) becomes manifest <u>before</u> dimensional reduction

$\longrightarrow$ example: $E_{6(6)}$: exceptional field theory

$E_{6(6)}$ exceptional field theory: duality symmetries

D=5 maximal supergravity (torus reduction of D=11 or IIB)

after proper dualization of the dof’s (different for D=11 / IIB)
the (bosonic sector of the) D=5 Lagrangian takes the $E_{6(6)}$ invariant form

$$\mathcal{L} = R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN}\partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,F_{\mu\nu}{}^M F^{\mu\nu\,N}$$
$$+ \; d_{KMN}\,F^M \wedge F^N \wedge A^K$$

[Cremmer]

$g_{\mu\nu}$: 5 x 5 external metric

$\mathcal{M}_{MN}$: 27 x 27 internal metric, parametrizing the coset $E_{6(6)}/USp(8)$

$A_\mu{}^M$: 27 vector fields $\longleftrightarrow$ 27 two-form fields $B_{\mu\nu\,M}$

d_{KMN} : symmetric $E_{6(6)}$ invariant tensor

exceptional field theory:

– same Kaluza-Klein reorganisation of the higher-dimensional fields

– keeping the dependence on all internal coordinates (non-abelian gauge structure)

$E_{6(6)}$ exceptional field theory: duality symmetries

exceptional field theory:

— same Kaluza-Klein reorganisation of the higher-dimensional fields

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the 27 vector fields in 5D $A_\mu{}^M$ descend from $(m, n = 1, \dots, 6)$ — 11D origin

– metric $A_\mu{}^m$: $\delta A_\mu{}^m = \partial_\mu \Lambda^m + \dots$ — internal diffeomorphisms

– 3-form $A_{\mu\, mn}$: $\delta A_{\mu mn} = \partial_\mu \Lambda_{mn} + \dots$

– 6-form $A_{\mu klmnp}$: $\delta A_{\mu klmnp} = \partial_\mu \Lambda_{klmnp} + \dots$

$\Big\}$ internal tensor gauge transformations

in Kaluza-Klein spirit:
embed all gauge parameters
into a single object

$$\left\{ \begin{array}{c} \Lambda^m(x^\mu, y^m) \\ \Lambda_{mn}(x^\mu, y^m) \\ \Lambda_{klmnp}(x^\mu, y^m) \end{array} \right\} \longrightarrow \Lambda^M(x^\mu, Y^M) \qquad \mathbf{27} \text{ of } \mathrm{E}_{6(6)}$$

generalized internal diffeomorphism

$\longrightarrow$ exceptional geometry

[Hull, Waldram, Pacheco, Hillmann, Berman, Perry, Godazgar, Godazgar, Coimbra, Strickland-Constable, Park, Blair, Malek, Musaev, Cederwall, Kleinschmidt, Thompson, Edlund, Karlsson, Aldazabal, Grana, Marques, Rosabal, ... , Hohm, HS]

$E_{6(6)}$ exceptional field theory

- generalized diffeomorphisms

$$\left\{ \begin{array}{c} \Lambda^{m}(x^\mu, y^m) \\ \Lambda_{mn}(x^\mu, y^m) \\ \Lambda_{klmnp}(x^\mu, y^m) \end{array} \right\} \rightarrow \begin{array}{c} \Lambda^{M}(x^\mu, Y^M) \\ \mathbf{27} \text{ of } \mathrm{E}_{6(6)} \end{array}$$

should embed standard internal diffeomorphisms

$$L_\Lambda V^m = \Lambda^n \partial_n V^m - \partial_n \Lambda^m V^n$$

generalized internal diffeomorphisms (naiv)

$$\mathcal{L}_\Lambda V^M \stackrel{?}{=} \Lambda^N \partial_N V^M - \partial_N \Lambda^M V^N$$

cannot be the correct answer:

– standard higher-dimensional diffeomorphisms (induce wrong algebra structure)

– not compatible with the group $E_{6(6)}$, e.g. invariance of d_{KMN}

$E_{6(6)}$ exceptional field theory

- generalized diffeomorphisms

$$\left\{\begin{array}{c}\Lambda^{m}(x^\mu, y^m) \\ \Lambda_{mn}(x^\mu, y^m) \\ \Lambda_{klmnp}(x^\mu, y^m)\end{array}\right\} \rightarrow \begin{array}{c}\Lambda^{M}(x^\mu, Y^M) \\ \mathbf{27} \text{ of } \mathrm{E}_{6(6)}\end{array}$$

should embed standard internal diffeomorphisms

$$L_\Lambda V^m = \Lambda^n \partial_n V^m - \partial_n \Lambda^m V^n$$

generalized internal diffeomorphisms

$$\mathcal{L}_\Lambda V^M \stackrel{!}{=} \Lambda^N \partial_N V^M - \left[\partial_N \Lambda^M\right]_{\rm adj} V^N$$

[Coimbra, Strickland-Constable, Waldram]

$$= \Lambda^N \partial_N V^M + \kappa \, (\mathbb{P}_{\rm adj})^N{}_P{}^M{}_Q \, (\partial_N \Lambda^P) \, V^Q$$

compatible with $E_{d(d)}$ structure (respects invariant tensors)

$E_{6(6)}$ exceptional field theory

- generalized diffeomorphisms

$$\left\{\begin{array}{c} \Lambda^{m}(x^\mu, y^m) \\ \Lambda_{mn}(x^\mu, y^m) \\ \Lambda_{klmnp}(x^\mu, y^m) \end{array}\right\} \rightarrow \begin{array}{c} \Lambda^{M}(x^\mu, Y^M) \\ \mathbf{27} \text{ of } \mathrm{E}_{6(6)} \end{array}$$

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compatible with $E_{d(d)}$ structure (respects invariant tensors)

- derivatives

$$\partial_M \longrightarrow \left\{\begin{array}{r} \partial_m = \frac{\partial}{\partial y^m} \\ \partial^{mn} \rightarrow 0 \\ \partial^{klmnp} \rightarrow 0 \end{array}\right.$$

covariant formulation: section constraints

[Berman, Perry, Godazgar, Godazgar, Coimbra, Strickland-Constable, Waldram, Cederwall, Kleinschmidt, Thompson]

$$Y^{MN}{}_{PQ} \, \partial_M \otimes \partial_N = 0 \qquad \left\{\begin{array}{rcl} Y^{MN}{}_{PQ} \, \partial_M \partial_N \Phi &=& 0 \\ Y^{MN}{}_{PQ} \, \partial_M \Phi_1 \, \partial_N \Phi_2 &=& 0 \end{array}\right.$$

with an $E_{d(d)}$ invariant tensor $Y^{MN}{}_{PQ} = d_{PQR} \, d^{RMN}$

projecting onto a subrepresentation of $\mathbf{27} \otimes \mathbf{27} \longrightarrow \mathbf{27}' + \mathbf{351} + \widetilde{\mathbf{351}}$

$E_{6(6)}$ exceptional field theory

- generalized diffeomorphisms

$$\left.\begin{array}{c}\Lambda^{m}(x^\mu, y^m)\\ \Lambda_{mn}(x^\mu, y^m)\\ \Lambda_{klmnp}(x^\mu, y^m)\end{array}\right\} \rightarrow \begin{array}{c}\Lambda^{M}(x^\mu, Y^M)\\ \mathbf{27}\text{ of }E_{6(6)}\end{array}$$

generalized internal diffeomorphisms

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[Coimbra, Strickland-Constable, Waldram]

$$= \Lambda^N \partial_N V^M + \kappa\,(\mathbb{P}_{\text{adj}})^N{}_P{}^M{}_Q\,(\partial_N \Lambda^P)\,V^Q$$

- section constraints

$$\boxed{Y^{MN}{}_{PQ}\,\partial_M \otimes \partial_N = 0 \qquad \left\{\begin{array}{rcl} Y^{MN}{}_{PQ}\,\partial_M \partial_N \Phi &=& 0\\ Y^{MN}{}_{PQ}\,\partial_M \Phi_1\,\partial_N \Phi_2 &=& 0\end{array}\right.}$$

solution (D=11 supergravity)

$$\partial_M \longrightarrow \left\{\begin{array}{rl} \partial_m &= \frac{\partial}{\partial y^m}\\ \partial^{mn} &\rightarrow 0\\ \partial^{klmnp} &\rightarrow 0\end{array}\right.$$

$$27 \longrightarrow 6 + 15 + 6$$

under $\mathrm{GL}(6)$

inequivalent solution (IIB supergravity)

$$\partial_M \longrightarrow \left\{\begin{array}{rl} \partial_i &= \frac{\partial}{\partial \tilde{y}^i}\\ \partial^{ijk} &\rightarrow 0\\ \partial^{i\,\alpha} &\rightarrow 0\\ \partial^{\alpha} &\rightarrow 0\end{array}\right.$$

$$27 \longrightarrow (2,1) + (2,5') + (1,5) + (1,10)$$

under $\mathrm{GL}(5) \times \mathrm{SL}(2)$

$E_{6(6)}$ exceptional field theory

- generalized diffeomorphisms

$$\left.\begin{array}{c} \Lambda^m(x^\mu, y^m) \\ \Lambda_{mn}(x^\mu, y^m) \\ \Lambda_{klmnp}(x^\mu, y^m) \end{array}\right\} \rightarrow \begin{array}{c} \Lambda^M(x^\mu, Y^M) \\ \mathbf{27} \text{ of } \mathrm{E}_{6(6)} \end{array}$$

generalized internal diffeomorphisms

$$\mathcal{L}_\Lambda V^M \overset{!}{=} \Lambda^N \partial_N V^M - \left[\partial_N \Lambda^M\right]_{\mathrm{adj}} V^N$$

[Coimbra, Strickland-Constable, Waldram]

$$= \Lambda^N \partial_N V^M + \kappa \, (\mathbb{P}_{\mathrm{adj}})^N{}_P{}^M{}_Q \, (\partial_N \Lambda^P) \, V^Q$$

- closure of the algebra

  — determines κ

  — up to section condition $\quad Y^{MN}{}_{PQ} \, \partial_M \otimes \partial_N = 0$

  — $\exists$ trivial gauge parameters

  — E-bracket $\quad [\Lambda_2, \Lambda_1]^M_{\mathrm{E}} = (\mathcal{L}_{\Lambda_1}\Lambda_2 - \mathcal{L}_{\Lambda_2}\Lambda_1)^M = 2\Lambda^K_{[2}\partial_K \Lambda^M_{1]} - 10 d^{MNP} d_{KLP} \, \Lambda^K_{[2} \partial_N \Lambda^L_{1]}$

  (not associative ! Jacobiator is a trivial gauge parameter)

- $\underbrace{\text{infinite-dimensional}}_{Y^M}$ $\underbrace{\text{local}}_{x^\mu}$ gauge structure of ExFT

covariant derivatives $\quad \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$

$E_{6(6)}$ exceptional field theory

- external covariant derivatives $\mathcal{D}_\mu \;=\; \partial_\mu - \mathcal{L}_{A_\mu}$

- non-abelian field strength $F_{\mu\nu}{}^M \equiv 2\partial_{[\mu}A_{\nu]}{}^M - \big[A_\mu, A_\nu\big]_{\rm E}^M$

  does not transform covariantly (Jacobiator of E-bracket)

$$\delta F_{\mu\nu}{}^M \;=\; \text{"covariant"} \;+\; d^{MNK}\partial_N\Big(\cdots\Big)_K$$

- covariant field strength : coupling to two-forms $\longrightarrow$ tensor hierarchy

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu}A_{\nu]}{}^M - \big[A_\mu, A_\nu\big]_{\rm E}^M + 10\, d^{MNK}\partial_K B_{\mu\nu N}$$

covariant under gauge transformations

$$\delta A_\mu{}^M \;=\; \mathcal{D}_\mu\Lambda^M - 10\, d^{MNK}\partial_K\Xi_{\mu N}$$

$$\delta B_{\mu\nu M} \;=\; 2\mathcal{D}_{[\mu}\Xi_{\nu]M} + d_{MKL}\Lambda^K\mathcal{F}_{\mu\nu}{}^L - d_{MKL}\, A_{[\mu}{}^K\,\delta A_{\nu]}{}^L + \mathcal{O}_{\mu\nu M}$$

$$d^{MNK}\partial_K\mathcal{O}_{\mu\nu N} = 0$$

requires 27 2-forms $B_{\mu\nu\, M}$, present in D=5 supergravity!

(as duals to the 27 vector fields)

$E_{6(6)}$ exceptional field theory: dynamics

invariant action functional

$E_{6(6)}$ exceptional field theory: dynamics

- recall D=5 supergravity
  [Cremmer]

$$\mathcal{L} \;=\; R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN} \partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,F_{\mu\nu}{}^M F^{\mu\nu\,N} + \; d_{KMN}\,F^M \wedge F^N \wedge A^K$$

- invariant ExFT action [Hohm, HS]

$$\mathcal{L} \;=\; \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu\,N} + \mathcal{L}_{\rm top} \;-\; V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

- covariantized for invariance under generalized diffeomorphisms

$$\mathcal{D}_\mu \;=\; \partial_\mu - \mathcal{L}_{A_\mu}$$

$$\mathcal{F}_{\mu\nu}{}^M \equiv 2\partial_{[\mu} A_{\nu]}{}^M - \big[A_\mu, A_\nu\big]_{\rm E}^M + 10\,d^{MNK}\,\partial_K B_{\mu\nu\,N}$$

two-form field equations completed by the topological term

$$S_{\rm top} \;=\; \int d^{27}Y \int_{\mathcal{M}_6} \big(d_{MNK}\,\mathcal{F}^M \wedge \mathcal{F}^N \wedge \mathcal{F}^K - 40\,d^{MNK} \mathcal{H}_M \wedge \partial_N \mathcal{H}_K\big)$$

boundary term of a six-dimensional bulk, implies vector-tensor duality

$E_{6(6)}$ exceptional field theory: dynamics

- recall D=5 supergravity [Cremmer]

$$\mathcal{L} \;=\; R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN}\partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,F_{\mu\nu}{}^{M}F^{\mu\nu\,N} + d_{KMN}\,F^{M}\wedge F^{N}\wedge A^{K}$$

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- "potential"

$$V_{\rm pot} \;=\; \frac{1}{24}\,\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\,(12\,\partial_L\mathcal{M}_{NK} - \partial_N\mathcal{M}_{KL}) \\ -\frac{1}{2}\,g^{-1}\partial_M g\,\partial_N\mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}^{MN}g^{-1}\partial_M g\,g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\,\partial_N g_{\mu\nu}$$

– invariant under generalized diffeomorphisms

– generalised (internal) curvature scalar

$E_{6(6)}$ exceptional field theory: dynamics

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- external diffeomorphisms
  - all terms are separately invariant under internal diffeomorphisms
  - relative coefficients uniquely fixed by external diffeomorphisms $\xi^\mu(x,Y)$

$$\begin{aligned}
\delta\mathcal{M}_{MN} &= \xi^\mu\,\mathcal{D}_\mu\mathcal{M}_{MN} \\
\delta A_\mu{}^M &= \xi^\nu\,\mathcal{F}_{\nu\mu}{}^M + \mathcal{M}^{MN} g_{\mu\nu}\partial_N\xi^\nu \\
\delta B_{\mu\nu\,M} &= \frac{1}{16}\,\xi^\rho e\varepsilon_{\mu\nu\rho\sigma\tau}\mathcal{M}_{MN}\mathcal{F}^{\sigma\tau N} - d_{MKL}\,A_{[\mu}{}^K\,\delta A_{\nu]}{}^L
\end{aligned}$$

$E_{6(6)}$ exceptional field theory: dynamics

- recall D=5 supergravity [Cremmer]

$$\mathcal{L} \;=\; R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN}\partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,F_{\mu\nu}{}^M F^{\mu\nu\,N} + \, d_{KMN}\,F^M \wedge F^N \wedge A^K$$

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- embedding of supergravity
  - upon explicit solution of the section condition by

    the theory coincides with the full D=11 supergravity !

$$\partial_M \;\longrightarrow\; \begin{cases} \partial_m = \frac{\partial}{\partial y^m} \\ \partial^{mn} \to 0 \\ \partial^{klmnp} \to 0 \end{cases}$$

  - upon different solution of the section condition by

    the theory coincides with the full IIB supergravity !

$$\partial_M \;\longrightarrow\; \begin{cases} \partial_i = \frac{\partial}{\partial \tilde{y}^i} \\ \partial^{ijk} \to 0 \\ \partial^{i\,\alpha} \to 0 \\ \partial^{\alpha} \to 0 \end{cases}$$

$E_{6(6)}$ exceptional field theory: dynamics

manifestly duality covariant formulation of maximal supergravity

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

D=5+27 with section condition

dictionary

dictionary

D=11 sugra

IIB sugra

- bosonic sectors of maximal supergravity determined by bosonic symmetries
- IIA and IIB supergravity accommodated in the same framework
- what can we do with it ..?

$E_{6(6)}$ exceptional field theory: dynamics

manifestly duality covariant formulation of maximal supergravity

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D=5+27 with section condition

dictionary

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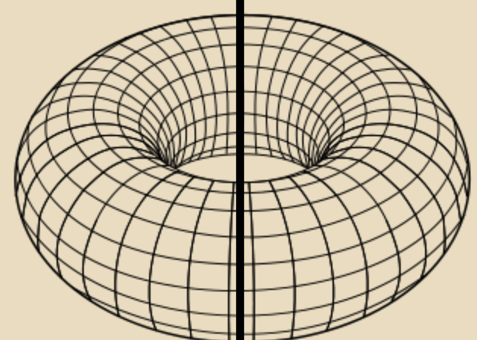
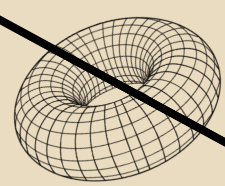
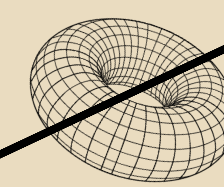


D=11 sugra

IIB sugra





T^6

T^5

D=5 maximal sugra

global $E_{6(6)}$

[Cremmer]

makes the symmetry enhancement after torus reduction manifest

$E_{6(6)}$ exceptional field theory: dynamics

manifestly duality covariant formulation of maximal supergravity

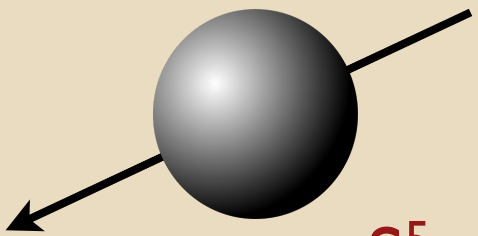
ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

D=5+27 with section condition

dictionary

IIB sugra



S^5 x AdS_5

D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

[Gunaydin, Romans, Warner]

also allows a compact description of complicated reductions

$E_{6(6)}$ exceptional field theory: dynamics

manifestly duality covariant formulation of maximal supergravity

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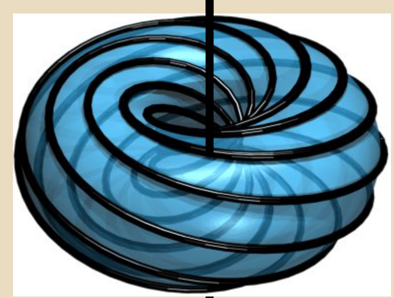
$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

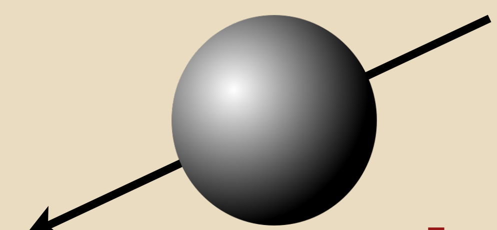
D=5+27 with section condition

dictionary

captured by a twisted torus (Scherk-Schwarz) reduction of ExFT



IIB sugra



S^5 x AdS_5

[Kaloper, Myers, Dabholkar, Hull, Reid-Edwards, Dall'Agata, Prezas, HS, Trigiante, Hohm, Kwak, Aldazabal, Baron, Nunez, Marques, Geissbuhler, Grana, Berman, Musaev, Thompson, Rosabal, Lee, Strickland-Constable, Waldram, Dibitetto, Roest, Malek, Blumenhagen, Hassler, Lust, Cho, Fernández-Melgarejo, Jeon, Park, Guarino, Varela, Inverso, Ciceri, ..., ...]

D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

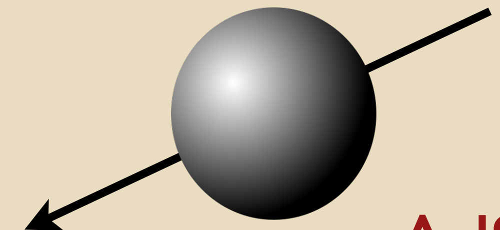
[Gunaydin, Romans, Warner]

also allows a compact description of complicated reductions

- consistent truncation on $AdS_5 \times S^5$

consistent truncation on $AdS_5 \times S^5$

IIB sugra



$AdS_5 \times S^5$

D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

[Gunaydin, Romans, Warner]

- $AdS_5 \times S^5$: maximal supersymmetric solution of IIB
- fluctuations around the background: D=5 gauged supergravity
- explicit reduction formulas: highly non-trivial

$$
\begin{aligned}
ds^2 \;&=\; \Delta^{-2/3}(x,y)\, g_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right) \\
G^{mn}(x,y) &= \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)
\end{aligned}
$$

D=11 on $AdS_4 \times S^7$: [de Wit, Nicolai] 1987

D=11 on $AdS_7 \times S^4$: [Nastase, van Nieuwenhuizen, Vaman] 1999

IIB on $AdS_5 \times S^5$: over the years, shown for various sub-sectors...

consistent truncation on $AdS_5 \times S^5$

ExFT

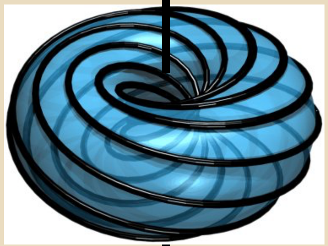
dictionary

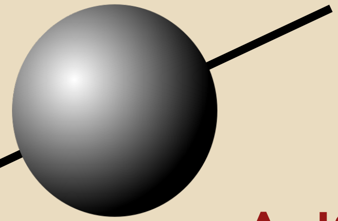
IIB sugra

$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\,M_{KL}(x)\,U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,A_\mu{}^K(x)$$

$$\mathcal{B}_{\mu\nu\,M}(x,Y) \;=\; \rho^{-2}(Y)\,U_M{}^K(Y)\,B_{\mu\nu\,K}(x)$$





$AdS_5 \times S^5$

D=5 maximal sugra
global $E_{6(6)}$
gauge group SO(6)

- consistent truncation via generalized Scherk-Schwarz ansatz in ExFT
  in terms of an $E_{6(6)}$—valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$
  - system of consistency equations $\left[(U^{-1})_M{}^P(U^{-1})_N{}^L\,\partial_P U_L{}^K\right]_{\mathbf{351}} \;\stackrel{!}{=}\; \rho\,X_{MN}{}^K$
  - generalized parallelizability [Lee, Strickland-Constable, Waldram]
  - no general classification of its solutions (Lie algebras vs Leibniz algebras)

consistent truncation on AdS$_5$ x S^5

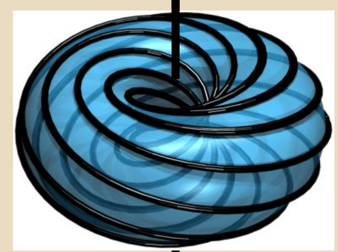
ExFT

dictionary

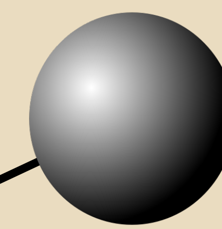
$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\,M_{KL}(x)\,U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,A_\mu{}^K(x)$$

$$\mathcal{B}_{\mu\nu\,M}(x,Y) \;=\; \rho^{-2}(Y)\,U_M{}^K(Y)\,B_{\mu\nu\,K}(x)$$



IIB sugra



AdS$_5$ x S^5

D=5 maximal sugra

global E$_{6(6)}$

gauge group SO(6)

- AdS$_5$ x S^5 twist matrix $U \in \mathrm{SL}(6)$ associated to SO(6) structure constants
  - background AdS$_5$ x S^5
  - full reduction formulas of IIB on AdS$_5$ x S^5
    in terms of sphere harmonics and the fields of D=5 maximal supergravity

$$U \;=\; \begin{pmatrix} \delta_i{}^j\, a(y^2) & y^i\, b(y^2) \\ y^i\, c(y^2) & d(y^2) \end{pmatrix}$$

consistent truncation on AdS₅ x S⁵

- e.g. metric (standard Kaluza-Klein form)

$$ds^2 \;=\; \Delta^{-2/3}(x,y)\, \mathrm{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right)$$

$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)$$

- e.g. 4-form (after reconstructing all components, in Kaluza-Klein basis)

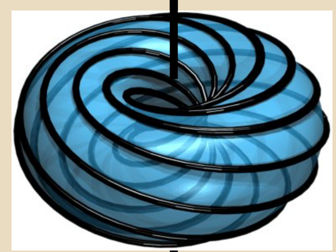
$$\begin{aligned}
C_{klmn} &= \tilde{C}_{klmn} + \frac{1}{16}\tilde{\omega}_{klmnp}\Delta^{4/3} m_{\alpha\beta}\tilde{G}^{pq}\partial_q(\Delta^{-4/3}m^{\alpha\beta}),\\
C_{\mu kmn} &= \frac{\sqrt{2}}{4}\mathcal{Z}_{[ab]kmn}A_\mu{}^{ab},\\
C_{\mu\nu mn} &= \frac{\sqrt{2}}{4}\mathcal{K}_{[ab]}{}^k\mathcal{Z}_{[cd]kmn}A_{[\mu}{}^{ab}A_{\nu]}{}^{cd},\\
C_{m\mu\nu\rho} &= -\frac{1}{32}\mathcal{K}_{[ab]m}\left(2\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}M_{ab,N}F^{\sigma\tau N} + \sqrt{2}\varepsilon_{abcdef}\Omega^{cdef}_{\mu\nu\rho}\right) - \frac{1}{4}\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{Z}_{[ef]mkl}(A_{[\mu}{}^{ab}A_\nu{}^{cd}A_{\rho]}{}^{ef}),\\
C_{\mu\nu\rho\sigma} &= -\frac{1}{16}\mathcal{Y}_a\mathcal{Y}^b\left(\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D^\tau M_{bc,N}M^{Nca} + 2\sqrt{2}\varepsilon_{cdefgb}F_{[\mu\nu}{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{ga}\right)\\
&\quad + \frac{1}{4}\left(\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{K}_{[ef]}{}^n\mathcal{Z}_{[gh]kln} - \mathcal{Y}_h\mathcal{Y}^j\varepsilon_{abcegj}\eta_{df}\right)A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{gh} + \Lambda_{\mu\nu\rho\sigma}(x).
\end{aligned}$$

$$\begin{aligned}
D_{[\mu}\Lambda_{\nu\rho\sigma\tau]} &= -\frac{1}{80}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D_\lambda(M^{Nac}D^\lambda M_{bc,N})\\
&\quad + \frac{1}{40}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}F^{\kappa\lambda N}\left(M_{bc,N}F_{\kappa\lambda}{}^{ac} - \frac{1}{2}\sqrt{10}\varepsilon_{\alpha\beta}\eta_{db}M^{d\alpha}{}_N B_{\kappa\lambda}{}^{\alpha\beta}\right)\\
&\quad + \frac{1}{100}\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}\mathcal{Y}_a\mathcal{Y}^b(10M^{ac,fd} + \mathcal{X}^{(af)ec,d}{}_e)\eta_{cd}\eta_{bf}\\
&\quad + \frac{1}{32}\sqrt{2}\varepsilon_{abcdef}F_{[\mu\nu}{}^{ab}F_{\rho\sigma}{}^{cd}A_{\tau]}{}^{ef} + \frac{1}{16}F_{[\mu\nu}{}^{ab}A_\rho{}^{cd}A_\sigma{}^{ef}A_{\tau]}{}^{gh}\varepsilon_{abcdeh}\eta_{fh}\\
&\quad + \frac{1}{40}\sqrt{2}A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_\sigma{}^{gh}A_{\tau]}{}^{ij}\varepsilon_{abcegi}\eta_{df}\eta_{hj},
\end{aligned}$$

- **proves the consistent truncation of IIB on AdS₅ x S⁵**

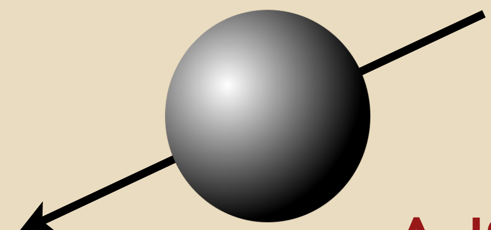
consistent truncation on AdS_5 x S^5

ExFT



dictionary

IIB sugra



AdS_5 x S^5

D=5 maximal sugra

global $E_{6(6)}$

gauge group SO(6)

- AdS_5 x S^5 twist matrix $U \in \mathrm{SL}(6)$ associated to SO(6) structure constants
  - background AdS_5 x S^5
  - full reduction formulas of IIB on AdS_5 x S^5

$$U = \left(\begin{array}{c:c} \delta_i{}^j\, a(y^2) & y^i\, b(y^2) \\ \hdashline y^i\, c(y^2) & d(y^2) \end{array}\right)$$

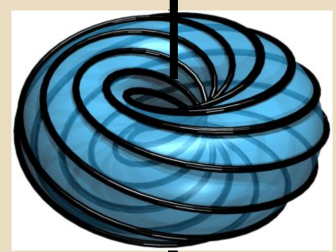
- **proves the consistent truncation of IIB on AdS_5 x S^5**

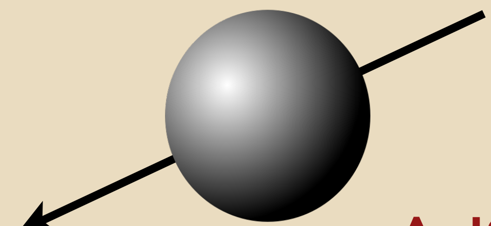
consistent truncation on $AdS_5 \times S^5$

ExFT

dictionary

IIB sugra





$AdS_5 \times S^5$

D=5 maximal sugra

global $E_{6(6)}$

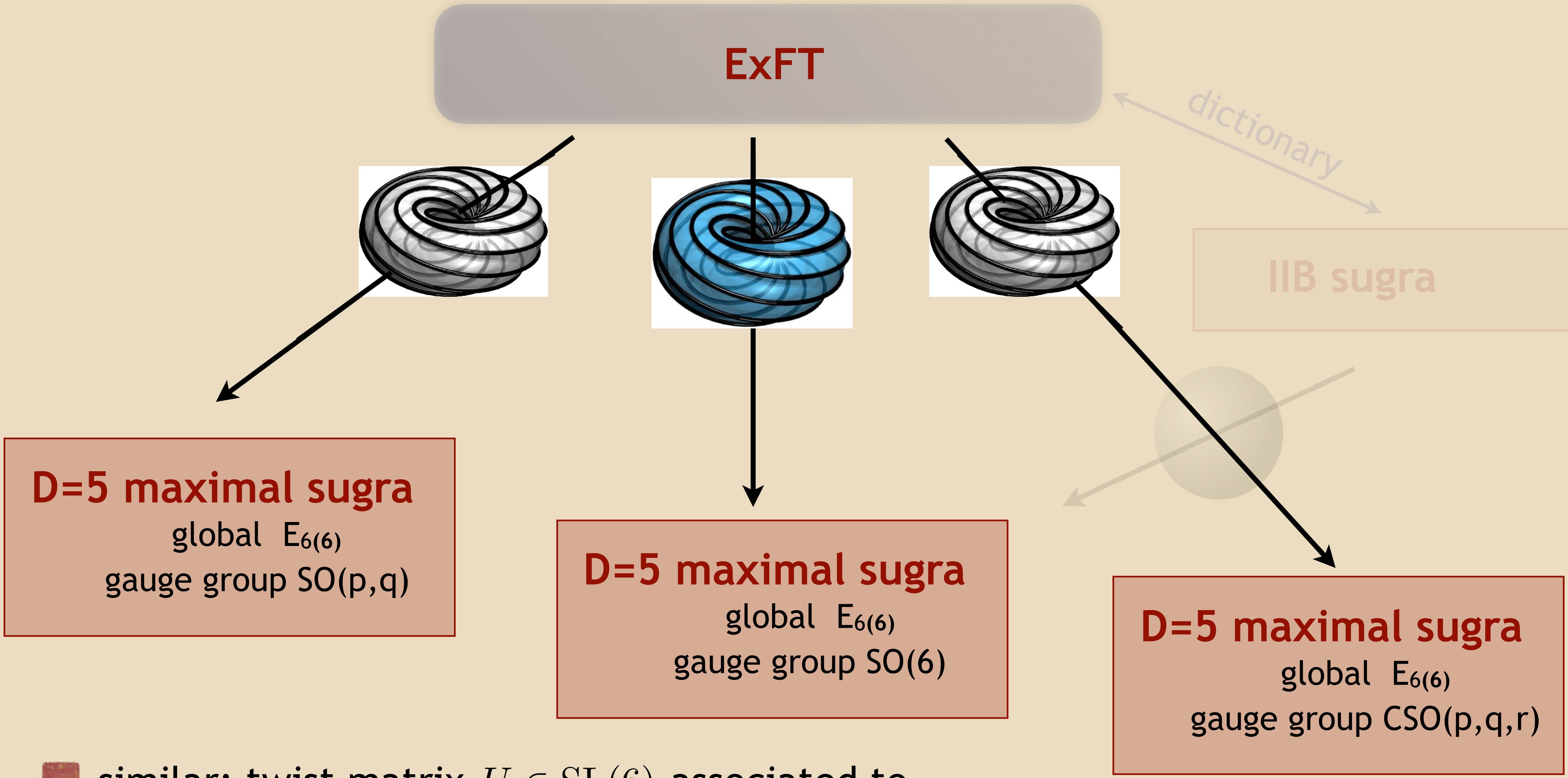
gauge group SO(6)

- similar: twist matrix $U \in \mathrm{SL}(6)$ associated to SO(p,q) and CSO(p,q,r) structure constants

  built from Killing vectors on SO(p,q)/SO(p–1,q)

$$U = \left(\begin{array}{c:c} \delta_i{}^j\, a(y^2) & y^i\, b(y^2) \\ \hdashline y^i\, c(y^2) & d(y^2) \end{array}\right)$$

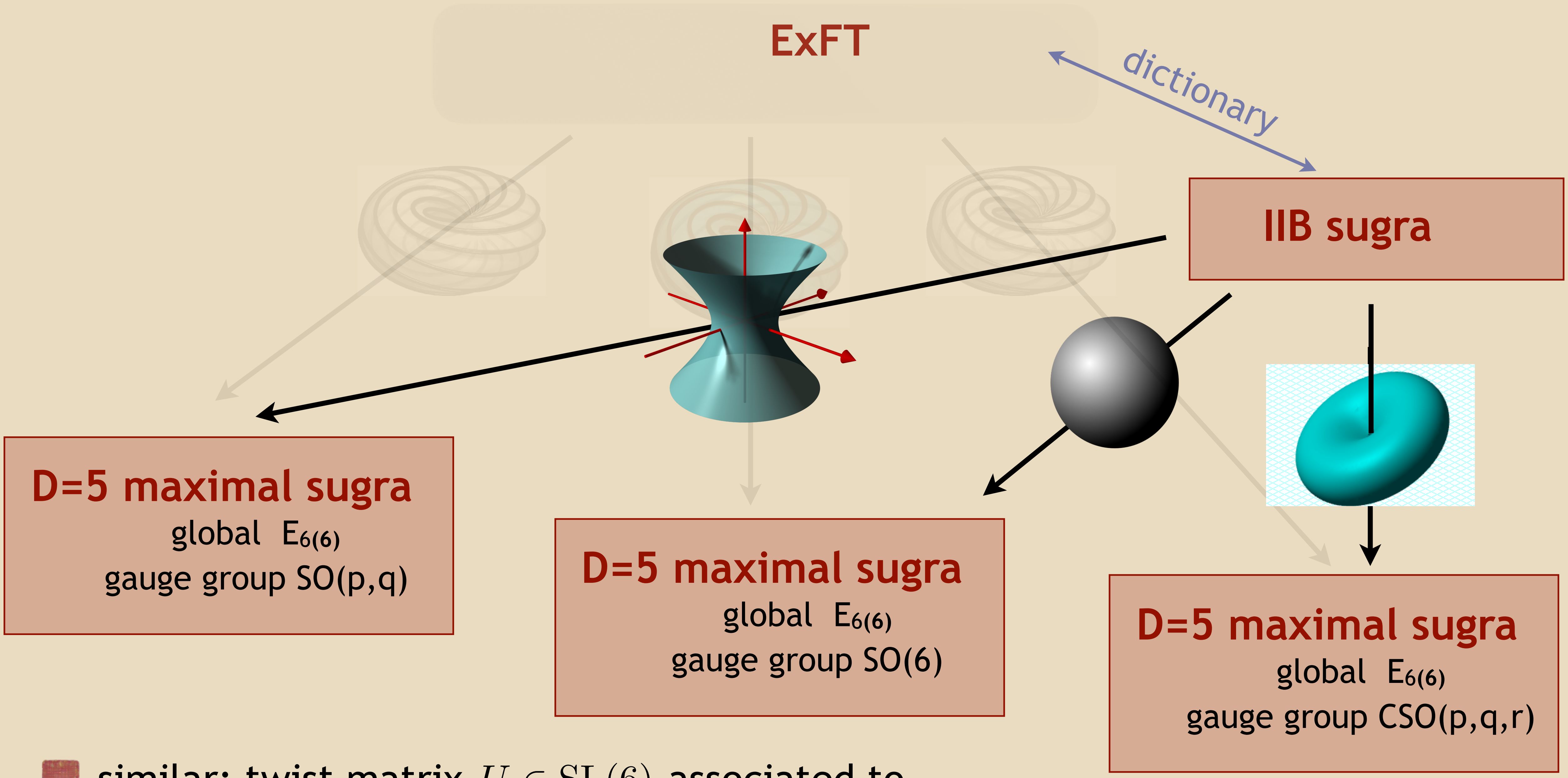
hyperboloid compactifications



- similar: twist matrix $U \in \mathrm{SL}(6)$ associated to SO(p,q) and CSO(p,q,r) structure constants

  built from Killing vectors on $\mathrm{SO(p,q)/SO(p{-}1,q)}$

hyperboloid compactifications



- similar: twist matrix $U \in \mathrm{SL}(6)$ associated to SO(p,q) and CSO(p,q,r) structure constants
  - background: (warped) hyperboloids [Hull, Warner] [Baron, Dall'Agata]
  - in general no IIB solutions, still consistent truncations!

other examples of consistent truncations

- consistent truncations with smaller isometry groups:
  [Inverso, HS, Trigiante, Malek]

  products of spheres and hyperboloids $S^p \times S^q$, $S^p \times H^q$

  specific D=4 construction, based on electric/magnetic split of internal coordinates

  inducing dyonic gaugings $(\mathrm{SO}(p,q) \times \mathrm{SO}(p',q')) \ltimes N$ [Dall'Agata, Inverso]

  with interesting vacua and solutions

- consistent truncations with less supersymmetry
  [Malek]

  embedding of half-maximal supergravity into ExFT

  construction and classification of supersymmetric AdS vacua

  half-maximal supersymmetric AdS vacua induce consistent truncations around

  $\longrightarrow$ [talks by Malek, Vall Camell]

other applications / developments

- ExFT for all finite-dimensional exceptional groups $E_{d(d)}$, d<9

  [Hohm, HS] [Abzalov, Bakhmatov,Musaev, Hohm, Wang, Berman,Blair, Malek,Rudolph]

  based on the different splits external/internal coordinates $\{x^\mu, y^m\} \longrightarrow \{x^\mu, Y^M\}$

- ExFT embedding of massive IIA theory

  [Ciceri, Guarino, Inverso] [Cassani, de Felice, Petrini, Strickland-Constable, Waldram]

  — by deformations of ExFT

  — by Scherk-Schwarz reduction violating the section conditions

  —> more general theme: consistent theories from reductions violating section constraints

- ExFT embedding of 'generalized IIB' theory

  [Baguet, Magro, HS]

  — background from η-deformed $AdS_5 \times S^5$ sigma model

  — T-dual of IIA with non-isometric dilaton

other applications / developments

- unifying framework for brane solutions

  [Berman, Rudolph, Bakhmatov, Kleinschmidt, Musaev, Otsuki, Fernandez-Melgarejo, Kimura, Sakatani]

  1/2 BPS branes from a single ExFT solution, organisation of exotic branes

- orbifolds and orientifolds in ExFT

  [Blair, Malek, Thompson]

  unified approach in terms of generalized orbifolds (O-folds)

- exceptional string sigma model

  [Arvanitakis, Blair]

  string sigma model with ExFT background fields

- ExFT loop calculations

  [Bossard, Kleinschmidt]

  duality covariant graviton amplitudes

- underlying mathematical structures

  [Cederwall, Palmkvist][Hohm, Kupriyanov, Lüst, Traube]
  [Cagnacci, Codina, Marques][Arvanitakis]

  L_∞-algebras, Borchers superalgebras, tensor hierarchy algebras

conclusions part I

- **exceptional field theory**

  - based on generalized diffeomorphisms in exceptional geometry
  - unique theory with generalized diffeomorphism invariance
    in all coordinates (modulo section condition)
  - upon an explicit solution of the section condition
    the theory coincides with full D=11 supergravity or full D=10 IIB supergravity
  - powerful tool for vacua and consistent truncations

- **tool for analyzing existing theories**
  — or hints towards a more fundamental structure ..?

  - weaken / relax section constraints
  - decrease number of external dimensions $\longrightarrow$ unifying picture

$\longrightarrow$ **part II: exceptional field theory for affine algebras**