A Proof Theoretic Non Scalar Account of Scalar Inferences

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April 22, 1999

Handout of a talk given at the preferably Non-Lexical Semantics Paris Conference, 28 30 May 1998

1 Modes of contrast

Aim of the talk: provide an explanation for the existence of systematic asymmetries in sentence pairs of the form X mais/but Y vs *Y' where we feel that, in ‘some’ sense X is conducive to Y'. In which sense?

1.1 Data

Constraints on linking in sentence pairs, Some of them already noted in Anscombe and Ducrot work on linguistic argumentation, Most of them noted more recently in (Jayez 1988) and (Ducrot 1995). 3 types of data: degree based modifiers, determiners, VPs, Adjectives

(1) a, Jean est intelligent, mais (moins vs *plus) que son frère
John is intelligent, but (less vs more) than his brother
b, La saison est froide, et même (plus vs *moins) froide que
The season is cold, and even (more vs less) cold than
l'année dernière
last year

c, Jean est marié, mais (depuis peu vs *longtemps)
Jean is married, but (since little vs long)
(John married a girl, but it's recent vs not recent)

Other examples: practically every degree adjective,
Prepositions
(2) a, C'est presque 100 FF, c'est (cher vs *bon marché)
It's almost 100 FF, it's (expensive vs cheap
b, Jean est arrivé après moi, mais il était (à l'heure vs John arrived after me, but he was (on time vs
*en retard)
late)
(intended interpretation for *: not that the fact of John being
late caused the fact of his arriving after me)
Adverbs
(3) a, Jean a (bien vs *à peine) compris le problème. Il est
John has (well vs hardly) understood the problem. He is
intelligent
b, J'ai *juste/seulement un rhume, Ça m'empêchera de
I have just a cold. It will prevent me from
venir
coming
Similar examples with presque, almost and degree adverbs (beaucoup, much, etc.)
Determiners
(4) a, Seuls Jean et ses amis sont venus, ça fait (peu vs
Only John and his friends came, it makes (few vs
*beaucoup)
a lot) of people
b, Quelques étudiants se sont inscrits, mais (pas tous vs
Some students registered, but (not all of them vs
?la plupart)
most of them)
Similar observations with certains, un certain nombre de, certain, etc,
Verbs

(5) a. Jean a marché; mais (pas longtemps vs *longtemps)
   John walked, but (not for a long time vs for a long time)

   b. Jean est allé jusqu'à la plage, mais (ça n' est pas loin vs
   John went to the beach, but (it's not far vs
   * c'est loin)
   it's far)

Remarks

Some examples redeemed if ironical (e.g., (3-a)),
some examples redeemed by ne que or only.
some examples redeemed by omitting mais or but,
phenomenon distinct from negative or positive polarity.

1.2 Scalar expectations

These examples are not all on a par. Some basic distinctions. For opposition
in general, see (Rudolph 1996).

- Lakoff (1971): semantic opposition (contrast in other terminologies
  ...).

(6) John is short but Mary is tall

- Again Lakoff (1971): denial of expectation (d.o.e.)

(7) John is tall but he is no good at basketball

- Indirect opposition (Auscom ete & Ducrot 1977). p mais q felicitous
  only w.r.t. r such that p favours r and q favours non r.

(8) Jean est grand mais peu rapide
   ('John is tall but slow')

   'Jean est grand ('John is tall') favours 'il est bon au basket' ('he is
   good at basketball')

   'Jean est peu rapide' ('John is slow') favours the contrary.

- Pourtant (≈ yet) conveys d.o.e. (Jaye 1981, 1988).

(9) a. Jean est grand, pourtant il n’ est pas bon au basket

   b. Jean est grand, pourtant il est lent

What the examples (1-a) (5-b) share

They are neither semantic opposition nor d.o.e. examples. They become

"or * when one puts par contre, in contrast, or pourtant, instead of mais.

(10) a. Jean est intelligent, ?par contre il l’est moins que son frère

   b. John is intelligent, ?in contrast he is less intelligent than his

   brother

   c. Jean est intelligent, ?pourtant il l’est moins que son frère

They accept in general although.

(11) a. (Although) John is tall, (but/yet) he is no good at basketball

   b. (Bien que) John soit 1 est très grand, mais/pourtant il ne vaut
   rien au basket

But there are also non d.o.e., mais with the same paraphrase.

(12) a. Bien que Jean soit intelligent, il l’est moins que son frère

   (Although John is intelligent, he is less intelligent than his brother)

   b. Bien qu’il ait marché jusqu’à la plage, ça ne lui a pas pris très
   longtemps

   (Although he walked to the beach, it didn’t take a long time)

Where they differ

1. Clause order matters or not bien que X, Y = Y, bien que X?

(13) a. Bien que Jean soit intelligent, il l’est moins que son frère

   (Although John is intelligent, he is less intelligent than his brother)

   b. Jean est moins intelligent que son frère, bien qu’il soit intelligent

   (John is less intelligent than his brother, although he is intelli-

   gent)

   c. Bien que certains étudiants se soient inscrits, tous ne l’ont pas
   fait

   (Although certain students registered, not all of them did)

   d. Tous les étudiants se sont inscrits, bien que certains
   l’aient fait

   (Not all students have registered, although some students did)

   e. Bien qu’il soit marié, il ne l’est pas depuis longtemps

   (Although he is married, it’s recent)

   f. Il n’est pas marié depuis longtemps, bien qu’il ? le soit

   (He married recently, although he is married)

2. Some exemples remain odd when mais is omitted,
2 Possible explanations for SSE

2.1 Monotonicity based

In Generalized Quantification theory, we might represent *John is more/less intelligent than his brother* as an inclusion between the set of intelligence degrees of John and that of his brother.

\[ \text{John is more intelligent than his brother} \equiv ID_J \supset ID_{JB} \]

So, monotonicity properties of *more/less ADJ than x* = those of \( \supset \) and \( \subseteq \). *Intelligent* \( \approx \) 'having more intelligence degrees than the average or than what is expected for a given intelligence threshold'. Then, monotonicity properties of *intelligent* = those of \( \supset \). In this respect, it could be argued that \( (1-\text{a}) \) is odd with *less* because it opposes two occurrences of the same monotonicity profile (\( \supset \)). Problem: the hypothesis rather a description than an explanation.

2.2 Graded rules

Ellisad (1993) proposes to associate monotonicity property with the notion of orientation argumentative studied by Anscombe et Ducrot, Upward monotony corresponds to positive orientation, downward monotony to negative one.

But orientation = ?

Ducrot (1988) proposes that scalar configurations are interpreted by graded proportionality rules. Two aspects:

- lexical selection of rules, e.g., *more* selects rules of the form \{the more X, the more/less Y\}, *intelligent* selects rules of the form \{the more one is intelligent, the more/less one \ldots\}
- existence of appropriate rules.

When no plausible rule satisfying the lexical selection criterion can be found, we get an anomaly. However, the commonsense rule we should apply to the contrast in (1-a) is not felicitous, being of the ([strange] sort (the more x is intelligent, the more x is more intelligent than other people).

2.3 Probability

The probability that John is more intelligent than his brother given that he is intelligent is, other things being equal, \( \geq \) to the probability that John is more intelligent than his brother.

Ok, this would account for the ‘expectation’ relation, but accommodation of ‘contrary’ premises must be blocked. Accommodation (Lewis 1979) allows one to let in additional premises which are required to interpret an inferential relation.

\[ \text{0. a. John came, but he didn't speak to Mary} \]
\[ \text{0. b. John came, but he *spoke to Mary} \]

John coming leads to the expectation that he spoke to Mary only under additional assumptions, for instance that John met Mary, that he had something to tell to Mary, etc.

Why is not the same mechanism possible in the symmetric case (16-b): John came + he didn’t meet Mary \neg he didn’t speak to Mary?

Proposal for SSE: plunge lexically scalar items into a general inference system (non-scalar account of lexically scalar inferences).
3 Proof-theoretic account

3.1 Intuitive description

Two claims
a. Lexical representations
b. Constraint on inference

Lexical representation
In (1a), John is intelligent is ‘represented’\(^4\) as:
\[ J_{\text{int}} \geq l_{\text{int}} \]
where \( J_{\text{int}} \) is the degree of John’s intelligence, and \( l_{\text{int}} \) is an arbitrary intelligence threshold; people whose intelligence degree is \( \geq l_{\text{int}} \) are intelligent. People whose intelligence degree is below may or may not be intelligent.

Constraint on proofs
\( J_B = \) John’s brother. If the intelligence of John’s brother is below \( l_{\text{int}} \), then John is more intelligent than his brother (\( J_{\text{int}} \geq J_B \)). By adding the fact that \( J_B < l_{\text{int}} \), we have a proof of ‘John is more intelligent than his brother’ from ‘John is intelligent’.

Can we prove that ‘John is less intelligent than his brother’ by adding something? Yes,

(17) a. If \( J_{\text{int}} \geq l_{\text{int}} \) and \( J_B < l_{\text{int}} \), then \( J_{\text{int}} < J_B \)
b. If \( J_{\text{int}} \geq l_{\text{int}} \), \( J_B < l_{\text{int}} \) and \( J_B < l_{\text{int}} \), then \( J_{\text{int}} < J_B \)

The proofs are (classically) correct, but the underlined formulas can be suppressed without altering the conclusion. We don’t need them, actually. The constraint is (18).

(18) Constraint on proofs, intuitive form
No proof with needless premises is allowed.

The linguistic constraints are explained by noting that the marked forms correspond to potential proof excluded by (18). Problem: no connection between the structure of proofs and (18) ⇒ the situation in (19)

(19) \[ J_{\text{int}} \geq l_{\text{int}} \cdot J_{\text{int}} \leq l_{\text{int}} \cdot J_B < l_{\text{int}} \cdot J_B = J_B \]

(19) would license (20), because ‘John is less intelligent than his brother’ entails that John and his brother have not the same degree of intelligence,\(^5\)

(20) John is intelligent, but he is \( ^{??} \) not less intelligent than his brother. Comes from the \( x \geq y \) \( x \leq y \) \( x = y \) stuff.

Lexical representation again
It could be argued that this problem is caused by the representation of ‘John is intelligent as \( J_{\text{int}} \geq l_{\text{int}} \). Suppose we adopt instead the (stronger) coding
\[ J_{\text{int}} > l_{\text{int}} \]

This representations allows for a different account which says that the proofs we want to get rid of are ‘bad’ because they contain useless (irrelevant) premises or contradictory premises. The latter case is illustrated by the new versions of (17b) and (19).

(17b) If \( J_{\text{int}} > l_{\text{int}} \), \( J_{\text{int}} \leq l_{\text{int}} \) and \( l_{\text{int}} < J_B \), then \( J_{\text{int}} < J_B \)

(19') \[ J_{\text{int}} > l_{\text{int}} \cdot J_{\text{int}} \leq l_{\text{int}} \cdot J_B = l_{\text{int}} \cdot J_B = J_B \]

In (17b) and (19'), the two premises \( J_{\text{int}} > l_{\text{int}} \) and \( J_{\text{int}} \leq l_{\text{int}} \) are mutually inconsistent. However, there are cases in which the weaker form \( i.e., \geq \) or \( \leq \) is required and which give rise to anomalous texts, such as (21)

(21) John’s intelligence is not superior to the average, but he is \( ^{??} \) less intelligent than his brother

If the first sentence is represented by \( J_{\text{int}} \leq l_{\text{int}} \), we must block (correct) proofs such as, where the useless formulas are underlined.

(22) a. \( J_{\text{int}} \leq l_{\text{int}} \cdot J_{\text{int}} \geq l_{\text{int}} \cdot J_B \leq l_{\text{int}} \cdot J_B \)
b. \( J_{\text{int}} \leq l_{\text{int}} \cdot J_{\text{int}} \geq l_{\text{int}} \cdot J_B < l_{\text{int}} \cdot J_B \)

The strategy I adopt is to devise a very general ‘proof killer’ which might be too powerful, I leave the question whether it must be further constrained (by invoking pragmatic factors, for instance) to another work. Equality comparatives (as, as, autant, etc.) raise an interesting problem, I conjecture that it can be addressed by adopting Ducot’s assumption that those comparatives have two levels of information (see section 4.1).

3.2 Formal representation

Diagnostic
Where does the problem with (19) come from?
\( x \geq y \) is \( x > y \) \( x = y \)

the conjunction of \( x \geq y \) and \( x \leq y \) uses the possibility offered by classical logic that absurdity leads to just anything (\( ex \) falso sequitur quodlibet,
absurdity = maximal information in a Boolean lattice, etc.)

More specifically, consider a Gentzen system for classical propositional logic, such as GSC from [Grandy 1977, p. 24] and add the rules for order,

(23) **Order rules**

1. \( \Sigma, x < y, y < z \vdash x < z, \Gamma \)
2. \( \Sigma, x = y, y < z \vdash x < z, \Gamma \)
3. \( \Sigma, x < y, y = z \vdash x < z, \Gamma \)
4. \( \Sigma, x > y, y > z \vdash x > z, \Gamma \)
5. \( \Sigma, x = y, y > z \vdash x > z, \Gamma \)
6. \( \Sigma, x > y, y = z \vdash x > z, \Gamma \)
7. \( \Sigma, x = y, y = z \vdash x < z, \Gamma \)
8. \( \Sigma, x = y \vdash \neg(x < y), \Gamma \)
9. \( \Sigma, x = y \vdash \neg(x > y), \Gamma \)
10. \( \Sigma, x > y \vdash \neg(x < y), \Gamma \)
11. \( \Sigma, x < y \vdash \neg(x > y), \Gamma \)

We want to prove that \( x \geq y, x \leq y \vdash x = y \). In classical logic, this is done as follows,

\[
\begin{align*}
\Sigma, x > y & \vdash x > y \\
\Sigma, x < y & \vdash x < y \\
x > y & \vdash \neg(x < y) \vdash x = y \\
\neg(x < y) & \vdash \neg(x > y) \vdash \neg(x > y) \\
\end{align*}
\]

\( \vdash x = y, (x > y \lor x = y) \vdash x = y \)

We have a proof of \( x > y, x < y \vdash x = y \) if we have a proof of \( x > y, \neg(x < y) \vdash x = y \)

We apply 7 and \( \neg \rightarrow \) in GSC,

\( x > y, \neg(x < y) \vdash x = y \)

What we are actually using: absurdity leads anywhere. Introduce inconsistent information and say just what you like.

**Medicine**

Accept that there is something right in Grice’s maxims and Sperber & Wilson (1986) relevance: we do not just introduce information in this way.

What we do (sometimes): if \( A \) leads to absurdity, reject \( A \).

What we don’t do: introduce \( A \) and \( \neg A \) and deduce \( B \) if it has no special relation with \( A \) or \( \neg A \).

This suggests no Left Weakening allowed,

(24) **Left Weakening**

\[
\begin{align*}
\Sigma, x & \vdash \Gamma \\
\Sigma & \vdash x \vdash \Gamma \\
\end{align*}
\]

No Left Weakening = every information which is introduced explicitly must be used.

If John is intelligent, maybe he is intelligent enough to be more intelligent than his brother

\( J_{int} \geq int \_ int > J_B rut \vdash \neg J_{int} \geq J_B rut \)

If John is intelligent, maybe he is intelligent enough to be less intelligent than his brother

\( J_{int} \geq int, \neg J_{int} \leq int \_ int > J_B rut \vdash \neg J_{int} < J_B rut \)

Systems without Left Weakening do not have the property that absurdity leads anywhere, E.g., in \( L_1 \) (Thistlewaite et al., 1988), we have the following property.

(25) **L1** Let \( L_{1, \neg} \) \( L_1 \) plus the non logical axioms corresponding to the order rules on (23) under their no LW form for instance with \( x > y, y > z \vdash x > z \) corresponding to \( \Sigma, x > y, y > z \vdash x > z, \Gamma \), etc.

Let \( \Sigma \) be a multiset containing only formulas of form \( u > w, u < w, u = w \), or disjunction of such formulas. Let \( x \geq y \) abbreviate \( \{x > y\} \lor \{x = y\} \), etc.

No judgment of form \( x > y, \Sigma \vdash x < z, x > y, \Sigma \vdash x \leq z, x > y, \Sigma \vdash x = z, x > y, \Sigma \vdash x \geq z, x > y, \Sigma \vdash x = z, x < y, \Sigma \vdash x > z, x \leq y, \Sigma \vdash x \leq z, x \leq y, \Sigma \vdash x = z \) is provable in \( L_{1, \neg} \).

**Proof**

No proof of length 0 exists (no order axiom has the corresponding form).

Induction step. The only two rules which are relevant are \( P_r^c \) and \( P_v^c \),

\[
\begin{align*}
\Sigma, \neg X & \vdash P_r^c \\
\Sigma, \neg (X \lor Y) & \vdash P_v^c \\
\Sigma, X & \vdash P_v^c \\
\end{align*}
\]

We have the following possibilities,

\( \vdash \) as a premise.
4 Extensions

4.1 Background and foreground in scalar expressions

In (Jayez 1987), it is proposed that *presque* NP_q(x), where NP_q is a quantitative NP has the following representation,

\[\Sigma, a, \neg(x > y), x < z\]

\[\Sigma, a \lor b, \neg(x > y), x < z\]

\[\Sigma, a, \neg(x > y), x = z\]

\[\Sigma, a \lor b, \neg(x > y), x = z\]

\[\Sigma, a, \neg(x > y), x \leq z\]

\[\Sigma, a \lor b, \neg(x > y), x \leq z\]

\[\Sigma, \neg(x > y), x < z\]

\[\Sigma, \neg(a \lor b) \lor (x > y), x \leq z\]

\[\Sigma, \neg(a \lor b) \lor (x > y) \lor (x < z)\]

If *cher* (expensive) corresponds to \(\geq l_{exp}\), the oddness in (2a) is readily explained by noting that the odd forms are instances of a judgment \(\Sigma, x \geq y \lor x < z\).

In general, one can assume that background information (= presupposition, in most cases) may not create bad configurations in the corresponding judgments. Only foreground information (= asserted information) is relevant. Similar proposal for only in (Horn 1996).

- **Seulement (just)**
  (3b), ‘I have just a cold’ = ‘I have no more than a cold’. What kind of reasoning can we appeal to?
  Let \(d\) be my disease degree.
  If \(d = l_{cold} \lor \neg \text{come}\) is the rule we use, certainly \(d < l_{cold} \land \neg \text{come}\) is not relevant (the premise \(d < l_{cold}\) cannot be used).
  If the rule is scalar: \(d \geq l_{cold} \lor \neg \text{come}\), meaning whenever I have got something which is \(\geq\) to the point which prevents me from coming.
  I don’t come; the judgment \(d = l_{cold} \lor \neg \text{come}\) is again irrelevant.

- **A peine and Seul similar**

- **Determiners.**
  Just associate \(n \geq l\_queues\) and \(n \geq l\_certains\) with *queues* and *certains*.
  Again: \(n \geq l\_queues\). \(\Sigma \lor n < l\_peus\)
  \(n \geq l\_queues\). \(\Sigma \lor n \geq l\_peus\)

- **Contrasts like il est resté 77 tôt vs tard (‘He stayed early vs late’), mentioned in (Ducrot 1995).** Actually the phenomenon is general with atelic constructions.

(26) a. Il a lu (tard vs 77 tôt)
  (‘He read late vs early’)

b. Il a mangé des biscuits jusque (tard vs 77 tôt) dans la nuit
  (‘He ate cake till late vs early) in the night’

c. Il a continué (tard vs 77 tôt)
  (‘He continued (late vs early)’)

d. Il a été président jusqu’à après (tard vs 77 avant) ma naissance
  (‘He was president till after vs before my birth’)

Karttunen’s (1974) lateness effect, studied in de Swart (1996) are analogous.

(27) The princess slept until nine (at the latest vs 77 at the earliest),
Examples (26a) [26d] point to not until (de Swart 1996) and *finque* (Towena 1996).
tard is associated with \( t \geq l_{\text{late}}, \) but with \( t \leq l_{\text{new}} \). The temporal semantic of states and activities resembles that of until, hence the affinity of these examples with temporal \( \text{jusqu'à}. \)

(28) **Intrinsic semantic of atelic constructions**

If \( \phi \) is atelic, asserting that \( \phi \) entails that \( \phi \) holds until some point in time, or, equivalently that \( \phi \) does not cease before some point in time.

Warning: again, assertion/presupposition distinction has to be used. We assert that \( \phi \) does not cease before \( t \) and we presuppose that it ceases at \( t \).

So, \( \text{It a lu} \) (He read) asserts that he did not stop reading before some point in time: \( \exists! (\text{end(reading)} \geq t) \). Now, \( \text{tard (late)} \) gives the position of the beginning or end of an event w.r.t. to a lateness threshold. Just try to compose the constraints by binding the threshold to the quantifier.

\[ \text{It a lu + tard = } \{ \exists! (\text{end(reading)} \geq t) \} \cup \{ t = l_{\text{late}}, \text{end(reading)} \geq l_{\text{late}} \} = \{ \text{end(reading)} \geq l_{\text{late}} \} \]

\[ \text{It a lu + tôt = } \{ \exists! (\text{end(reading)} \geq t) \} \cup \{ t = l_{\text{late}}, \text{end(reading)} \leq l_{\text{late}} \} = \{ \text{end(reading)} \geq l_{\text{late}}, \text{end(reading)} \leq l_{\text{late}} \} \]

Sentences are ok when the beginning is considered, because, presumably, there is no intrinsic semantic restriction comparable to (28) for beginnings.

### 4.2 Ducrot's notion of déréalisation

In his 1995 paper, Ducrot shows that the range of phenomena pertaining to the general problem is extremely vast. He calls déréalisant (actuality demoter or a demoter) any modifier which attacks the 'essence' of an object. For instance, in (1c), the recency of the marriage makes it a sort of not genuine marriage.

The present proposal leads to distinguish at least 3 different cases.

1. Some scalar inferences exploit the intrinsic semantic properties of items (SSE). The typical cases are \( \text{intelligent, presque, etc.} \)

2. The marriage case could correspond to an indirect opposition case, in the sense of Anscombe & Ducrot (1977). \( \text{John is married} \sim \text{John has the properties one expects to hold of a married man}. \text{John’s marriage is recent} \sim \text{John has not the properties one expects to hold of a married man} \)

3. It could also correspond to an a demotion in the following sense, Things, events, situations have properties which make them perceptible. They can be seen, heard, tasted, counted, They have duration, they occur at some determinate time, etc. An a trace is a trace of the existence of an object in a cognitive (perceptual, conceptual, etc.) system. An a demoter removes an a trace.

Example: duration. Let \( e \) be an event, if \( e \) occurs it has some duration \( \geq \) the duration threshold (the threshold under which an event no longer exists w.r.t. some recognizing system). So, \( \text{occurs}(e) + \text{duration}(e) \geq l_{\text{duration}} \).

If we say that \( e \) was short, we introduce an information such as \( \text{duration}(e) \leq l_{\text{short}} \). Now, there is a scalar expectation corresponding to: \( \text{duration}(e) \leq l_{\text{short}} \sim l_{\text{short}} \sim \text{duration} \sim l_{\text{duration}} \)

Finally, this gives an indirect opposition pattern: the event occurs \( \sim \) its duration is not below the duration threshold, the event is short \( \sim \) its duration is below the duration threshold.

(29) **a demoters**

In a form \( p \text{ mais } q \) \( q \) is an a demoter of \( q \) if there exists a relevant proof from \( q \) to the negation of an a trace of \( q \)

Examples of a demotion:

On occurrence time. Things which happen happen at some time (!), postponing them is a manner of suggesting they might not occur at all, \( \text{It viendra, mais pas tout de suite} \) (He will come, but not immediately). An a trace of the event is the fact that it occurs before \( \infty \) (the end of time): \( \text{event occurrence} < \infty \). But we have \( \text{event occurrence} \geq l_{\text{immediately}} \sim l_{\text{immediately}} > \infty \sim \text{event occurrence} > \infty \).

Similar examples with occurrence frequency (something which happens, but not often), or intensity (something which happens, but at a very low level),

### 5 Conclusion

Summary: lexical information (position w.r.t. thresholds) is dealt with in a general inference system. This permits a uniform treatment for superficially different cases.

Remaining problems (those I am aware of!):

- status of equality expressions (see Ducrot vs de Cornulier controversy) like \( \text{suitant que} \)
- syntax-semantic interface for expressions triggering SSE

**References**

Notes

1 As noted by Lakoff (1971) d.o.e. uses of but can be paraphrased by using although. The same observation goes for bien que. The reverse is not true, Examples which are ok with although or bien que are not necessarily denial of expectation examples. See (1-3).

2 Other constructions in French seem to license pourtant because they exploit a high degree reading.

(30) a. Pour intelligent qu'il soit, Jean ne l’est (pourtant/quand même) pas autant que son frère
b. Jean a beau être intelligent, il ne l’est (pourtant/quand même) pas autant que son frère
c. Avec toute son intelligence, Jean 
   n’arrive (pourtant/quand même) pas à égaler son frère

Examples (30a) (30c) mean ‘John’s intelligence is terrific, yet his brother is even more intelligent’. This is a direct opposition pattern, Such differences are connected with the difference observed by Fradin (1977) between propositional concessive sentences (bien que) and what he calls extensional concessive sentences, whose concession operator ranges through a set of degrees, In English, think of constructions like No matter how John is intelligent, X.

3 In addition, some speakers don’t like the ‘subordinate clause first’ order, i.e. (13-e)

4 That is, any acceptable representation for the adjective must entail this scalar property.

5 Moreover, (19) does not license a perfectly natural sentence like John is intelligent, but he is less intelligent than his brother, since John being not less intelligent than his brother does not entail that John and his brother have not the same degree of intelligence.

6 The Gentzen systems for relevance logics use multisets and non oriented judgments, That is, a judgment like X, Σ ⊢ y is represented as Σ, → X, Y, So a (true judgment) like A ⊢ (A → B) → B is → A, ¬ (A → B), B.

7 The existence of different levels of information is still an open question (cf, Atlas, 1996).

8 There is more to be said about determiners, however, See the discussions in (Jayez 1988) and Corblin (1997).