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Effective magnetic permeability of a turbulent fluid with macroferroparticles

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Abstract. A fully developed turbulent flow is capable to mix and homogenize a suspension of heavy macroscopic particles even at a high concentration of particles. If the particles are ferromagnetic, a kind of "turbulent ferrofluid" can be obtained. In the present work, we present a direct measurements of the effective magnetic permeability in a turbulent fluid with suspended ferromagnetic particles of typical size 0.01-0.1 mm and volume fraction c up to 25%. We show that the effective permeability can be fitted by the linear law $\mu_{\text{eff}} = 1 + 5.3c$ for $c \leq 10\%$. For higher volume fractions the permeability exceeds this linear relation.

PACS. 47.65 Magnetohydrodynamic fluids

Rapid Note

1 Introduction

Fluids with relatively high magnetic permeability exhibit various remarkable properties and are intensively studied in the framework of ferrohydrodynamics [1]. The typical ferrofluid is a colloid solution of Fe_3O_4 particles in kerosene. The size of a ferromagnetic particle is about 10^{-8} m. There are two reasons why the particles should be so small. Firstly, this size provides a mono-domain structure of particles. Secondly, such particles can be suspended in the fluid by Brownian motion. The stability of ferrofluids is of crucial importance in their production. The addition of surfactants allows one to avoid clustering and get ferrofluids of practically unlimited period of validity. The permeability of ferrofluids reach $\mu_{\text{eff}} \sim 10$, under particle volume fraction about 20%.

Much effort has been spent on getting conducting ferrofluids, however, no stable fluid is currently available (see, *e.g.*, [2] and references therein). Among the problems for which conducting ferrofluids might be very desirable is the MHD dynamo problem. The study of the dynamo action is a challenging and costly procedure to be carried in laboratory conditions, since this phenomenon is a non-linear instability. Dynamo action takes place when the stretching of the magnetic field lines overcome the Joule dissipation, i.e. provided that the magnetic Reynolds number Rm exceeds some critical value Rm_*. This threshold depends on the particular geometry of the flow. One of the lowest theoretical estimates [3], $Rm_* = 17.7$, was obtained for the so-called Ponomarenko dynamo, corresponding to the screw motion of a rigidly rotating cylinder in the infinite conducting medium [4]. However, to achieve in laboratory conditions even such a relatively low magnetic Reynolds number, one needs to generate a flow with an extremely high hydrodynamic Reynolds number. Indeed, the ratio of the two Reynolds numbers is given by the magnetic Prandtl number

$$P_m = \frac{Rm}{Re} = \frac{\nu}{\nu_m} = \nu\sigma\mu_0\mu, \tag{1}$$

where ν is the viscosity, ν_m is the magnetic viscosity, σ is the conductivity, μ is the magnetic permeability, and μ_0 is the permeability of vacuum. For a high electric conductivity, one tends to use molten metals as working fluids. Among these fluids is Sodium which has one of the best conductivities $\sigma \sim 10^7$ S/m, so that $\mu_0\sigma \sim 10$, resulting in $P_m \sim 10^{-5}\mu$, at best. It means that the critical magnetic Reynolds number can be obtained and exceeded only in a fully developed turbulent flow ($Re > 10^6/\mu$). The development of a fluid with an electrical conductivity equal to that of liquid metals and a high relative permeability may be of great practical importance because of the possibility to maximize the accessible range of Rm and minimize the turbulence level.

The fact that the turbulent flows are required to achieve the high magnetic Reynolds numbers provides a new way of looking at the problem of obtaining conducting ferrofluids. It implies the use of the strength of pressure fluctuations (dispersive grain pressure [5]) to mix and homogenize the suspension of heavy macroscopic particles. This idea was put forward in preliminary studies in dynamo projects such as the Perm [6] and VKS

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(von Kármán - Sodium) [7] experiments. In the Perm project, the flow is a screw motion in a toroidal channel. The possibility to mix a high-concentration suspension of heavy macro particles was proved in a water experiment, in which the titanium powder or glass beads with diameter about 1mm (up to 20% of volume fraction) were suspended by the turbulent screw flow in the toroidal channel [8]. A diphasic fluid made of iron beads in liquid Gallium has been generated in a von Kármán flow generated in the gap between counter-rotating discs by Martin *et al.* [9]. Volume fractions of 6.35 mm iron beads equal to 30% and 45% were tested at varying rotation speeds in the range [0, 50] Hz. Effective permeabilities ranging from to 2.3 to 4.8 were estimated from the magnitude of magnetic induction caused by the flow motion. The values, at such high concentration, were found to be consistent with a variation $\mu \propto 1/(1-(c/c_{\rm cp})^{1/3}))$, where $c_{\rm cp} = 0.74$ is the close packing concentration. Note that the method used in [9] does not separate two effects which could affect the magnetic induction, *i.e.* the magnetic permeability variation and the flow modification caused by high concentration of iron beads.

The purpose of this paper is to measure the effective permeability of a turbulent mixture as a function of volume fraction for the whole range of accessible concentrations. We have used iron powder instead of spherical beads and water as a working fluid. The use of water is justified by the similarity of the hydrodynamic characteristics of sodium and water (density, kinematic viscosity, etc.) and by the fact that the electric conductivity plays no part in our measurements. As explained below, in the present work the permeability is obtained by performing direct measurements of magnetic fluxes.

2 Experimental setup

Our experiments were carried out with the setup shown in Figure 1. The suspension of ferromagnetic particles is prepared in a cylindrical tank (1) of volume about 5 liters. The bottom of the tank is connected with a vertical tube (2) where the measurements are made. The inner diameter of the tube is 21.6 mm. The entrance of the tube is a funnel (3) with a 6-blade flow diverter (4) of diameter 50 mm. The function of this funnel is to generate a screw flow in the measuring tube. The diverter is precisely the same as the one used in the experiments with screw flows in the toroidal channel studied under the dynamo project [6]. The tube (2) was manufactured from fiberglass. On the tube inner wall there is a one-row measuring coil (5) made of 100 turns of wire of diameter 0.28 mm. An excitation coil (6) (800 spires of wire wound around the tube) produces an alternating magnetic field (H = 357 A/m, $f = 50$ Hz).

The fluid is a water solution of ethylenglycol to avoid the corrosion of iron powder. The size of an individual ferromagnetic particle is in the range 0.03 – 0.13 mm, with an average value of about 0.06 mm. A snapshot of the ensemble of particles is shown in Figure 2 where one can note the size dispersion and anisotropy of the shapes. The question

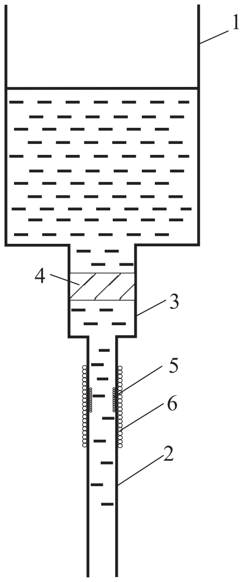

Fig. 1. Experimental set-up.

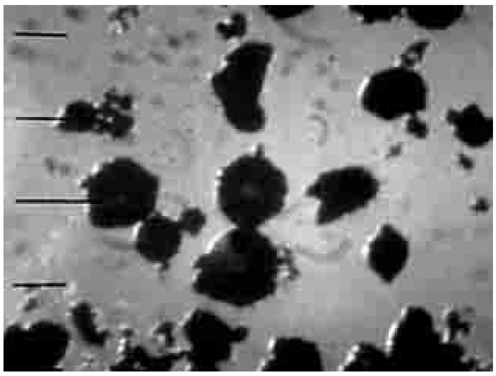

Fig. 2. Ferromagnetic particles. Ticks at left correspond to 0.1 mm.

of the maximum volume fraction with such polydisperse anisotrpic particles is still an open one. For monodipserse particles of spherical shape, the maximum volume fraction of the close-packing lattice is $c_{\rm sph} = 0.7404$ (see *e.g.* [11]). For a polydisperse micture of spherical grains, one expects the random-packing concentration is 0.65, while the loose-packing concentration is even less [12]. For strongly anistropic particles, the maximum volume fraction may become even lower. In our ferromagnetic particles, we measure $c_{\rm pow} = 0.42$.

The mechanical mixer provides the homogeneity of the suspension of iron powder in the upper vessel. Once this operation has been done, the vertical channel is opened and the electromotive force E_m is measured as the suspension passes through the coil (5). The effective permeability of the fluid-iron powder mixture is defined as

$$\mu_{\rm eff} = \frac{E_m}{E_0}, \tag{2}$$

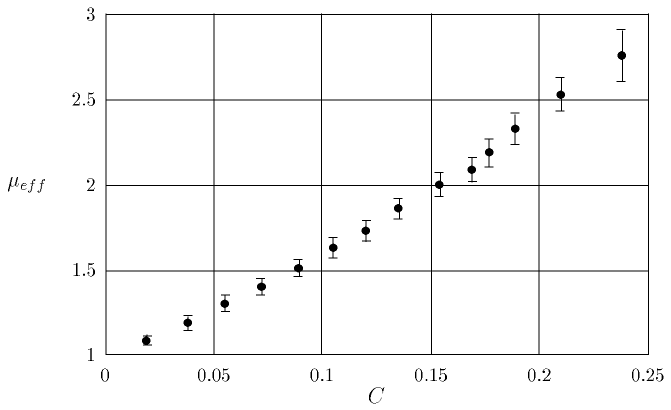

Fig. 3. Effective permeability of turbulent water suspension *versus* volume fraction.

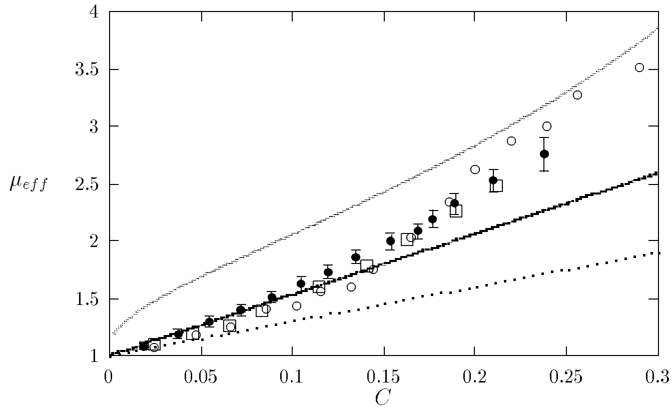

Fig. 4. Effective permeability *versus* volume fraction: black circles – water suspension, open circles – ferromagnetic particles in sand, boxes – carbonil iron powder in sand. Lines correspond to different fitting (see text).

where E_0 and E_m are the electromotive forces induced in coil (5) for the empty tube and the tube with mixture, respectively. We have performed 8 measurements for each volume fraction.

3 Results

Measurements were made for the water-iron powder mixture at volume fractions up to 0.25. At higher concentrations, it was impossible to obtain a homogeneous suspension of particles. The variation of the effective permeability with volume fraction is shown in Figure 3. It is seen that μ_{eff} grows as the volume fraction increases, slightly faster than linear.

To check whether the magnetic characteristics of the water suspension mixture depend on the flow properties, we have performed similar permeability measurements for a mixture consisting of the same iron powder and sand. In this case, the measurements can be extended to larger volume fractions (up to 0.3). We have also studied the influence of the ferromagnetic properties of particles by using carbonil iron powder with typical grain size of about 0.005 mm. The results for ferromagnetic particles of both sizes are practically identical, which confirms that the governing parameter in this problem is the particle concentration rather than the particle size.

All results are plotted in Figure 4. It is seen that the permeability of the dry mixture is close to that of the water suspension. However the permeability of the dry mixture is slightly less at low concentrations and slightly higher at large concentrations. This can be interpreted by the influence of the mixing factor of the flow. At low particle concentration the flow provides a more homogeneous distribution of ferromagnetic particles, which results in a higher effective permeability. At high concentrations the particles begin to coalesce forming chains. In the sand these chains are stable, and in the turbulent flow they are destroyed by the flow. It should be noted that the permeability of the sand mixture systematically increases by a factor of 10 to 20% if the tube is shaken. This increase of μ_{eff} with shaking may also be related to the formation of chains.

The results obtained can be confronted with some theoretical predictions for the permeability of suspension (or mixture). The most straightforward prediction concerns the limit of low concentrations of particles. The asymptotic case of vanishing concentration was considered by Landau and Lifshitz [10] for the electric permittivity of a suspension of spherical particles and can be adjusted to the case under discussion. This prediction reads:

$$\mu_{\text{eff}} = 1 + 3c, \tag{3}$$

where c is the volume concentration of ferromagnetic particles. Equation (3) can be obtained as follows. Let B be the microscopic value of magnetic induction, and H is the microscopic value of the magnetic field. Then, locally $B(x) = \mu(x)H(x)$, where $\mu = \mu_0$ is observed with the probability $(1-c)$, and $\mu = \mu_1 \gg \mu_0$ is obtained with the probability $c \ll 1$. Introducing the mean magnetic induction $\mathcal{B}$ averaged over the distribution of ferromagnetic particles gives $\mathcal{B} = \mu_{\text{eff}} H$. On averaging, the magnetic field is calculated in a very small fraction c of space, in which each particle can be viewed as an 'isolated island' subjected to a prescribed external magnetic field. This results in $\mu_{\text{eff}} = 1 + Ac$, where A is a constant factor related to the geometrical shape of ferromagnetic particles. In the case of spherical particles, one obtains $A = 3$. In Figure 4, this expression corresponds to the dotted line. It is clear that this line essentially underestimates the experimental data. This difference is accounted for by the complicated form of the particles of iron powder which modifies the value of the constant A (the linear behavior is only ensured as long as the particles may be regarded as isolated). A simple way to take into account the variations in particle shapes is to introduce an effective concentration c_{eff}, renormalized to the maximum concentration. Specifically, we define:

$$c_{\text{eff}} = c\,\frac{c_{\text{pow}}}{c_{\text{sph}}}, \tag{4}$$

and replace it in equation (3), which gives:

$$\mu_{\text{eff}} = 1 + 3\frac{c_{\text{sph}}}{c_{\text{pow}}}c \approx 1 + 5.3c. \tag{5}$$

This expression is shown in Figure 4 as a thick solid line. It is a better fit than equation (3) and gives a reasonable approximation of the data up to $c \approx 0.1$. At higher volume fractions, equation (5) also underestimates the experimental data.

For large concentrations, Martin *et al.* [9] suggested the following scaling:

$$\mu_{\text{eff}} = \frac{1}{1 - (c/c_{\text{sph}})^{1/3}}\,. \tag{6}$$

It is shown in Figure 4 as a thin solid line. This curve yields a reasonable estimation of the sand mixture at highest concentrations. We note that for the self-consistency it would be necessary to replace c by c_{eff} in equation (6); however it gives highly overestimated values for μ_{eff}. This leads us to conclude that the scaling (6) is applicable to a very large c in the vicinity of a limiting concentration only.

Two further comments are in order. First we should note that all estimates given by equations (3, 5), and (6) do not take into account the possible clustering of ferromagnetic particles. The small deviation of our results for ferromagnetic particles-sand mixture compared to the water suspension is very likely to be due to the formation of chains. Second is the fact that the scaling (5) lies between the data for the dry mixture and that for the water suspension. It could be a consequence of the small underestimation of c_{pow}. Indeed, $c_{\text{sph}} = 0.74$ is the theoretical value for the close-packing lattice and an experimental measurement would most probably give a lower value.

Rapid Note

4 Discussion

Our results show that the mixture of ferroparticles in liquid sodium could, in principle, enlarge the magnetic Reynolds number up to a factor of 2 preserving the hydrodynamic properties of the flow. This enlargement can be very important for realization of dynamo experiments in laboratory conditions. Of course, the effective viscosity of this mixture also grows with the volume fraction c, but for the dynamo experiments it is not important because the hydrodynamical Reynolds number is extremely large.

The use of diphasic media in dynamo experiments can also be supported by a more general motivation. Real astrophysical magnetic fields are generated in very inhomogeneous media. For instance, an interstellar medium, whose motion produces galactic magnetic fields, consists of 3 phases (cool, warm and hot) with very different temperatures, conductivities, etc. [13]. The available galactic dynamo theory makes only first steps towards the consideration of the multiphase nature of an interstellar medium. Since the estimations of magnetic permeability and electric permittivity appear to be so mutually instructive, one can expect that the experimental study of magnetic permeability will shed light on this obscure problem.

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