Universal intermittent properties of particle trajectories in highly turbulent flows

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We present a collection of eight data sets, from state-of-the-art experiments and numerical simulations on turbulent velocity statistics along particle trajectories obtained in different flows with Reynolds numbers in the range $R_\lambda \in [120 : 740]$. Lagrangian structure functions from all data sets are found to collapse onto each other on a wide range of time lags, revealing a universal statistics, and calling for a unified theoretical description. Parisi-Frisch Multifractal theory, suitable extended to the dissipative scales and to the Lagrangian domain, is found to capture intermittency of velocity statistics over the whole three decades of temporal scales here investigated.

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The presence of intrinsic chaotic fluctuations distributed over a wide range of length and time scales makes fluid turbulence a challenge both for theory and experiments [1–3]. Most of our understanding of turbulence relies on phenomenological facts based on experimental or numerical observations. The distortion of material fluid elements into intricate geometries is a characteristic of turbulent flows [5], which contributes to enhance the mixing properties and remains largely unexplained. Such a problem calls for a Lagrangian approach to turbulence: a Lagrangian coordinate system moves with the fluid and, therefore, removes the advection by the large-scale structures of the flow.

The main difficulty of Lagrangian investigations stems from the necessity to resolve the wide range of time scales which drive different particle behaviours: from the longest, T_L, given by the stirring mechanism, to the shortest τ_η, typical of viscous dissipation. Indeed the ratio, $T_L/\tau_\eta \sim R_\lambda$, grows with the Taylor Reynolds number, R_λ, that ranges from hundreds to thousands in laboratory flows. Some aspects of Lagrangian statistics have been experimentally measured: particle accelerations [6], velocity fluctuations in the inertial range [7, 8] and two-particle dispersion [9, 29]. Others, connected to the entire range of motions, have long been restricted to numerical simulations [10–14]. A fundamental open question is about the degree of *universality* of velocity statistics at various temporal scales. Universality is the first requirement for building a general theory of turbulent statistics and, if proved, may open the possibility for effective stochastic modeling [15] in many applied situations. Particle velocity statistics is indeed at the core of any model of turbulent mixing and transport [3, 4] as, e.g., the formation of rain clouds by water droplets [16].

This Letter aims to investigate universality of velocity temporal fluctuations by quantitatively comparing data obtained from the most advanced laboratory [7–9] and numerical [10–13, 17] experiments. Main outcome of our analysis is the data collapse on a common functional form, providing evidence for the desired universality.

We analyse the probability distribution of velocity fluctuations at all scales, focusing on moments of these distributions, namely the Lagrangian Velocity Structure Functions (LVSF) of positive integer order p:

$$S_i^{(p)}(\tau) = \langle [v_i(t+\tau) - v_i(t)]^p \rangle = \langle (\delta_\tau v_i)^p \rangle, \qquad (1)$$

where $i = x, y, z$ are the three velocity components, and the average is defined over the ensemble of particle trajectories. As stationarity and homogeneity is assumed, moments of velocity increments only depend on the time lag τ. In the inertial range, for $\tau_\eta \ll \tau \ll T_L$, non-linear energy transfer governs the dynamics. Thus, from a dimensional viewpoint, only the scale τ and the energy transfer ϵ should enter the structure functions. The only admissible choice is $S_i^{(p)}(\tau) \sim (\epsilon\tau)^{p/2}$, but it does not take into account the fluctuating nature of dissipation. Empirical studies have indeed shown that the tails of the probability density functions of $\delta_\tau v$ become increasingly non-Gaussian at decreasing τ/T_L, leading to intermittency and to anomalous scaling exponents, meaning a breakdown of the dimensional law, i.e.

$$S_i^{(p)}(\tau) \sim \tau^{\xi(p)} \tag{2}$$

with $\xi(p) \neq p/2$.

When dissipative effects become dominant, typically for scales $\tau \sim \tau_\eta$ and smaller, the power-law behaviour (2) breakdown and refined arguments have to be employed, as we will see in the following. However, the dependence of the exponents on the forcing mechanism, on the degree of anisotropy/non-homogeneity and on Reynolds number remains poorly understood. This is the problem of *universality* in Lagrangian turbulence.

The statistics of velocity fluctuations at varying time lag τ can be quantitatively captured by the logarithmic derivatives of $S_i^{(p)}(\tau)$ versus $S_i^{(2)}(\tau)$ [18–20]. This defines the local scaling exponents

$$\zeta_i(p,\tau) = \frac{\mathrm{d}\log S_i^{(p)}(\tau)}{\mathrm{d}\log S_i^{(2)}(\tau)} \,. \tag{3}$$

For statistically isotropic flows, all components are equivalent, and the spread among them quantifies the statistical/systematic departure from a perfect isotropic ensemble. The τ-dependence of $\zeta_i(p,\tau)$ allows for a scale-by-scale characterisation of intermittency.

Figure 1 shows the local exponents of order $p = 4$ from a collection of eight data sets (Table I and II) for different Reynolds numbers (see caption). Two observations are in order. First, the data show a strong variation around the dissipative time $\tau/\tau_\eta \sim O(1)$ that depends on the Reynolds number, and a clear tendency toward a plateau for larger lags $\tau > 10\tau_\eta$. Second, all data sets, with comparable Reynolds numbers, agree well in the whole range of time lags. The relative scatter increases only for large τ, due to the combined effects of the lack of statistics, the anisotropy of the flow and of different R_λ. In particular, finite volume effects in experimental particle tracking can produce a small – but systematic – downward shift of the points at long-lag times [20, 21]. In addition, the fact that, at comparable Reynolds numbers, all data sets recover the same behaviour by going to smaller and smaller time lags provides a strong indication of Lagrangian *universality* of the energy cascade. Such an agreement had not been observed before and is comparable with that found for the corresponding Eulerian quantities [30]. The quality of data shown in (Fig. 1) opens the possibility to quantitatively test phenomenological models for LVSF, scale-by-scale. Parisi-Frisch Multifractal (MF) model of the inertial range, and its generalization to the dissipative range [22–25], has proved to give a satisfactory description of Eulerian and Lagrangian fluctuations [14, 26–28]. It is thus appealing to search for a link between Eulerian and Lagrangian statistics [14, 26–28], since this points to a unique interpretation of turbulent fluctuations. Moreover, it would reduce the number of free parameters. According to the MF model, Eulerian velocity increments at inertial scales are characterised by a local Hölder exponent h, i.e. $\delta_r u \sim r^h$, whose probability $\mathcal{P}(h) \sim r^{3-D(h)}$ is weighted by the Eulerian fractal dimension $D(h)$ of the set where h is observed [1]. The dimensional relation $\tau \sim r/\delta_r u$ bridges Lagrangian fluctuations over a time lag τ to the Eulerian ones at scale r. Following Refs. [25, 26], it is shown in Ref. [28] how to extend the MF framework to get a unified description at all time scales for Lagrangian turbulence. Accordingly, Lagrangian increments display a continuous and differentiable behaviour at the transition from the dissipative to the inertial range,

$$\delta_\tau v(h) = V_0 \frac{\tau}{T_L} \left[\left(\frac{\tau}{T_L}\right)^\beta + \left(\frac{\tau_\eta}{T_L}\right)^\beta \right]^{\frac{2h-1}{\beta(1-h)}} \,, \tag{4}$$

EXP	R_λ	τ_η (s)	meas. vol. (η^3)	N_{tr}	Tech.	Ref.
1	124	8.5×10^{-2}	340^3	1.6×10^6	PTV	[8]
2	690	9×10^{-4}	1700^3	6.0×10^6	PTV	[7]
3	740	2×10^{-4}	6600^3	9.5×10^3	AD	[6]

TABLE I: Experiments. By columns: 1- Taylor Reynolds number; 2- Kolmogorov time scale τ_η; 3- measurement volume in unit of the Kolmogorov length scale η; 4- N_{tr} total number of Lagrangian trajectories measured; 5- measurement technique: Particle Tracking Velocimetry (PTV) and Acoustic Doppler (AD); 6- Reference.

DNS	R_λ	N^3	N_{tr}	Diss.	Tech.	Ref.
1	140	256^3	5×10^5	N	T	[10]
2	320	1024^3	5×10^6	N	T	[12]
3	400	2048^3	3×10^5	N	L	[9]
4	600	1856^3	1.6×10^7	C	L	[16]
5	650	2048^3	4×10^5	N	CS	[11]

TABLE II: Numerical simulations. By columns: 1- Taylor Reynolds number R_λ; 2- number of collocation points N^3; 3- total number of Lagrangian tracers N_p; 4- characteristic of dissipation: normal viscous terms (N), weakly compressible code (C); 5- interpolation technique for Lagrangian integration: linear interpolation (L), tricubic interpolation (T); cubic-splines (CS); 6- references.

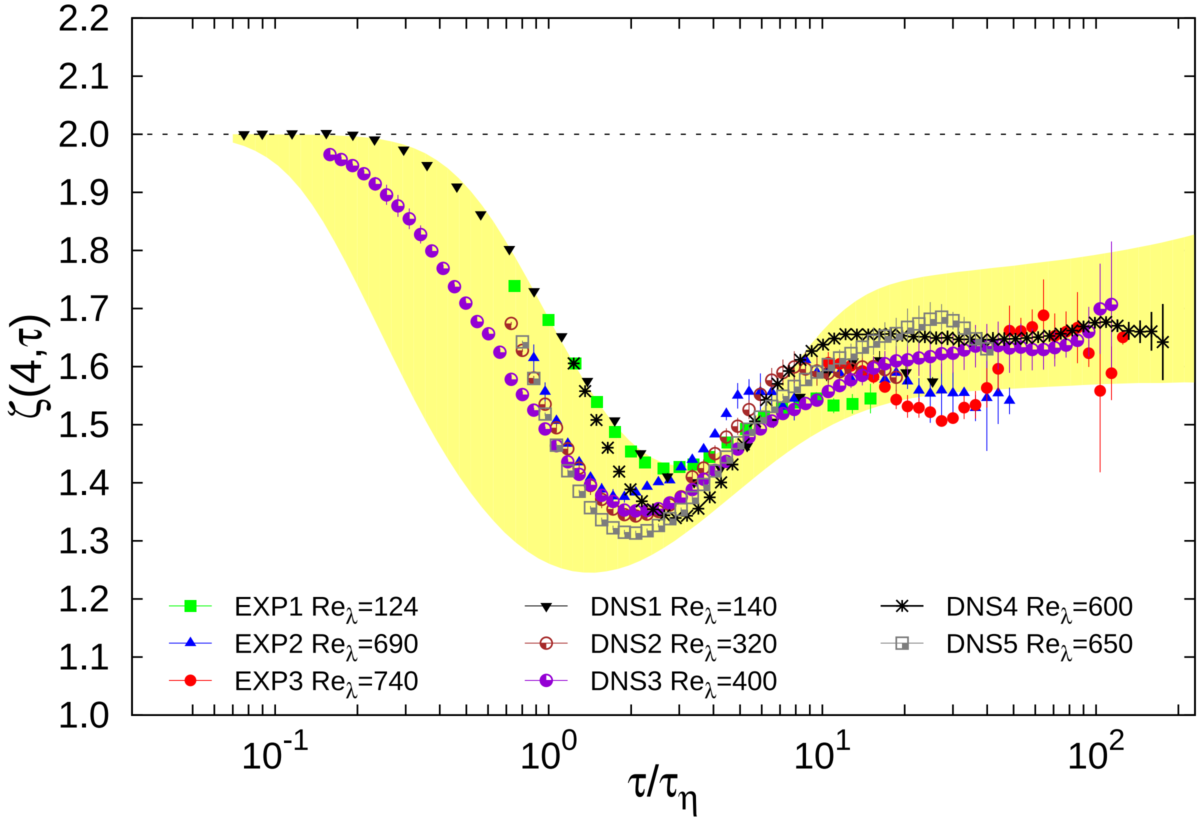

FIG. 1: Log-Lin plot of the fourth order local exponent, $\zeta(4,\tau)$, averaged over the three velocity components, as a function of the normalised time lag τ/τ_η. Data sets come from three experiments (EXP) (see table 1) and five direct numerical simulations (DNS) (see table 2). Error bars are estimated from the spread between the three components, except in EXP3 where only two components were measured. Each data set is plotted only in the time range where the known experimental/numerical limitations are certainly not affecting the results. In particular, for each data set, the largest time lag always satisfies $\tau < T_L$. The minimal time lag is set by the highest fully resolved frequency. The shaded area displays the prediction obtained by the MF model by using $D_L(h)$ or $D_T(h)$, with $\beta = 4$, for a range of $R_\lambda \in [150:800]$, comparable with the range of R_λ in the data. Notice that the MF predictions have been obtained by fixing equal to 7, the multiplicative constant in the definition of τ_η. The straight dashed line corresponds to the dimensional non-intermittent value $\zeta(4,\tau) = 2$. Notice that two DNS are sufficiently resolved to get the right dimensional scaling in the high frequency limit.

β being a free parameter controlling the crossover around $\tau \sim \tau_\eta$, and V_0 the root mean square velocity. In order to get a prediction for the behaviour of the LVSF, given by

$$\langle(\delta_\tau v)^p\rangle \sim \int dh P_h(\tau,\tau_\eta)[\delta_\tau v(h)]^p\,, \qquad (5)$$

we must consider in (4) the intermittent fluctuations of the dissipative scale [14, 26, 28], $\tau_\eta(h)/T_L \sim R_\lambda^{2(h-1)/(1+h)}$. The last necessary ingredient is to specify the probability of observing fluctuations of h, this is done in analogy to Eq. (4):

$$P_h(\tau,\tau_\eta) = \mathcal{Z}^{-1}(\tau)\left[\left(\frac{\tau}{T_L}\right)^\beta + \left(\frac{\tau_\eta}{T_L}\right)^\beta\right]^{\frac{3-D(h)}{\beta(1-h)}}\,, \qquad (6)$$

where $\mathcal{Z}$ is a normalizing function [28] and $D(h)$ the fractal dimension of the support of the exponents h. At this point, given the Reynolds number, we are left with two parameters (β and a multiplicative constant in the definition of τ_η), plus the function $D(h)$.

Eulerian Velocity Structure Functions (EVSF) have been measured in the last two decades (see Ref.[30] for a data collection) providing a way to estimate the function $D(h)$ based on empirical data. Many functional forms have been proposed in the literature [1] that are consistent with data, up to statistical uncertainties. Eulerian velocity statistics can be measured in terms of longitudinal or transverse fluctuations. Fluid velocity along particle paths is naturally sensitive to both kinds of fluctuations. We thus evaluated the LVSF in (5) using the fractal dimensions $D_L(h)$ and $D_T(h)$ obtained by fitting longitudinal [30] and transverse [31] EVSF, respectively.

The shaded area in (Fig. 1) represents the range of variation of the MF prediction computed from $D_L(h)$

or $D_T(h)$, measured in the Eulerian statistics, and at changing Reynolds numbers. This must be interpreted as our uncertainty. The prediction works very well: all data fall within the shaded area. The role of the parameters is clear. Changing β modifies the sharpness and shape of the dip region at τ_η – the larger β the more pronounced the dip; while, with increasing R_λ, the flat region at large lags develops a longer plateau. In the limit $R_\lambda \to \infty$ the MF model predicts $\zeta(4) \simeq 1.71$ from $D_L(h)$ and $\zeta(4) \simeq 1.59$ from $D_T(h)$ statistics.

For the Eulerian $D(h)$, we used the following log-Poisson [1] functional form,

$$D(h) = \frac{3(h-h^*)}{\log(\gamma)}\left[\log\left(\frac{3(h^*-h)}{d^*\log(\gamma)}\right)-1\right]+3-d^* . \quad (7)$$

Different couples of parameters, (h^*, γ), have been chosen to fit longitudinal and transverse Eulerian fluctuations. The third parameter $d^* = (1-3h^*)/(1-\gamma)$ is fixed by requiring that third order EVSFs scale linearly. For the longitudinal exponents [30], we used $(h^*_L = 1/9, \gamma_L = 2/3)$ [1]. For the transverse exponents, we used $(h^*_T = 1/9, \gamma_T = 1/2)$ which fits the data in Ref. [31]. The functions $D_L(h)$ and $D_T(h)$ are convex; the strongest fluctuations (smallest Hölder exponents h^{min}) are h^*_L and h^*_T. The maximum, $D_L(h^{max}_L) = D_T(h^{max}_T) = 3$, is attained for $h^{max}_L = 0.382$, and $h^{max}_T = 0.57$. Since only the range $h^{min} \le h \le h^{max}$ is relevant to the fit of EVSF of positive order [1], the extremes of integration in Eq. (5) have been set in the interval, $[h^*_L, h^{max}_L]$ and $[h^*_T, h^{max}_T]$, respectively. Modifying h^{max} alters the MF prediction for $\tau \sim T_L$, but does not change the behaviour of the curves in the inertial range.

This comprehensive comparison of the best available experiments and numerical simulations provides strong evidence of the universality of Lagrangian statistics. One important open question is the effect of a mean flow, as in turbulent jets [32] and wall bounded turbulence, where strong persistence of anisotropy may break the recovery of small-scale universality. We showed that a multifractal description is in good agreement with data, even in the dissipative range where intermittency is significantly increased. The multifractal description captures the intermittency at all scales with only a few parameters, independent of the Reynolds number. This is the universal feature of Lagrangian turbulence revealed by this study. There exists a long debate on the statistical importance of vortex filaments around dissipative time and length scales [1, 33]. Simulations [10, 19, 34] show that the dip region for $\tau \sim \tau_\eta$ can be depleted/enhanced by decreasing/increasing the probability of particles being trapped in vortex filaments. The multifractal model is able to capture the intermittency around τ_η with the help of the free parameter β. Different values of β should then correspond to different statistical weights of vortex filaments along particle trajectories.
Only further advances in both experimental techniques and numerical power will allow us to test the same questions here addressed also for the higher order statistics.

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