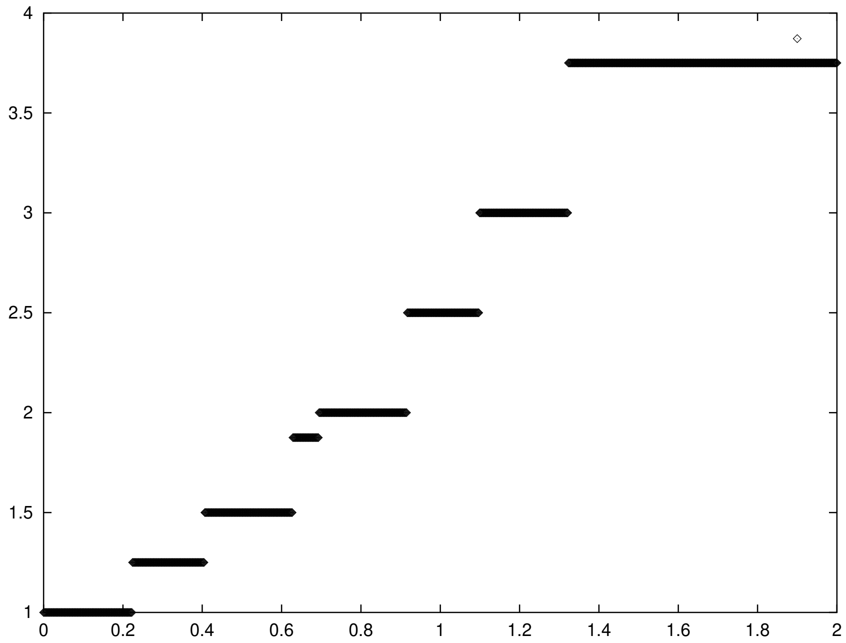

FIG. 1 - Value of E_3 versus t.

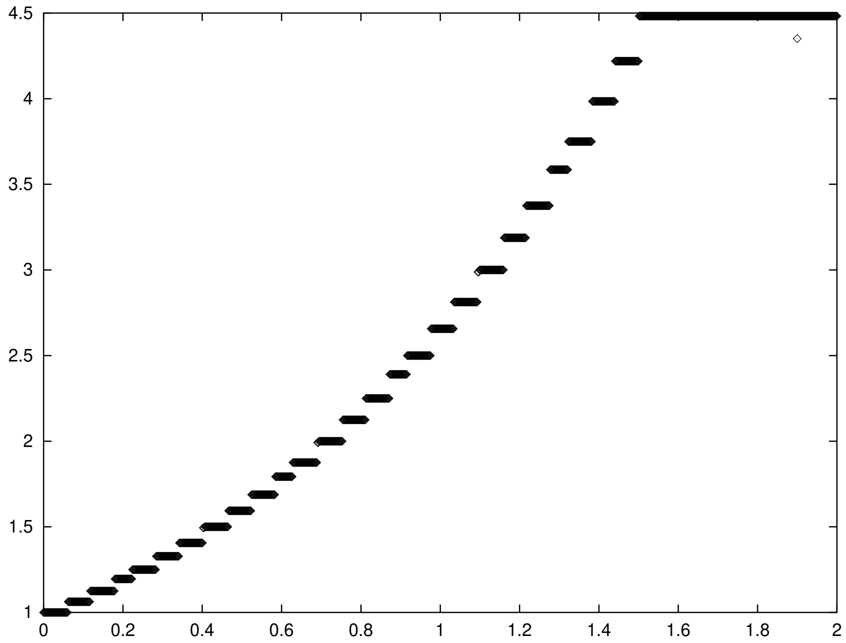

FIG. 2 - Value of E_5 versus t.

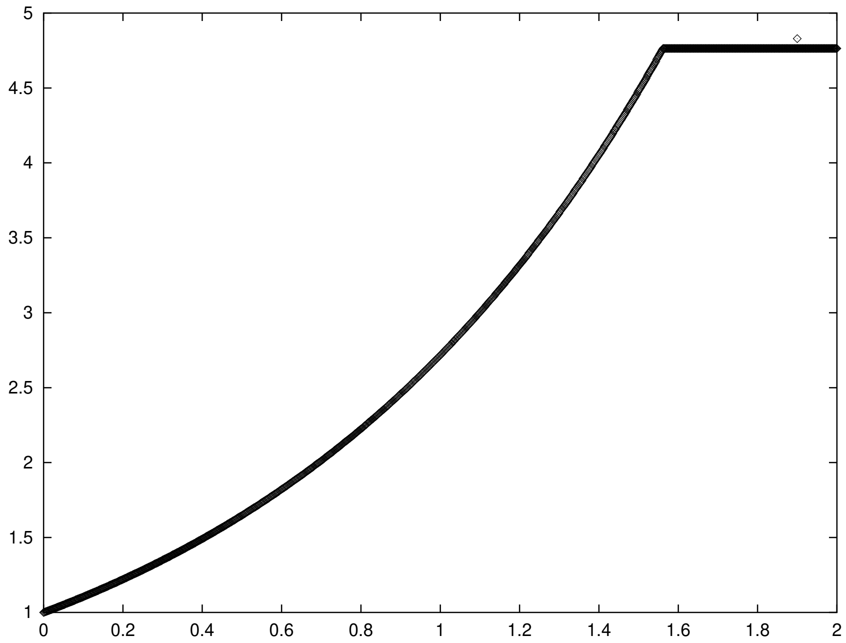

FIG. 3 - Value of E_{11} versus t.

Theorem 1 (restoring decomposition algorithm) *Let (w_n) be a decreasing sequence of positive real numbers such that the series $\sum_{i=0}^{\infty} w_i$ converges. If*

$$\forall n, w_n \leq \sum_{k=n+1}^{\infty} w_k \tag{1}$$

then for any $t \in [0, \sum_{k=0}^{\infty} w_k]$, the sequences (t_n) and (d_n) defined as

$$\begin{aligned} t_0 &= 0 \\ t_{n+1} &= t_n + d_n w_n \\ d_n &= \begin{cases} 1 & \textit{if } t_n + w_n \leq t \\ 0 & \textit{otherwise} \end{cases} \end{aligned} \tag{2}$$

satisfy $t = \sum_{n=0}^{\infty} d_n w_n = \lim_{n \to \infty} t_n$.

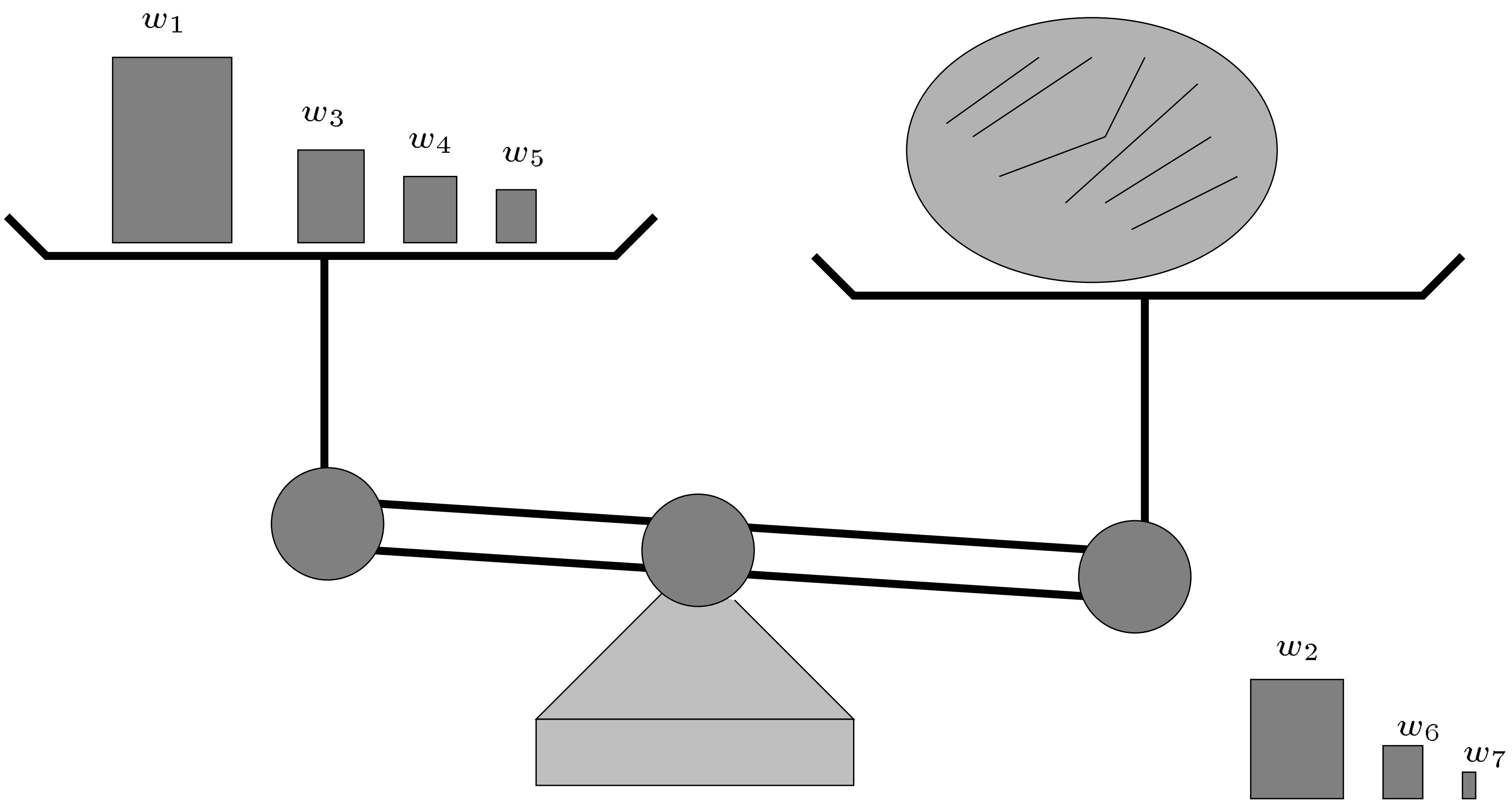

FIG. 4 - The restoring algorithm. The weights are either unused or put on the pan that does not contain the loaf of bread being weighed.

Theorem 2 (non-restoring decomposition algorithm) *Let (w_n) be a decreasing sequence of positive real numbers such that $\sum_{i=0}^{\infty} w_i$ converges. If*

$$\forall n, w_n \leq \sum_{k=n+1}^{\infty} w_k, \tag{3}$$

then for any $t \in [-\sum_{k=0}^{\infty} w_k, \sum_{k=0}^{\infty} w_k]$, the sequences (t_n) and (d_n) defined as

$$\begin{array}{lcl} t_0 & = & 0 \\ t_{n+1} & = & t_n + d_n w_n \\ d_n & = & \left\{ \begin{array}{rl} 1 & \textit{if } t_n \leq t \\ -1 & \textit{otherwise} \end{array} \right. \end{array} \tag{4}$$

satisfy $t = \sum_{n=0}^{\infty} d_n w_n = \lim_{n \to \infty} t_n$.

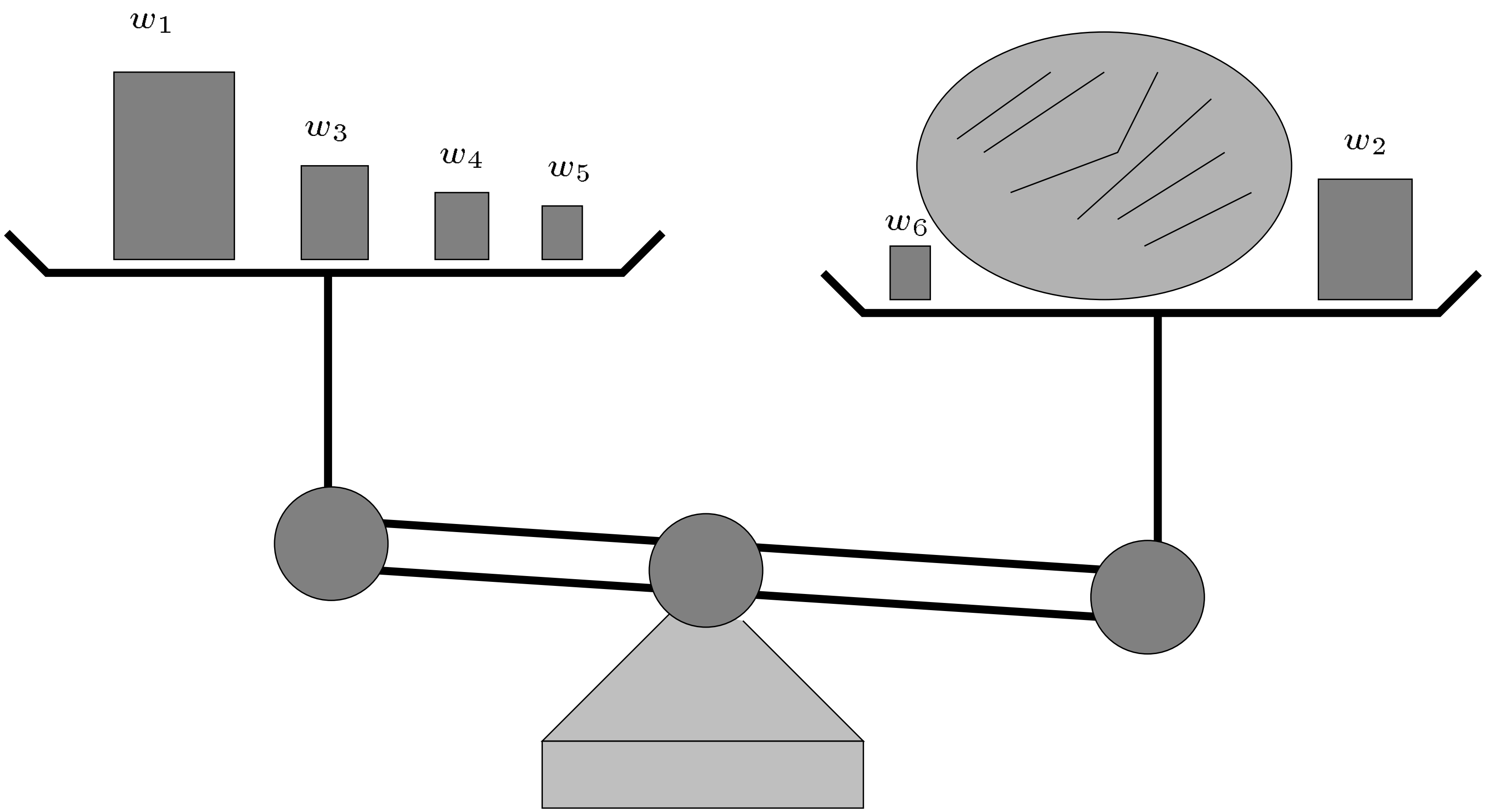

FIG. 5 - The non-restoring algorithm. All the weights are used, and they can be put on both pans.

Faster computation of exponentials

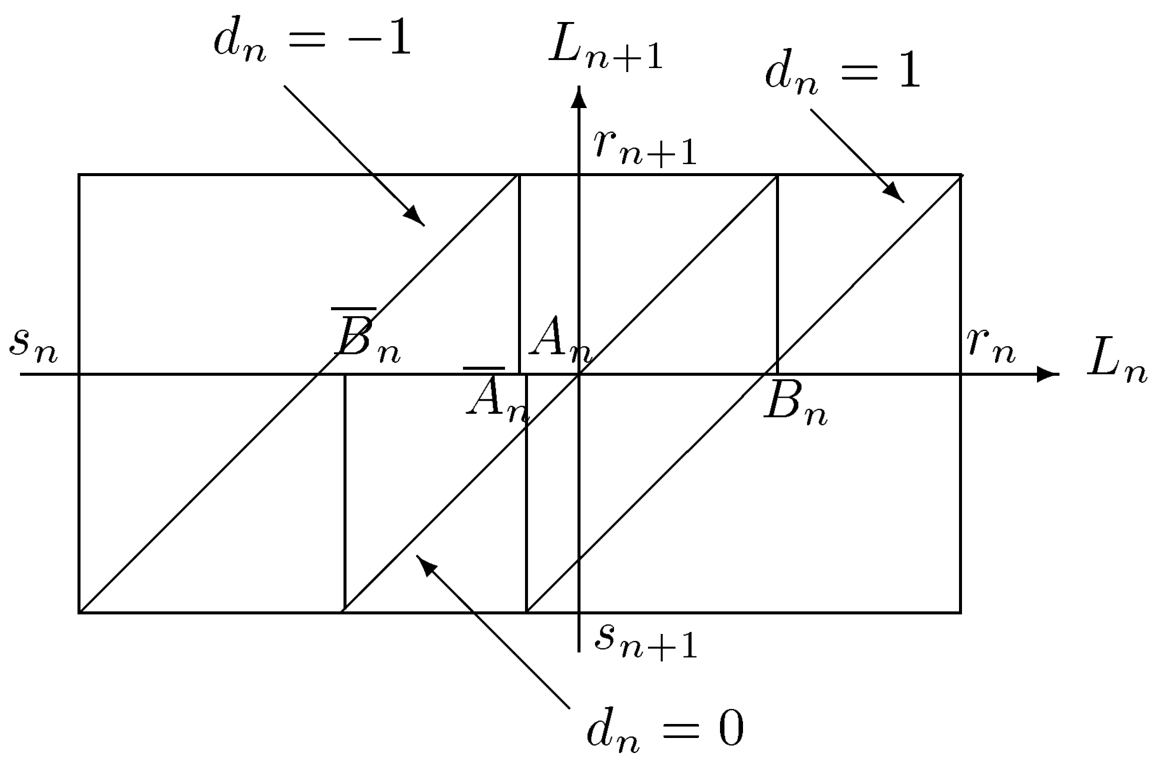

FIG. 6 - Robertson diagram of the “redundant exponential” algorithm.

The values A_n, B_n, $\overline{A}_n$, and $\overline{B}_n$ of Figure 6 satisfy:

$$\left\{\begin{array}{lcl} A_n & = & s_{n+1} + \ln\left(1 + 2^{-n}\right) \\ B_n & = & r_{n+1} \\ \overline{A}_n & = & r_{n+1} + \ln\left(1 - 2^{-n}\right) \\ \overline{B}_n & = & s_{n+1}. \end{array}\right. \tag{5}$$

Using these relations, one can show that:

$$\left\{\begin{array}{lcr} 2^n A_n & \leq & 0 \\ 2^n B_n & \geq & 1/2 \\ 2^n \overline{A}_n & \geq & -1/2 \\ 2^n \overline{B}_n & \leq & -1. \end{array}\right. \tag{6}$$

n	$2^n s_n$	$2^n r_n$	$2^n \overline{B}_n$	$2^n \overline{A}_n$	$2^n A_n$	$2^n B_n$
1	-2.484	1.738	-1.098	-0.460	-0.287	0.927
2	-2.196	1.854	-1.045	-0.190	-0.152	0.961
3	-2.090	1.922	-1.022	-0.088	-0.079	0.980
4	-2.043	1.960	-1.011	-0.043	-0.041	0.990
5	-2.021	1.980	-1.005	-0.021	-0.021	0.995
6	-2.011	1.990	-1.003	-0.010	-0.010	0.997
∞	-2	2	-1	0	0	1

TAB. 1 - First values and limit values of $2^n s_n$, $2^n r_n$, $2^n A_n$, $2^n B_n$, $2^n \overline{A}_n$, and $2^n \overline{B}_n$.

Faster computation of logarithms

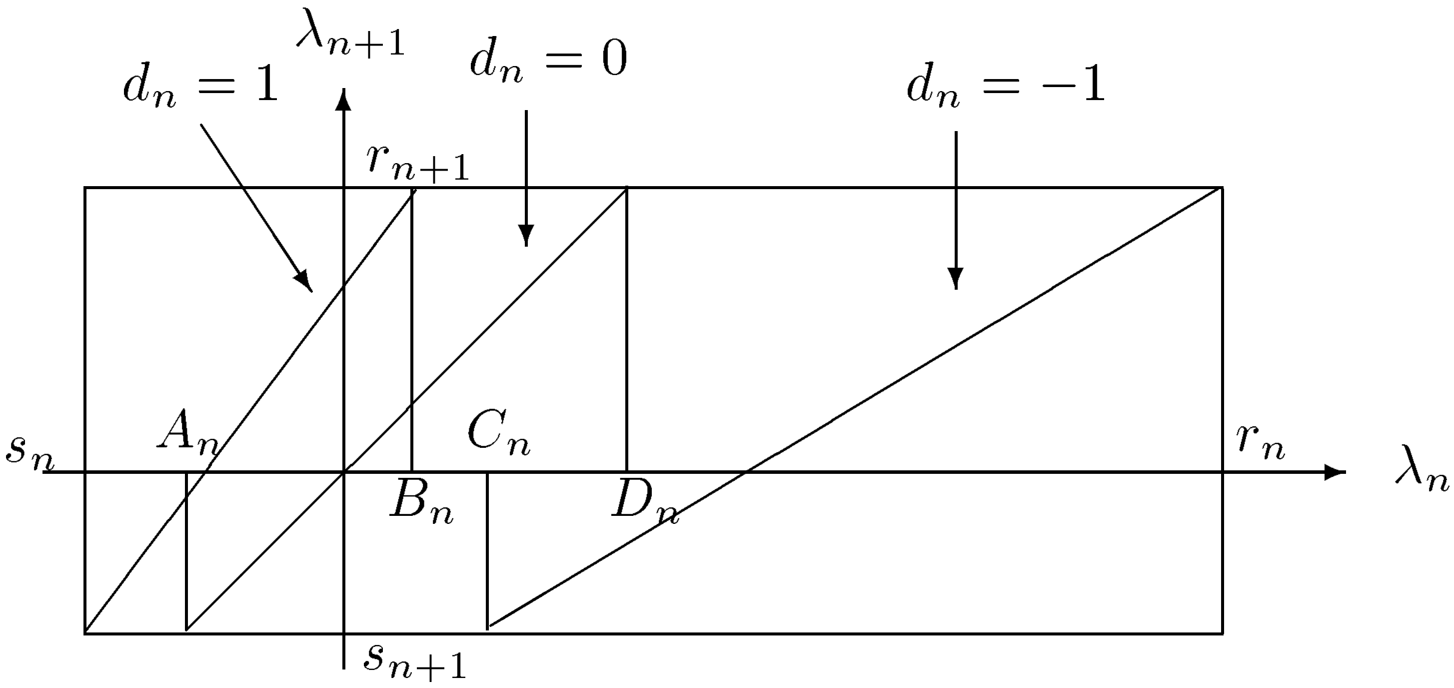

FIG. 7 - Robertson diagram for the logarithm. The three straight lines give $\lambda_{n+1} = \lambda_n(1 + d_n 2^{-n}) + d_n 2^{-n}$ for $d_n = -1, 0, 1$.

n	$2^n s_n$	$2^n r_n$	$2^n A_n$	$2^n B_n$	$2^n C_n$	$2^n D_n$
1	-1.161	4.925	-0.74	0.30	0.51	1.46
2	-1.483	2.925	-0.85	0.15	0.19	1.19
3	-1.709	2.388	-0.92	0.08	0.09	1.09
4	-1.844	2.179	-0.96	0.04	0.04	1.04
5	-1.920	2.086	-0.98	0.02	0.02	1.02
∞	-2	2	-1	0	0	1

TAB. 2 - First values and limits of $2^n s_n$, $2^n r_n$, $2^n A_n$, $2^n B_n$, $2^n C_n$, and $2^n D_n$.

Baker's Predictive Algorithm

i	$\ln(1+2^{-i})$
0	$0.101100010111001000010111111101111101000111\cdots$
1	$0.011001111100110010001111101100101111111001\cdots$
2	$0.001110010001111111101111100011110011010100\cdots$
3	$0.000111100010011100000111011011100010101011\cdots$
4	$0.000011111000010100011000011000000000100010\cdots$
5	$0.000001111110000010100110110000111001111000\cdots$

TAB. 3 - The first digits of the first 6 values $w_i = \ln(1+2^{-i})$. As i increases, w_i gets closer to 2^{-i}.

x	w_i	decomposition
$\frac{1}{10}$	2^{-i} (binary)	000011001100110011001
	$\ln(1+2^{-i})$	000011010001101011101
	$\arctan 2^{-i}$	000011001100111111001
$\frac{1}{1024}$	2^{-i} (binary)	00000000001000000000000000000000000000
	$\ln(1+2^{-i})$	00000000001000000000001111111111010101
	$\arctan 2^{-i}$	0000000000100000000000000000000010101

TAB. 4 - Comparison among the binary representations and the decompositions (given by the restoring algorithm) on the discrete bases $\ln(1+2^{-i})$ and $\arctan 2^{-i}$ for some values of x. When x is very small the different decompositions have many common terms.