Entanglement properties of lattice bosons from a variational wave function

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Introduction

State of a N-spin $\frac{1}{2}$ system: $|\psi\rangle = \sum_{\vec{\sigma}} C(\vec{\sigma})\,|\vec{\sigma}\rangle$

$\rightarrow 2^N$ variables

Mean field Ansatz: $C(\vec{\sigma}) = \Pi_i C_i(\sigma_i)$
$\rightarrow 2N$ variables

But:

$$\langle S_i^z S_j^z\rangle - \langle S_i^z\rangle\langle S_j^z\rangle = \begin{cases} \frac{1}{4} - \langle S_i^z\rangle^2 & \text{if } i = j \\ 0 & \text{if not} \end{cases}$$

Gradient algorithm

Aim: Find $|\psi\rangle$ which minimize $\langle\psi|H|\psi\rangle$, with Hamiltonian $H = -J\sum_i (S_{i+1}^x S_i^x + S_{i+1}^y S_i^y)$

and state

$$|\psi\rangle = \sum_{\vec{\sigma}} \Big(\prod_P C_P(\vec{\sigma}_P)\Big)|\vec{\sigma}\rangle$$

EPS algorithm

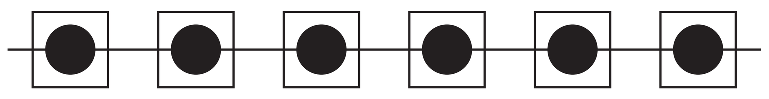

Figure 2: Mean field

$\rightarrow 2N$ variables

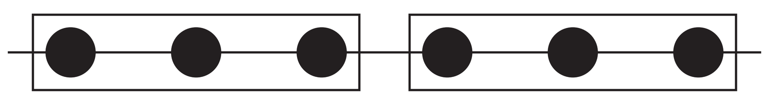

Figure 3: Disjoint plaquettes of size 3

$\rightarrow 2^3\frac{N}{3}$ variables

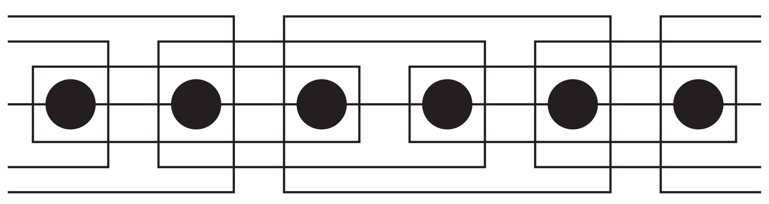

Figure 4: EPS with plaquettes of size 3

$\rightarrow 2^3 N$ variables

EPS delocalized

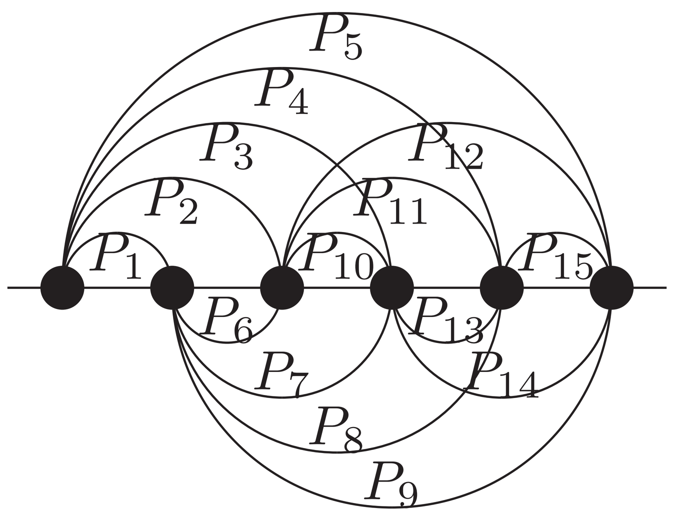

Figure 1: EPS with delocalized plaquettes of size 2

Energy

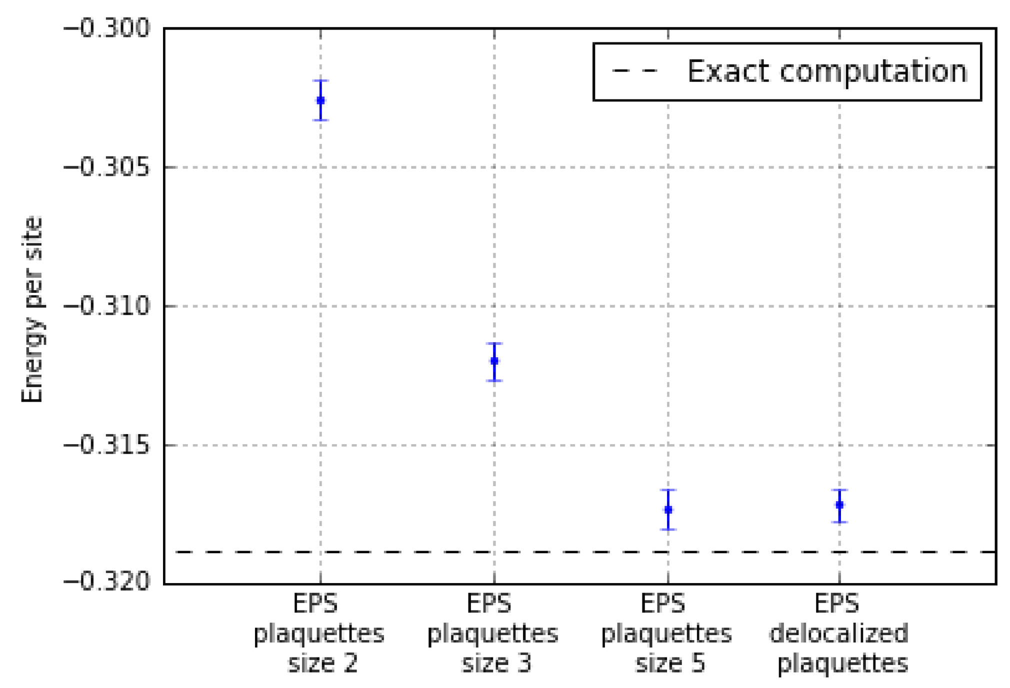

Figure 7: Energy of a system of size 30 founded with different EPS algorithms

Correlation

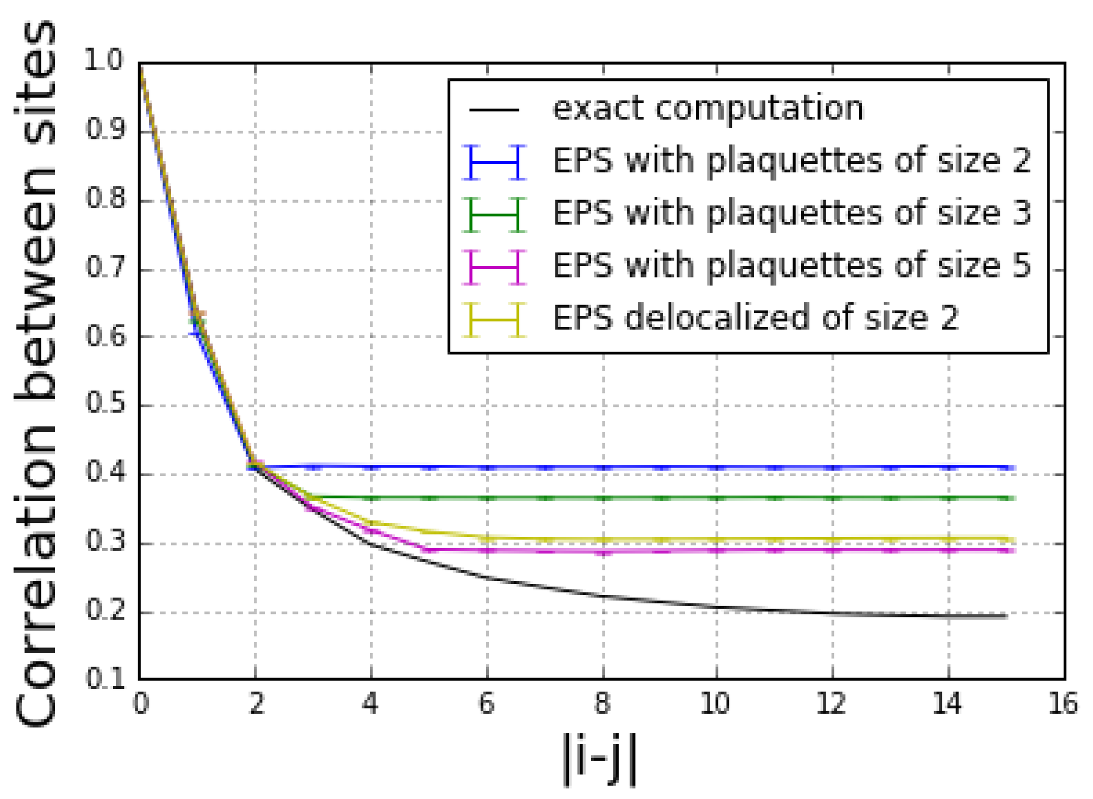

Figure 5: Correlation function along x axis between sites i and j of a system of size 30

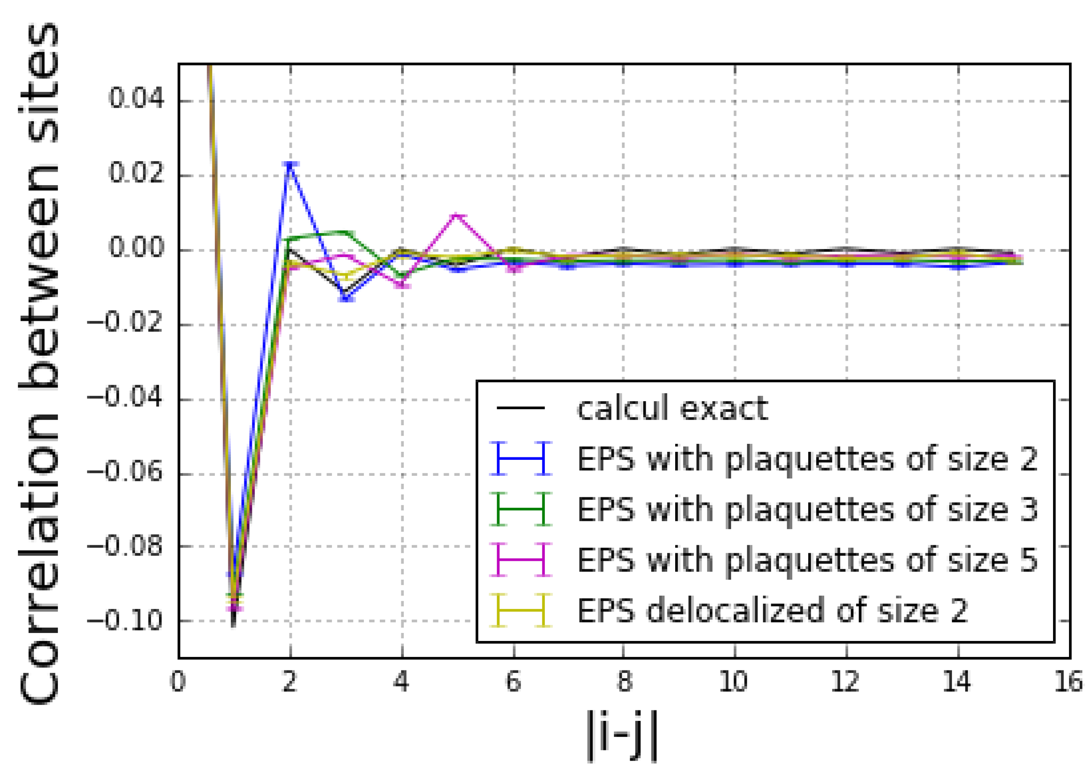

Figure 6: Correlation function along z axis between sites i and j of a system of size 30

Entanglement entropy

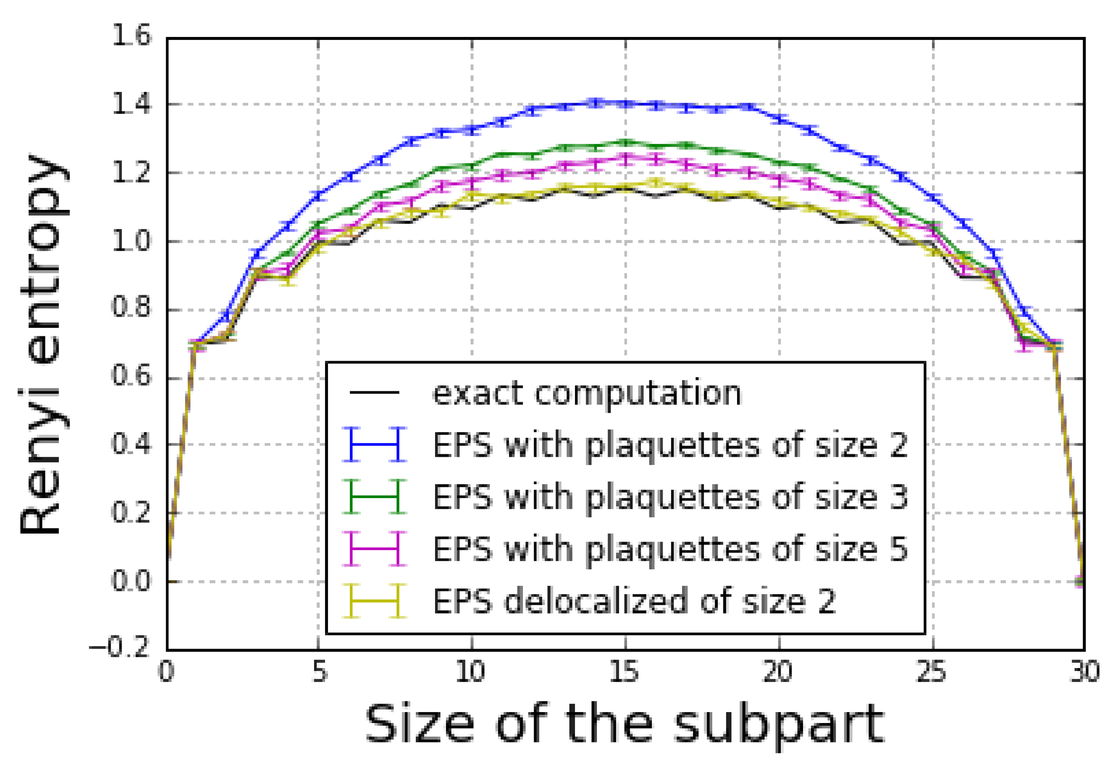

Figure 8: Entanglement entropy as a function of the size of the subsystem of a system of size 30

EPS with delocalized plaquettes size 3

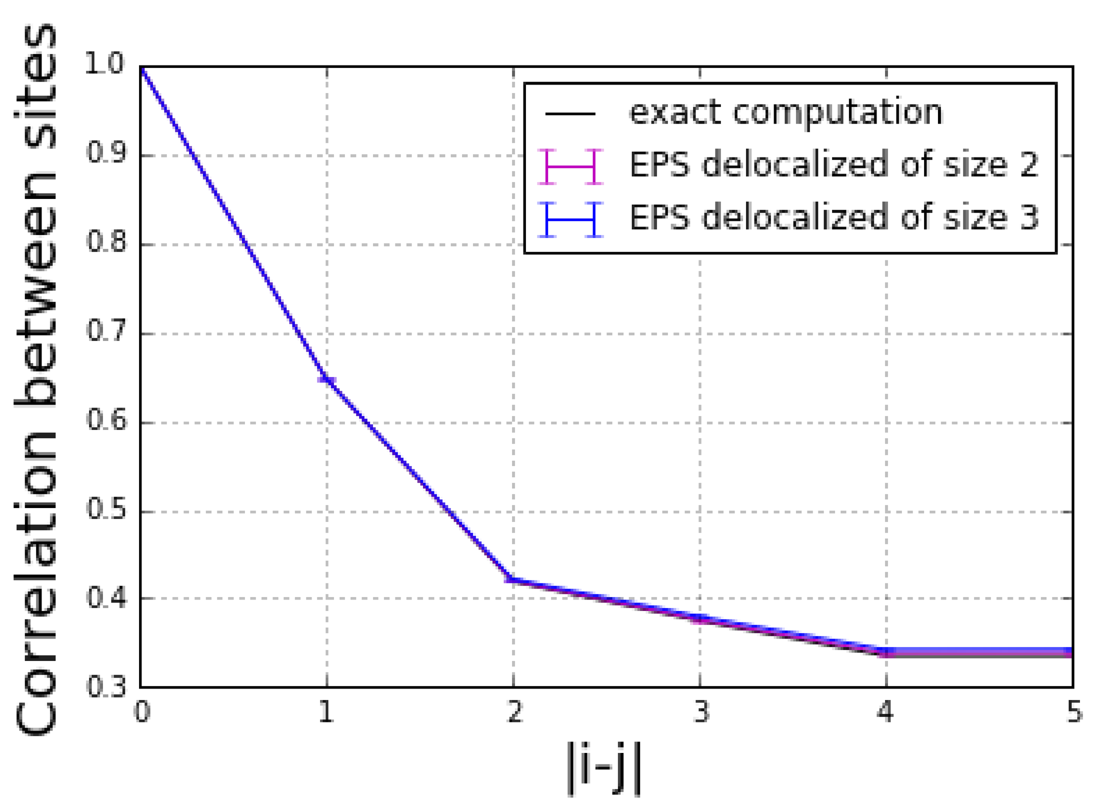

Figure 9: Correlation function between sites i and j of a system of size 10

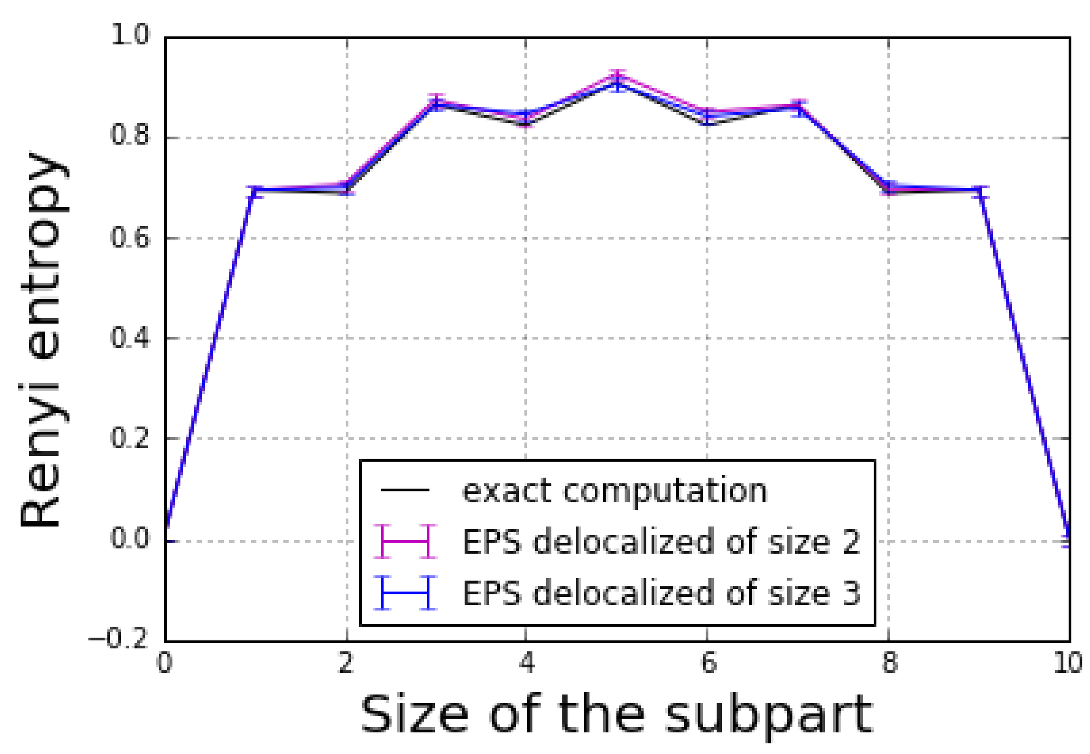

Figure 10: Entanglement entropy as a function of the size of the subsystem of a system of size 10

Conclusion

- EPS algorithm reproduces well short range correlations but fails to reproduce long range

  $\rightarrow$ Use EPS algorithm on short range correlations system

- EPS algorithm with delocalized plaquettes better reproduces the overall shape of the curve, but costs more calculation time

  $\rightarrow$ Find a compromise between delocalized and non delocalized

- EPS algorithm with delocalized plaquettes seems to have a correlation length

  $\rightarrow$ Does it depend on the number of step in the gradient algorithm ?

References

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