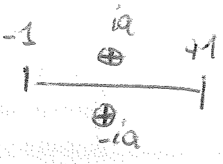
2. f is analytic at x if there exists a Taylor series that converges to f at x. (Stronger than C^∞)

$$f(x) = \frac{1}{1+a^2x^2}$$:



$\oplus$: singularities

A straight line on a log plot indicates exp. conv.
A straight line on a loglog plot indicates algebraic conv.

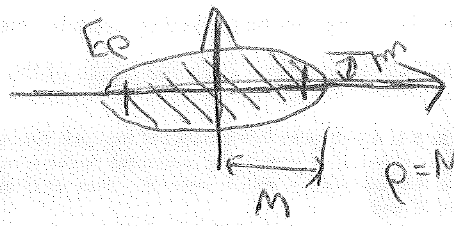
Proof of 8.1 (Main idea) (Cauchy integrals)

1. $$a_k = \frac{2}{\pi}\int_{-1}^{+1} \frac{f(x)\,T_k(x)}{\sqrt{1-x^2}}\,dx$$

2. Transplant $F(z)$ via $x = \frac{1}{2}(z+z^{-1})$ and you find:
$$a_k = \frac{1}{\pi i}\int_{|z|=1} z^{-1-k}\,F(z)\,dz$$

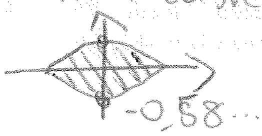
3. Deform the ~~rate~~ contour $|z|=1 \to |z|=\rho$

4. Use $|z^{-1-k}| = \rho^{-1-k}$ on $|z|=\rho$
Which converges exponentially.



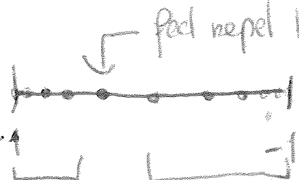
$\rho = M+m$

3.1 In fact equispaced interpolation converges if f is analytic in a "American football"



Essence: We need all pts of $[-1,+1]$ to be about equally spaced from grid in the sence of geometric mean.

feel repel by "electrons" (other pts) by the same magnitude on each side.



less electrons, but closer

more electrons, but larger.

4.1 $p(x)$ is actually a polynomial! (of degree n)