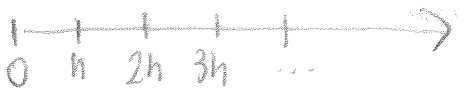
2.1 * IVP: marching (from left to right) approach.

$y' = f(t,y)$



We use, for example, finite differences with a step h. ~ In principle, $h \to 0$, in practice h is "small"))

- Adams, 1850, discovered/predicted Neptune using a higher than Euler's method method. (Adams-Bashford method, multistep). ODE113 (in Matlab)
- Runge-Kutta methods are ODE23 and ODE45 in Matlab.

* Chebfun uses ODE113 for IVPs, with small tolerance; then converts result to a chebfun.

* BVP: global (e.g. Chebyshev) approach.  Using this kind of grid.

3 Idea: data on Chebyshev grid; compute a polynomial interpolant; apply differential operator → discrete problem called "Chebyshev spectral method"

Given data: $y_0, \dots, y_{n+2}$ on (n+3)-points grid. Then we look for an interpolant of degree n+2. Apply operator and sample on a (n+1)-points grid, and add two more boundary conditions. This idea comes from Toby Driscoll.

4 Tucker's function: $\sin(\cos x^2 + 10 \sin y^2) \to \cos x$

"It's contours at $z=0$ are beautiful"

1. LOW-RANK APPROXIMATION $f(x,y) = \sum_{k=1}^{n} u(x)\, v(y)$: rank n function, ~~rank 1 separable~~. rank 1 = separable.
2. BIVARIATE CHEBYSHEV SERIES $f(x,y) = \sum_{k=0}^{N} \sum_{j=0}^{M} a_{jk} T_j(x) T_k(y)$ "Tensor product of 1D chebyshev series"

Chebfun2 do the 1.

Note: SIAM 2002 100 digits challenge.

5 "CG as a direct alg. is worse than GE"

A step of GE is (Find biggest entry, substract off a rank 1 matrix). The low-rank approx. is the same! — but take fewer steps than the whole matrix.