ER02 Winter Research School – final exam

PART 1: NUMERICAL COMPUTING

Problem 1: What is the first term of the Chebyshev series of $\sin(\pi x)^2$?
(a) 0, (b) $1/\pi$, (c) 2, (d) π.

Problem 2: Which of these functions is analytic in $[-1, 1]$?
(a) e^{-1/x^2}, (b) $(1-x)\log(1-x)$, (c) $|x-i|$, (d) $\tan(2x)$.

Problem 3: Which problems are Runge-Kutta methods generally used for?
(a) ODE IVPs, (b) ODE BVPs, (c) eigenvalue problems, (d) PDEs.

Problem 4: What is the form of the colleague matrix for a polynomial $p(x)$?
(a) tridiagonal, (b) tridiagonal plus an extra row, (c) triangular.

Problem 5: How many years after Gauss quad. was Clenshaw-Curtis quad. invented?
(a) 16, (b) 116, (c) 146, (d) -16

Problem 6: If a Chebyshev series in x is transplanted to z, what does it become?
(a) a Taylor series, (b) a Laurent series, (c) a Fourier series.

Problem 7: How does Chebfun's rootfinding algorithm achieve $O(n^2)$ complexity?
(a) Fast eigenvalue algorithm, (b) Fast Fourier Transform, (c) recursion.

Problem 8: Do equispaced interpolants of $\log(1+x^2)$ in $[-1, 1]$ converge as $n \to \infty$?
(a) Yes, (b) No, (c) Yes in theory but no with rounding errors.

Problem 9: If $f(x)$ has n roots in $[-1, 1]$, what do we expect of the degree of its chebfun?
(a) $> n/2$, (b) $> n$, (c) $> n^2$.

Problem 10: Operation count for evaluation of degree n Cheb interpolant by the barycentric formula ?
(a) $O(n)$, (b) $O(n \log n)$, (c) $O(n^2)$.

PLEASE TURN OVER...

PART 2: RIGOROUS NUMERICAL COMPUTING

Problem 1: When working in base β with finite precision, is *round to nearest even* the best choice?
(a) Always, (b) Never, (c) Depends on the base β.

Problem 2: How many subnormal numbers are there in a floating point system with base $\beta = 2$ and precision $p = 3$?
(a) 6, (b) 7, (c) 8.

Problem 3: Let A = Associative, C = Commutative, and D = Distributive. Which of these properties hold for interval arithmetic?
(a) A and C and D, (b) A and C but not D, (c) Not C but A and D.

Problem 4: What is the interval image of sin over the interval $[0, \pi/2]$, i.e. what is $\sin([0, \pi/2])$?
(a) $[-1, 1]$, (b) $[0, 1/\sqrt{2}]$, (c) $[0, 1]$.

Problem 5: In extended interval arithmetic what is the interval image of e^{-x} over the interval $[-\infty, +\infty]$?
(a) It is not well-defined, (b) It is $[0, +\infty]$, (c) It is $[-\infty, +\infty]$.

Problem 6: What is a potential disadvantage of finite differences?
(a) They are affected by rounding/discretization errors, (b) they are very slow to compute, (c) Both (a) and (b).

Problem 7: Given a continuously differentiable function f together with an interval $\mathbf{x}$ containing the point x_0, does $(f(\mathbf{x} + h) - f(\mathbf{x}))/h$ enclose the real derivative $f'(x_0)$ for sufficiently small h?
(a) Yes - always, (b) No - never, (c) Sometimes.

Problem 8: What can you say when the interval Newton operator N_f maps $\mathbf{x}$ into itself?
(a) There are no zeros of f in $\mathbf{x}$, (b) There is a unique zero of f in $\mathbf{x}$, (c) There may be a zero of f in $\mathbf{x}$.

Problem 9: Deriving formulae for taylor arithmetic to standard functions such as e^x and $\sin x$ uses ...?
(a) The mean value theorem, (b) The chain rule, (c) The intermediate value theorem.

Problem 10: Let $u = (u_0, u_1, u_2)$ and $v = (v_0, v_1, v_2)$ represent two elements in second-order automatic differention mode. The arithmetic rule for multiplication is given by $u \times v = (u_0v_0, u_1v_0 + u_0v_1, ...)$. What should the third component be?
(a) $u_0v_2 + u_2v_0$, (b) $u_0v_2 + 2u_1v_1 + u_2v_0$, (c) u_2v_2.