Validated Numerics – Computer Lab 2

Problem 1: Write an interval bisection routine. Your program should take as parameters a function handle, the search domain $\mathbf{x}$, and a tolerance `TOL`. What are the results you get for the function $f(x) = \sin x - \cos x$ with $\mathbf{x} = [-10, +10]$ and `TOL` $= 0.0001$?

Problem 2: Write an interval Newton routine. Your program should take as parameters a function handle and a search domain $\mathbf{x}$. Make sure the search stops when two successive encosureas are identical. What are the results you get for the function $f(x) = \sin(\cos(x-3))$ with the search domains $\mathbf{x}_1 = [1, 2]$, $\mathbf{x}_2 = [1.5, 2.5]$, and $\mathbf{x}_3 = [2, 3]$?

Problem 3: Combine the ideas from the two previous problems and write a hybrid bisection/interval Newton routine. It should bisect the search domain into subintervals $\mathbf{x}_k$ until either $\mathtt{diam}(\mathbf{x}_\mathtt{k}) \leq$`TOL`, or $0 \notin f'(\mathbf{x}_k)$. In the latter case the subinterval should undergo an interval Newton search. Your program should prompt the user for the search domain $\mathbf{x}$, and a tolerance `TOL`. What are the results you get for the function $f(x) = \sin(\cos(x-3))$ with the search domain $\mathbf{x} = [-10, 10]$, and `TOL` $= 0.001$?

Problem 4: Write an interval optimizer. You may choose to use any or all of the midpoint, monotonicity, or convexity checks. Your program should take as parameters a function handle, a search domain $\mathbf{x}$, and a tolerance `TOL`. What are the results you get for the function

$$f(x) = x^2 - \frac{1}{2}e^{-(a(x-\frac{1}{2}))^2}$$

with $a = 10000$, $\mathbf{x} = [-10, +10]$ and `TOL` $= 10^{-10}$?

Problem 5: Write a uniform-step, interval integrator. Your program should take as parameters a function handle, the domain of integration $\mathbf{x}$, and a the number of subdomains N. What results do you get for the function

$$f(x) = e^{x+\sin x}$$

over the domain $\mathbf{x} = [-2, +2]$. and using $N = 10, 100, 1000$?

Problem 6: (Only if you have time!) Make the integrator above adaptive.