Qualitative Alternating Parity Tree Automata

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Outline

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Computational vs Reactive Programs

Computational Programs:

Run in order to produce a final result on terminaison.

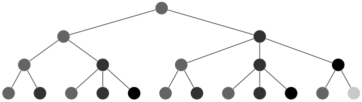
$$x \longrightarrow \boxed{\text{Program}} \longrightarrow y$$

Specified in terms of Inputs/Outputs.

Correctness: Hoare triples $= \{P\}C\{Q\}$

Reactive Programs:

Maintain an ongoing interaction with their environment.

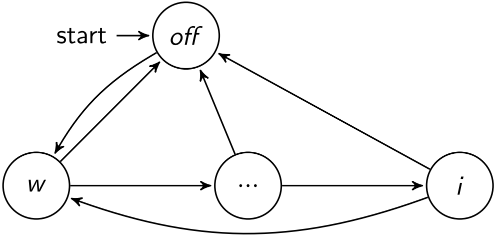


Specified in term of behavioural tree.

Correctness: Behavioural specifications:

- MSO
- Tree automata

A Reactive Program: Printer Driver



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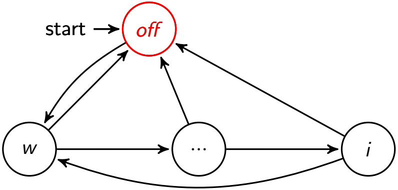

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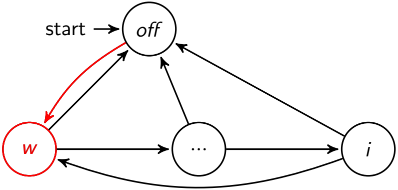

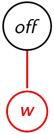

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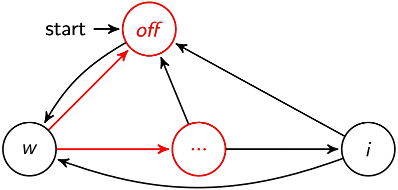

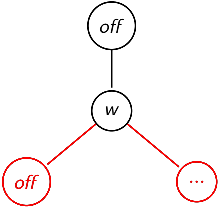

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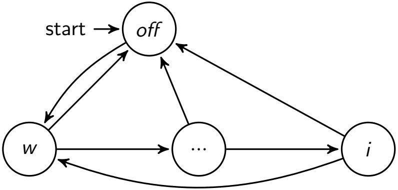
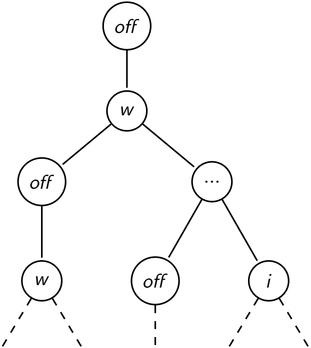
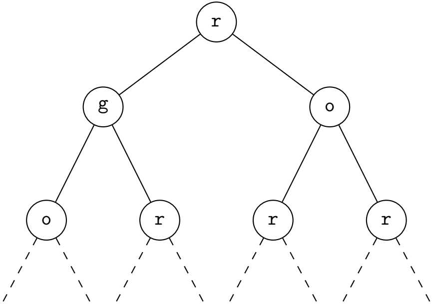




Example of Specification Checking

Specification: "$\exists^\infty r \Rightarrow \exists^\infty g$"



$Q = \{q_r, q_g, q_o\}$ $\mathcal{A}$

initial state: q_o

colours: $c(q_r) = 1$

$c(q_g) = 0$

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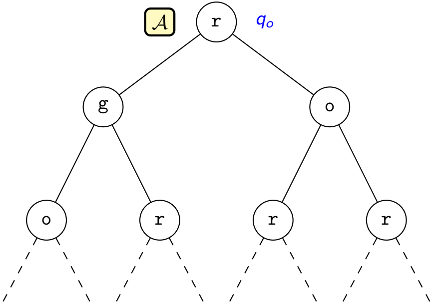

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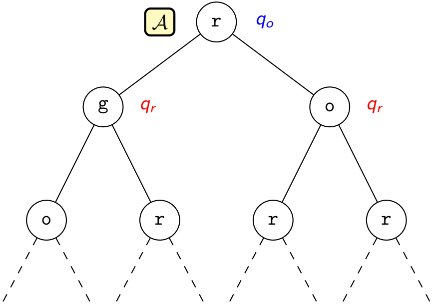

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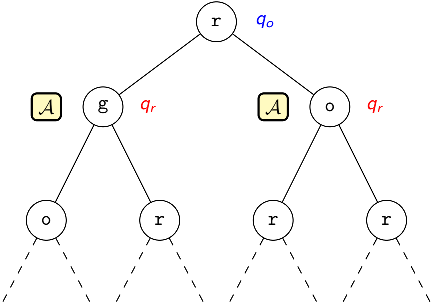

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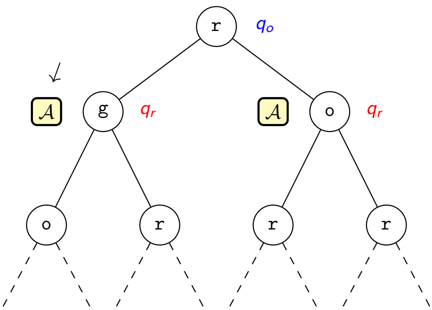

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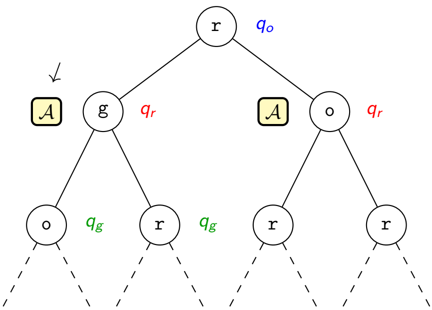

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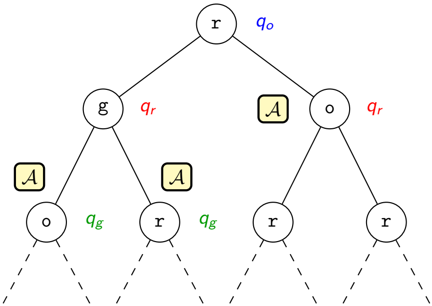

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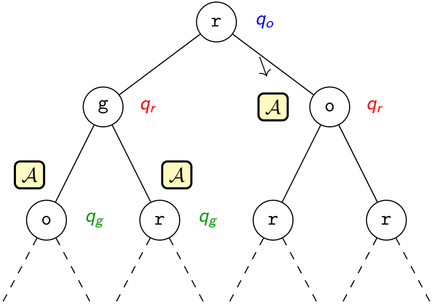

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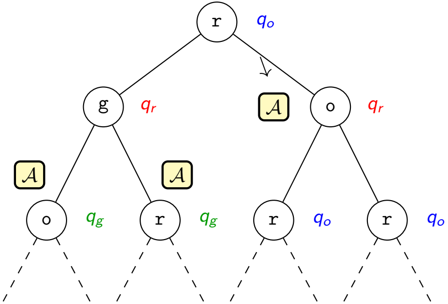

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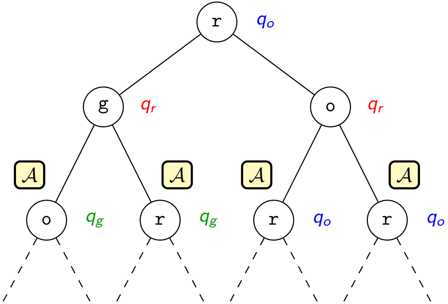

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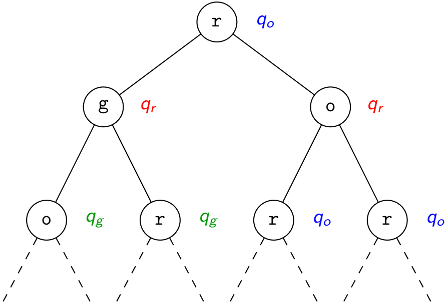

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A branch is accepting if the smaller colour seen infinitely many often is even (Parity).

A run is accepting if all its branches are accepting ($\forall$).

A tree is accepted if there exists an accepting run ($\exists$).

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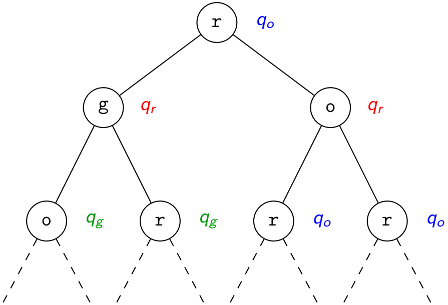

$$\exists \rho \; \forall B, \; Parity(B)$$

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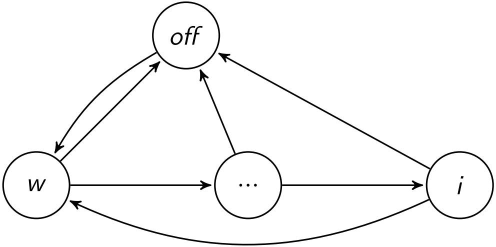
And if There Were Probabilities?



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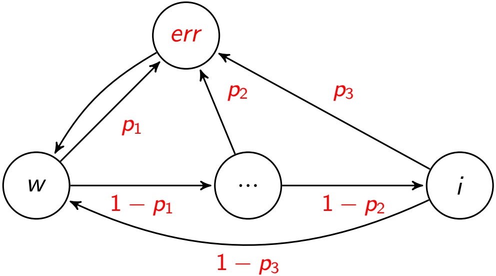

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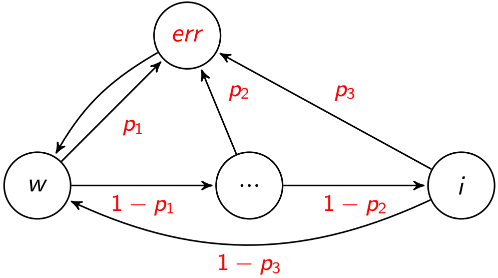


$$\exists\rho\ \forall B,\ Parity(B) \longrightarrow \exists\rho\ \forall^{=1} B,\ Parity(B)$$

Outline

Principle of the Alternation

Non-determinism:

$$\begin{tikzcd} C_1 \arrow[d] \\ C_{2.2} \arrow[d] \\ C_{3.4} \end{tikzcd}$$

Universality:

$$\begin{tikzcd} & C_1 \arrow[dl] \arrow[dr] & & \\ C_{2.1} \arrow[d] & & C_{2.2} \arrow[dl] \arrow[d] \arrow[dr] & \\ C_{3.1} & C_{3.2} & C_{3.3} & C_{3.4} \end{tikzcd}$$

Alternation:

$$\begin{tikzcd} C_1 \arrow[d] \arrow[dr] & \\ C_{2.1} \arrow[d] & C_{2.2} \arrow[d] \\ C_{3.1} & C_{3.4} \end{tikzcd}$$

Alternation in Tree Automata

$Q = Q_{\exists} \uplus Q_{\forall}$.

To construct a run:

- In $Q_{\exists}$, Eve chooses the next transition. $\rightarrow \sigma$
- In $Q_{\forall}$, Adam chooses the next transition. $\rightarrow \tau$

$$\exists \rho\ \forall B,\ Parity(B) \longrightarrow \exists \sigma\ \forall \tau\ \forall B,\ Parity(B)$$

$$\exists \rho\ \forall^{=1} B,\ Parity(B) \longrightarrow \exists \sigma\ \forall \tau\ \forall^{=1} B,\ Parity(B)$$

Outline

Classical Acceptance: the Simulation Theorem

Theorem (Simulation Theorem)

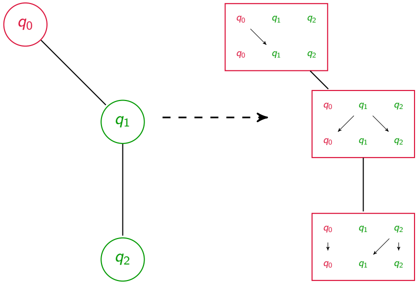
Let $\mathcal{A}$ be an alternating parity tree automaton. Then one can effectively construct a non-deterministic parity tree automaton $\mathcal{B}$ s.t. $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$.

Proof.

$$\exists\sigma\ \forall\tau\ \forall B,\ Parity(B).$$

$$\downarrow$$

$$\exists\sigma\ \forall B,\ \underbrace{\forall\tau\ Parity(B).}_{Parity'(B)}$$



$\square$

Qualitative Acceptance: the Simulation Theorem?

$$\exists\sigma\ \forall\tau\ \forall^{=1} B,\ \mathit{Parity}(B).$$

$$\not\Downarrow$$

$$\exists\sigma\ \forall^{=1} B\underbrace{,\ \forall\tau\ \mathit{Parity}(B)}_{\mathit{Parity}'(B)}.$$

Qualitative Acceptance: the Simulation Theorem?

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$$\not\downarrow$$

$$\exists\sigma\ \forall^{=1}B\underbrace{,\ \forall\tau\ \mathit{Parity}(B)}_{\mathit{Parity}'(B)}.$$

Conjuncture

Alternating qualitative parity tree automata are strictly more expressive than non-deterministic qualitative parity tree automata.

Other works

To show the conjuncture:

- Example of an interesting alternating language.
- Characterization of alternating languages: pumping lemmas.

Parallel work:

- Parity hierarchy.
- Non complementation.