Coinduction based algorithm to decide Büchi automata equivalence

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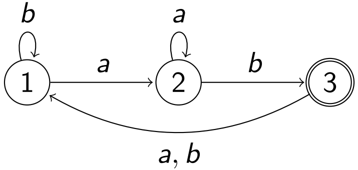
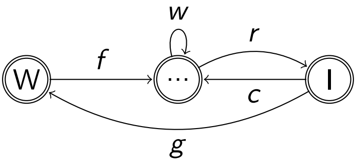
Introduction: Büchi automata



→ Accepts words having infinitely many *a*'s and infinitely many *b*'s

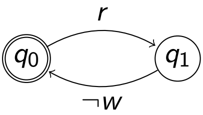
Motivations

PRINTER DRIVER



w: wait, r: request, c: cancel,
g: grant, f: finish

LTL FORMULA



$$\mathbf{G}[r \Rightarrow \mathbf{XF}\ \neg w]$$

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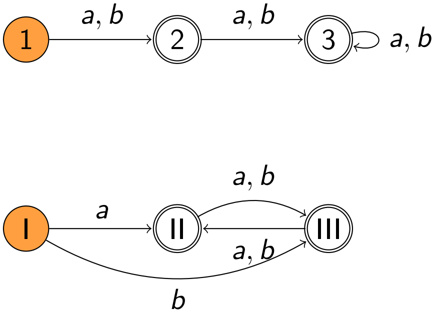
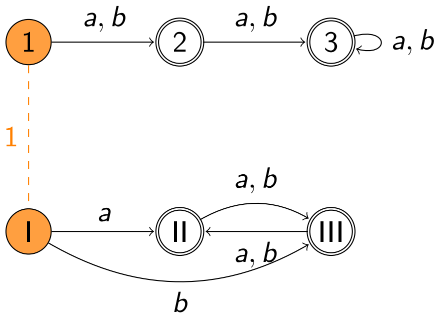
HK algorithm on DFAs

INPUT: 1 DFA, 2 states x and y ; OUTPUT: $x \sim y$?



HK algorithm on DFAs

IDEA: Assume $x \sim y$ and see if there is a contradiction



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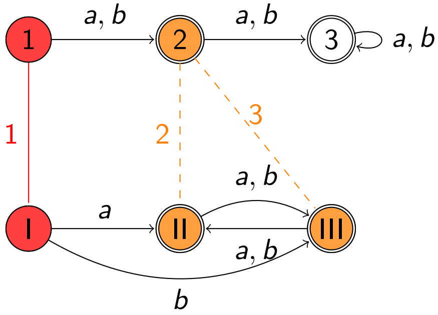

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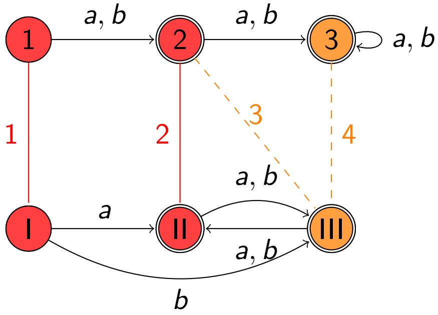

HK algorithm on DFAs

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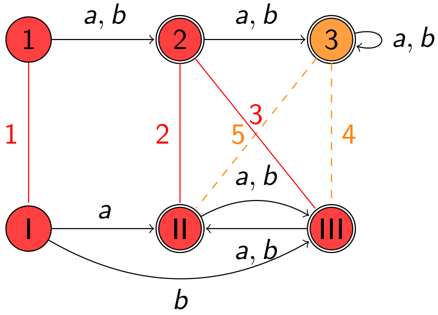

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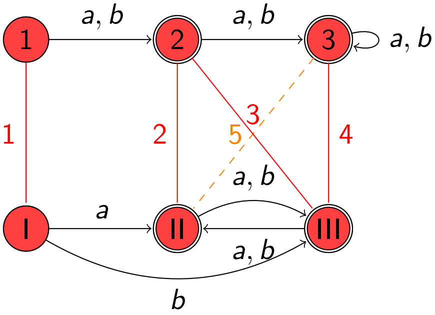
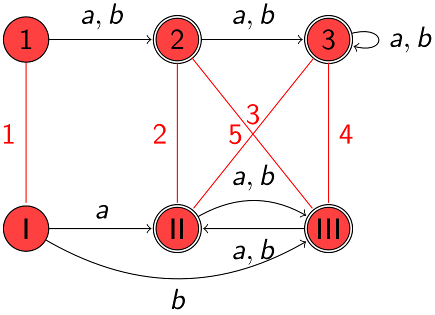


HK algorithm on DFAs

IDEA: Assume $x \sim y$ and see if there is a contradiction



HK algorithm on DFAs

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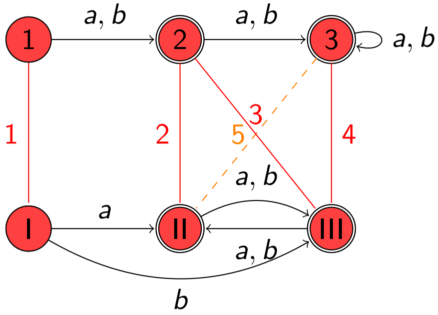
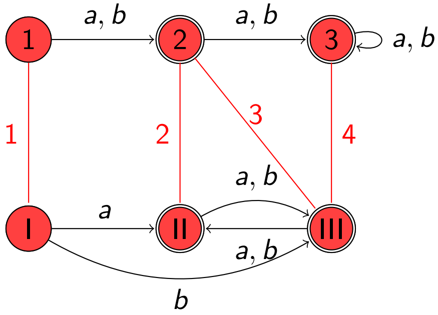


Relation closed under equivalence

HK algorithm on DFAs

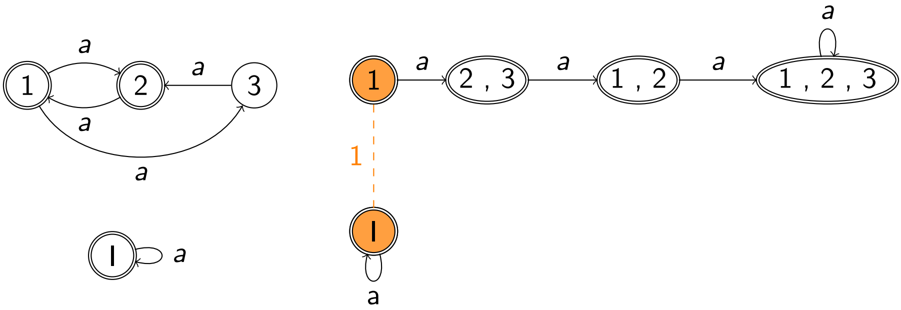
IDEA: Assume $x \sim y$ and see if there is a contradiction



Relation closed under equivalence

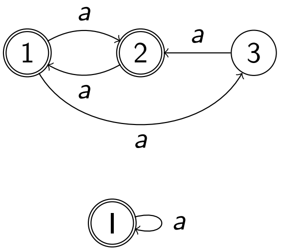
HK algorithm on NFAs

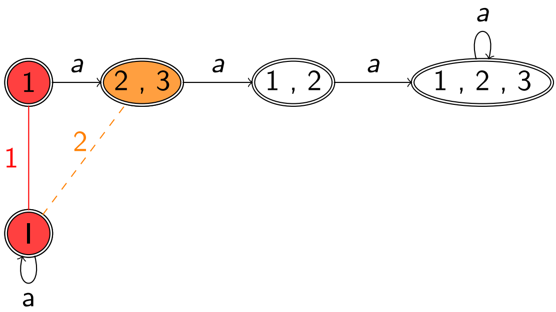
→ Run the algorithm on the powerset



HK algorithm on NFAs

$\rightarrow$ Run the algorithm on the powerset





HK algorithm on NFAs

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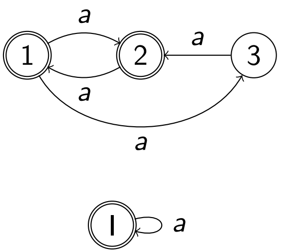

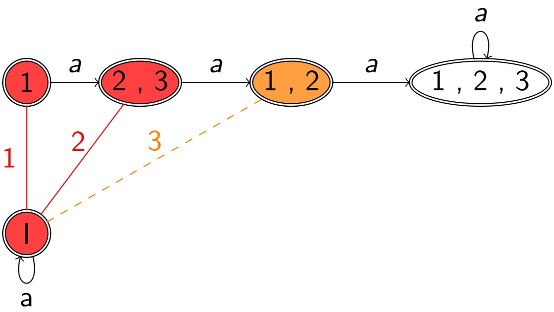

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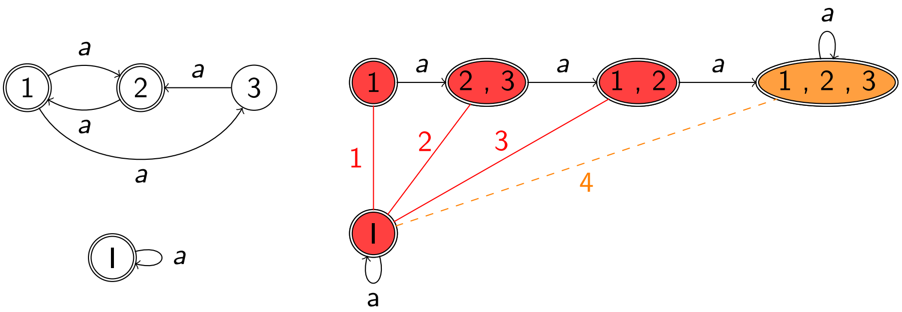

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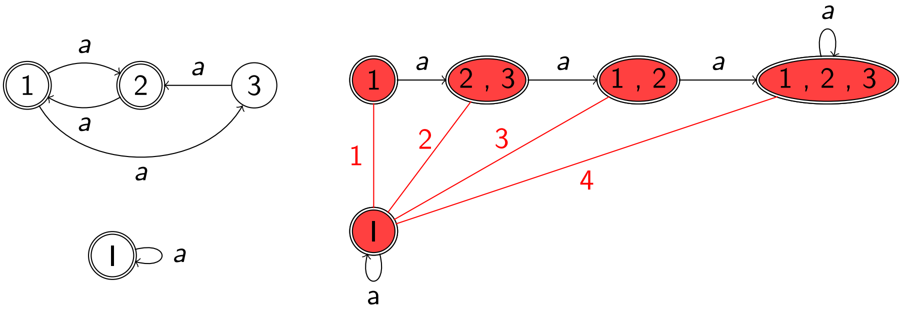
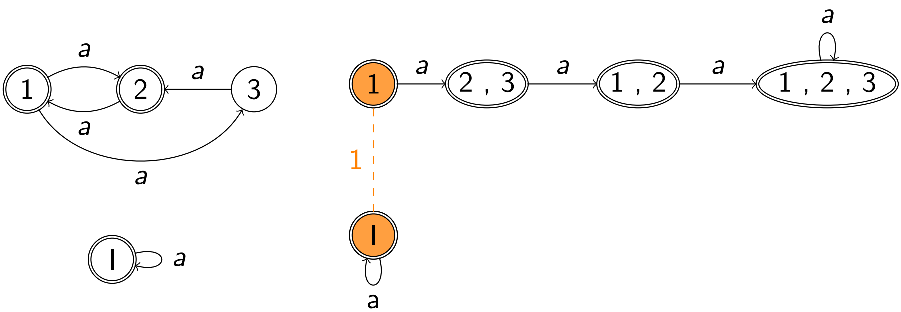
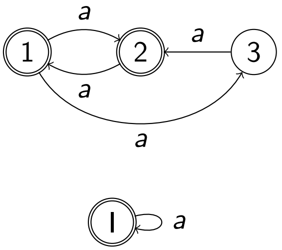
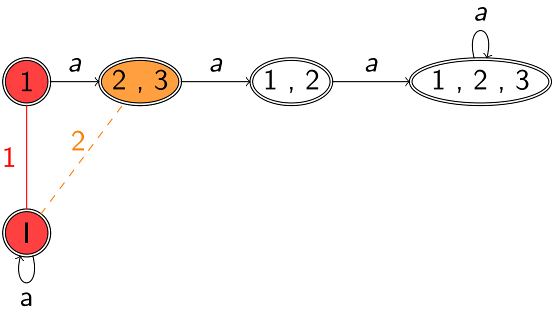


HKC algorithm on NFAs



HKC algorithm on NFAs





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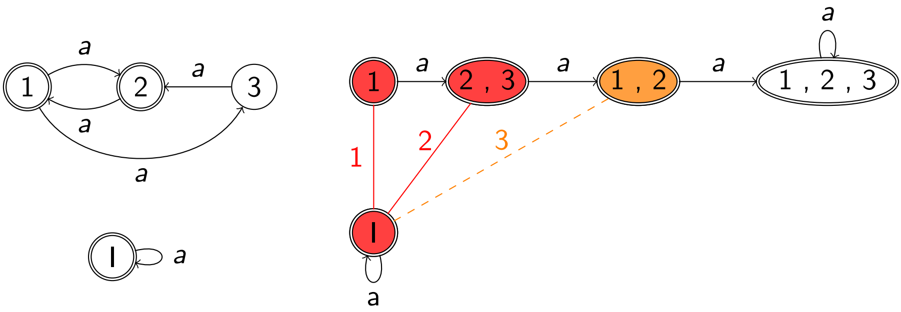

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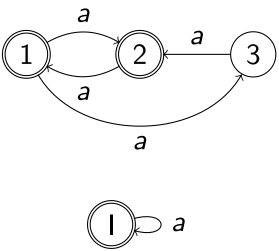
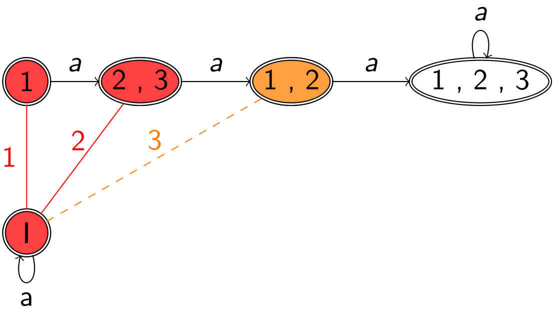
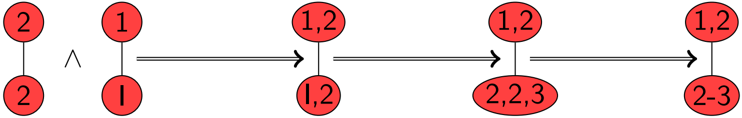
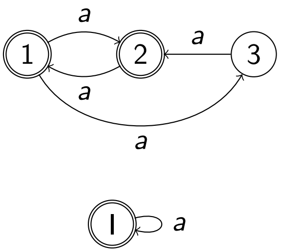
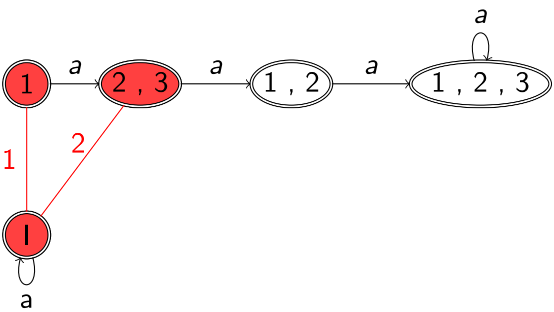


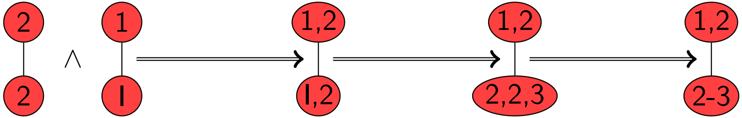




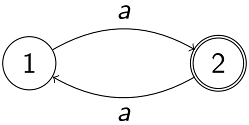
HKC algorithm on NFAs







HKC algorithm on Büchi automata?



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Ultimately periodic words

Words of shape $u \cdot v^\omega$

$u = u_0 u_1 u_2 u_3 u_4 u_5 u_6 u_7 \ldots$: accepted by a Büchi automaton

$q_0 q_1 q_2 \mathbf{q_f} q_4 q_5 q_6 \mathbf{q_f} q_8 \ldots$: an accepting run

$u_0 u_1 u_2 (u_3 u_4 u_5 u_6)^\omega$ accepted by $q_0 q_1 q_2 \mathbf{q_f} (q_4 q_5 q_6 \mathbf{q_f})^\omega$

$\rightarrow$ Any non-empty rational language has a ultimately periodic word

Equivalence of language equivalence

Corollary

$\mathcal{L}_1 = \mathcal{L}_2$ *iff* $UP(\mathcal{L}_1) = UP(\mathcal{L}_2)$ $\qquad$ $\mathcal{L}_1, \mathcal{L}_2$ *rationals*

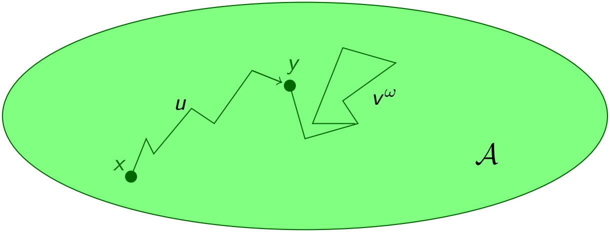
$UP((\mathcal{L}_1 \cup \mathcal{L}_2) \setminus (\mathcal{L}_1 \cap \mathcal{L}_2)) = \varnothing$

$\Rightarrow (\mathcal{L}_1 \cup \mathcal{L}_2) \setminus (\mathcal{L}_1 \cap \mathcal{L}_2) = \varnothing$

$\Rightarrow \mathcal{L}_1 = \mathcal{L}_2$

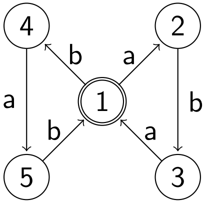
Rationality of ultimately periodic languages

$$UP(\mathcal{L}) \quad - \quad u \cdot v^\omega \quad \rightsquigarrow \quad u \cdot \$ \cdot v \quad - \quad \mathcal{L}_\$$$

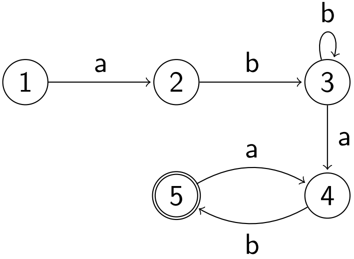


$$\mathcal{L}_\$ = \bigcup_y \mathcal{M}_{x,y} \cdot \$ \cdot \mathcal{N}_y$$

Issues when constructing $\mathcal{A}_{\mathcal{N}_y}$

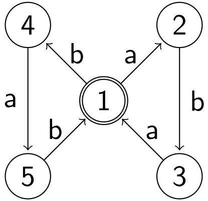


Need to read $(ab)^3$



Need to read *abab* and then *ab*

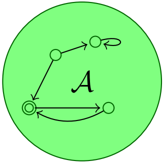
Construction of $\mathcal{A}_{\mathcal{N}_y}$



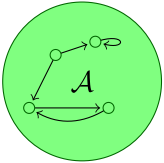
$$\begin{pmatrix} 1,0 \\ 2,0 \\ 3,0 \\ 4,0 \\ 5,0 \end{pmatrix} \xrightarrow{a} \begin{pmatrix} 2,0 \\ \perp \\ 1,1 \\ 5,0 \\ \perp \end{pmatrix} \xrightarrow{b} \begin{pmatrix} 3,0 \\ \perp \\ 4,1 \\ 1,1 \\ \perp \end{pmatrix}$$

$$1 \overset{ab}{\rightsquigarrow} 3 \;,\; 3 \overset{ab}{\rightsquigarrow} 4 \;,\; 4 \overset{ab}{\rightsquigarrow} 1$$

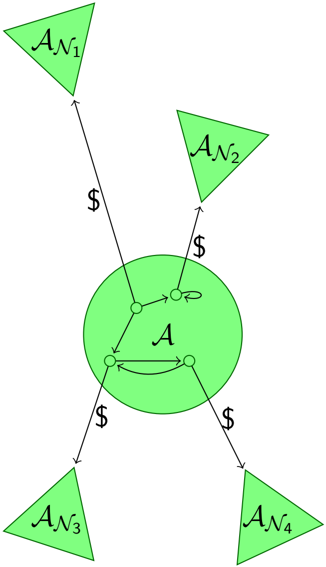
Construction of $\mathcal{A}_\$$



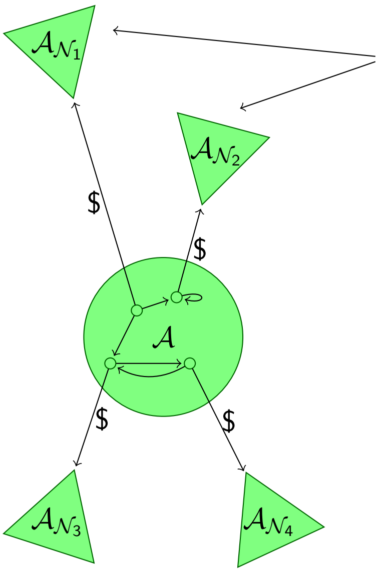
Construction of $\mathcal{A}_\$$



Construction of $\mathcal{A}_\$$



Construction of $\mathcal{A}_\$$



Same structure but
different accepting conditions

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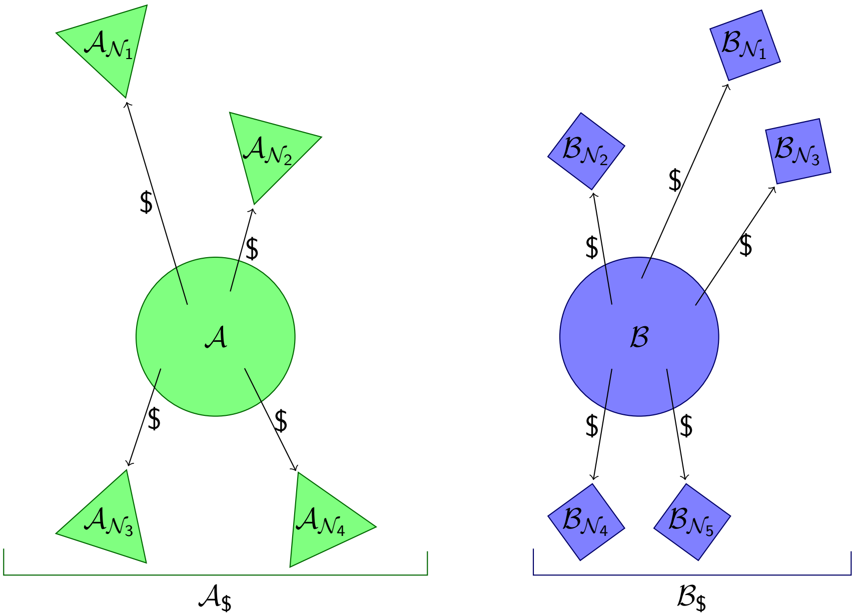
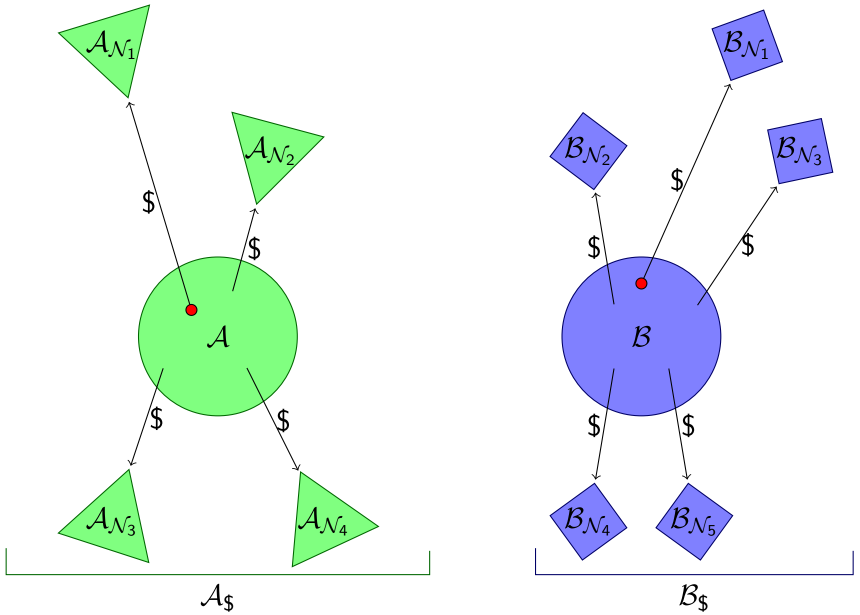
How the algorithm works

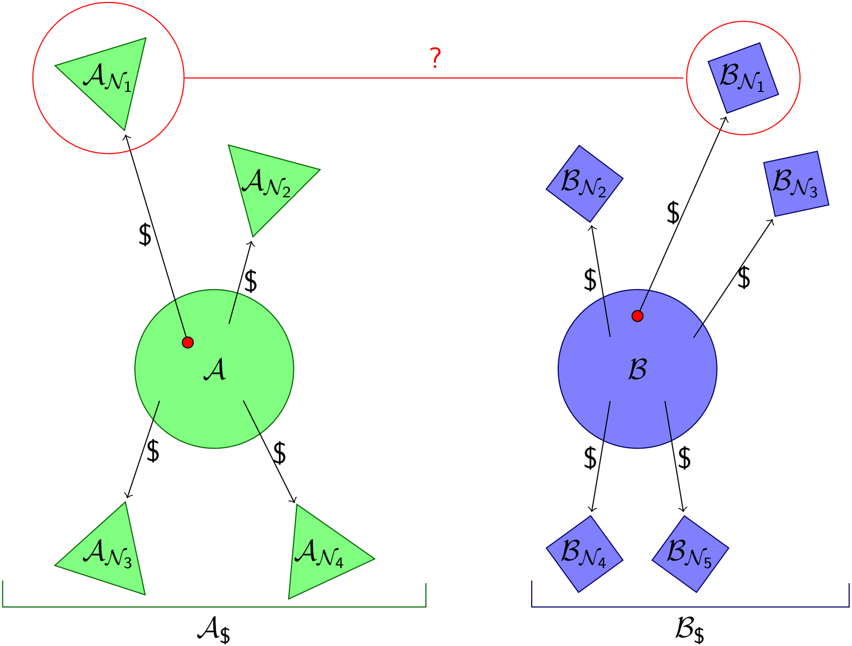
$\rightarrow$ We can run HKC on $\mathcal{A}_{\$}$

$\mathcal{A}$
$\mathcal{B}$



$\mathcal{A}_{\mathcal{N}_1}$
$\mathcal{A}_{\mathcal{N}_2}$
$\mathcal{A}$
$\mathcal{A}_{\mathcal{N}_3}$
$\mathcal{A}_{\mathcal{N}_4}$
$\mathcal{A}_{\$}$
$\mathcal{B}_{\mathcal{N}_1}$
$\mathcal{B}_{\mathcal{N}_2}$
$\mathcal{B}_{\mathcal{N}_3}$
$\mathcal{B}$
$\mathcal{B}_{\mathcal{N}_4}$
$\mathcal{B}_{\mathcal{N}_5}$
$\mathcal{B}_{\$}$


$\mathcal{A}_{\mathcal{N}_1}$
$\mathcal{A}_{\mathcal{N}_2}$
$\mathcal{A}$
$\mathcal{A}_{\mathcal{N}_3}$
$\mathcal{A}_{\mathcal{N}_4}$
$\mathcal{A}_{\$}$
$\mathcal{B}_{\mathcal{N}_1}$
$\mathcal{B}_{\mathcal{N}_2}$
$\mathcal{B}_{\mathcal{N}_3}$
$\mathcal{B}$
$\mathcal{B}_{\mathcal{N}_4}$
$\mathcal{B}_{\mathcal{N}_5}$
$\mathcal{B}_{\$}$



$\mathcal{A}_{\mathcal{N}_1}$
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$\mathcal{B}_{\mathcal{N}_1}$
$\mathcal{A}_{\mathcal{N}_2}$
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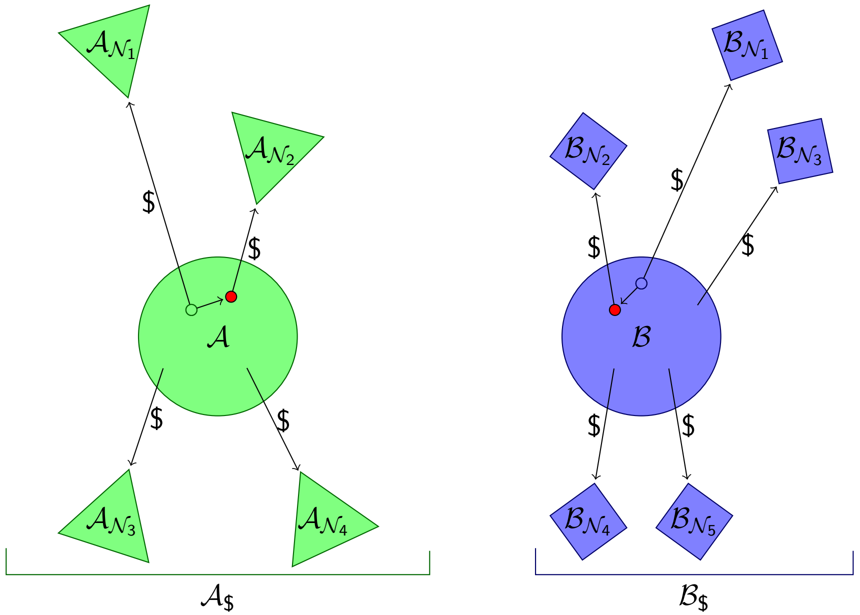
$\mathcal{B}_{\mathcal{N}_4}$
$\mathcal{B}_{\mathcal{N}_5}$
$\mathcal{A}_{\$}$
$\mathcal{B}_{\$}$



$\mathcal{A}_{\mathcal{N}_1}$
$\mathcal{A}_{\mathcal{N}_2}$
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$\mathcal{A}$
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$\mathcal{A}_{\mathcal{N}_3}$
$\mathcal{A}_{\mathcal{N}_4}$
$\mathcal{A}_\$$
$\mathcal{B}_{\mathcal{N}_1}$
$\mathcal{B}_{\mathcal{N}_2}$
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$\mathcal{B}$
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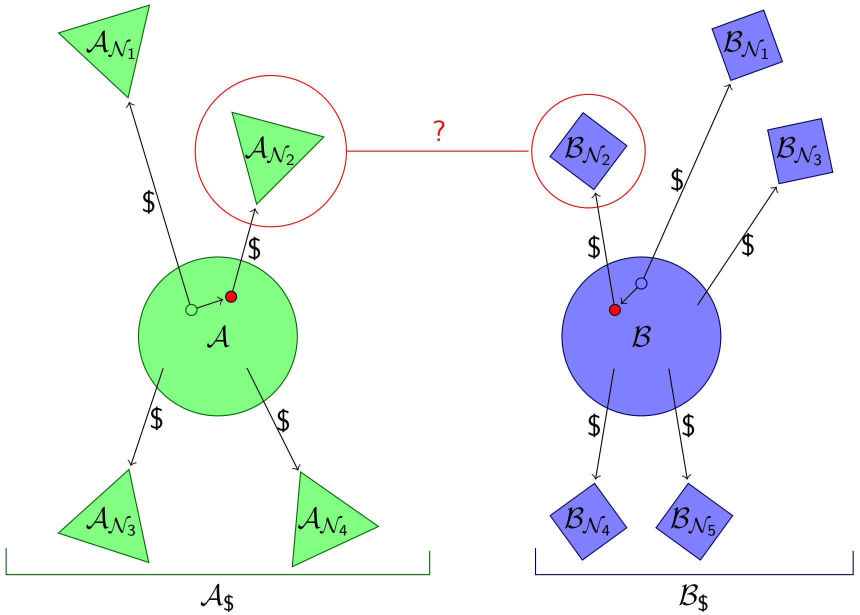
$\mathcal{B}_{\mathcal{N}_4}$
$\mathcal{B}_{\mathcal{N}_5}$
$\mathcal{B}_\$$



$\mathcal{A}_{\mathcal{N}_1}$
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$\mathcal{A}_{\mathcal{N}_4}$
$\mathcal{B}_{\mathcal{N}_4}$
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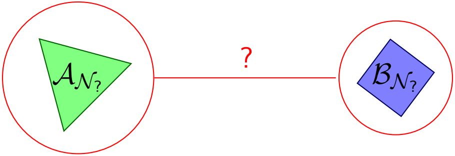
$\mathcal{A}_\$$
$\mathcal{B}_\$$

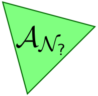
1st improvement: pre-processing



$$\hookrightarrow \{(q_1, r_1), (q_2, r_2), (q_3, r_3), ..., (q_n, r_n)\}$$

$$\forall i, \ \ o(q_i) = o(r_i)?$$

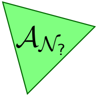
2nd improvement: state compression



: contains less states that we would think

$$\begin{pmatrix}1,0\\2,0\\3,0\\4,0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}4,0\\3,0\\3,1\\1,0\end{pmatrix} \qquad \begin{pmatrix}1,0\\2,0\\3,0\\4,0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}2,1\\3,0\\3,1\\3,1\end{pmatrix}$$

2nd improvement: state compression



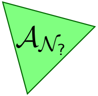
: contains less states that we would think

$$\begin{pmatrix}1,0\\2,0\\3,0\\4,0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}\mathbf{4},\mathbf{0}\\3,0\\3,1\\1,0\end{pmatrix}$$

$$\begin{pmatrix}1,0\\2,0\\3,0\\4,0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}2,1\\3,0\\3,1\\\mathbf{3},\mathbf{1}\end{pmatrix}$$

$$\begin{pmatrix}1,0\\2,0\\3,0\\4,0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}4,0\\3,0\\3,1\\3,1\end{pmatrix}$$

2nd improvement: state compression



: contains less states that we would think

$$\begin{pmatrix}1,0\\2,0\\3,0\\4,0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}4,0\\3,0\\3,1\\\mathbf{1},\mathbf{0}\end{pmatrix}$$

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$$\begin{pmatrix}1,0\\2,0\\3,0\\4,0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}2,1\\3,0\\3,1\\1,0\end{pmatrix}$$

2nd improvement: state compression

$\mathcal{A}_{\mathcal{N}_?}$: contains less states that we would think $2^{(2m)^m} \to \left(2^{2m}\right)^m$

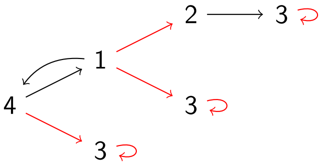
$$\begin{pmatrix} 1,0 \\ 2,0 \\ 3,0 \\ 4,0 \end{pmatrix} \xrightarrow{a} \begin{pmatrix} 4,0 \\ 3,0 \\ 3,1 \\ 1,0 \end{pmatrix}$$

$$\begin{pmatrix} 1,0 \\ 2,0 \\ 3,0 \\ 4,0 \end{pmatrix} \xrightarrow{a} \begin{pmatrix} 2,1 \\ 3,0 \\ 3,1 \\ 3,1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} (4,0),(2,1) \\ (3,0) \\ (3,1) \\ (1,0),(3,1) \end{pmatrix}$$

$$\begin{pmatrix} 1,0 \\ 2,0 \\ 3,0 \\ 4,0 \end{pmatrix} \xrightarrow{a} \begin{pmatrix} 2,1 \\ 3,0 \\ 3,1 \\ 1,0 \end{pmatrix}$$

1st issue: description of the automaton

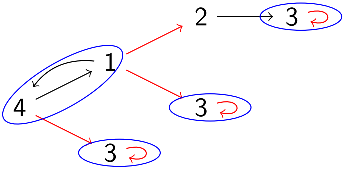
$$\begin{pmatrix} (4,0),(2,1),(3,1) \\ (3,0) \\ (3,1) \\ (1,0),(3,1) \end{pmatrix}$$



1st issue: description of the automaton

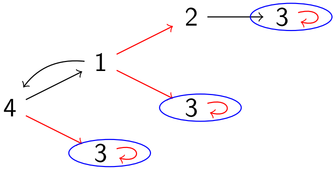
$$\begin{pmatrix} (4,0),(2,1),(3,1) \\ (3,0) \\ (3,1) \\ (1,0),(3,1) \end{pmatrix}$$

1. Compute the strongly connected components



1st issue: description of the automaton

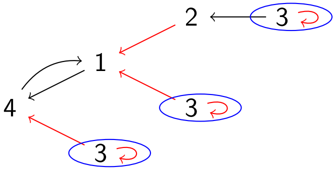
$$\begin{pmatrix} (4,0),(2,1),(3,1) \\ (3,0) \\ (3,1) \\ (1,0),(3,1) \end{pmatrix}$$



1. Compute the strongly connected components
2. Keep the ones having a final edge

1st issue: description of the automaton

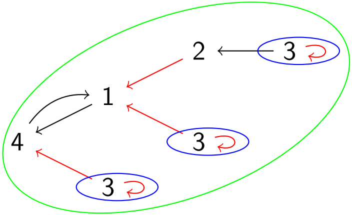
$$\begin{pmatrix} (4,0),(2,1),(3,1) \\ (3,0) \\ (3,1) \\ (1,0),(3,1) \end{pmatrix}$$



1. Compute the strongly connected components
2. Keep the ones having a final edge
3. Reverse the edges

1st issue: description of the automaton

$$\begin{pmatrix} (4,0),(2,1),(3,1) \\ (3,0) \\ (3,1) \\ (1,0),(3,1) \end{pmatrix}$$



1. Compute the strongly connected components
2. Keep the ones having a final edge
3. Reverse the edges
4. Compute the connected components

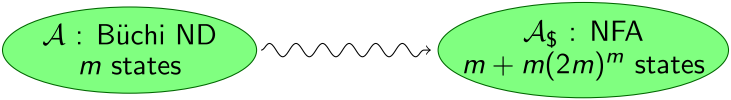
2nd issue: computation of the congruence

$\rightarrow$ The problem becomes NP-Complete

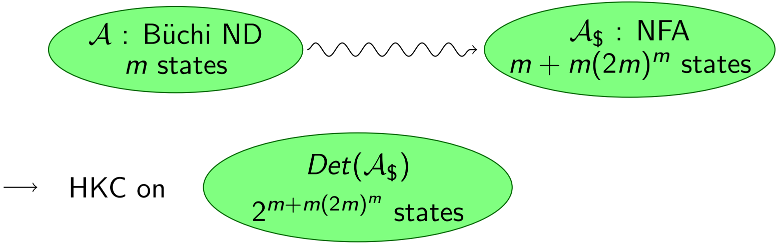
$\rightarrow$ Use of a SAT-Solver

Table of Contents

Summary



Summary


$\mathcal{A}$: Büchi ND
m states
$\mathcal{A}_\$$: NFA
$m + m(2m)^m$ states
$\longrightarrow$ HKC on
$Det(\mathcal{A}_\$)$
$2^{m+m(2m)^m}$ states

Summary

$\mathcal{A}$: Büchi ND, m states $\rightsquigarrow$ $\mathcal{A}_{\$}$: NFA, $m + m(2m)^m$ states

$\longrightarrow$ HKC on $Det(\mathcal{A}_{\$})$, $2^{m+m(2m)^m}$ states

$\longrightarrow$ HKC on $Det(\mathcal{A}_{\mathcal{N}_?})$, $2^{(2m)^m}$ states $+$ HKC on $Det(\mathcal{A})$, 2^m states

Summary

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$\longrightarrow$ HKC on $\mathcal{A}'_{\mathcal{N}_?}$: DFA, 2^{2m^2} states $+$ HKC on $Det(\mathcal{A})$, 2^m states

Future Work

- Implementation and comparison with existing methods
- Further improvements of the algorithm
- Extension to other automata classes

Thank You

- Filippo Bonchi and Damien Pous.
  Checking NFA equivalence with bisimulations up to congruence.
  In *Principle of Programming Languages (POPL)*, pages 457–468, Roma, Italy, January 2013. ACM.
  16p.

- Hugues Calbrix, Maurice Nivat, and Andreas Podelski.
  Ultimately periodic words of rational ω-languages.
  In *International Conference on Mathematical Foundations of Programming Semantics*, pages 554–566. Springer, 1993.