TD 1: Automata and Turing Machines

1 Automata

Exercise 1. *True or False ?*

Which of the following statements are true ?

1. All regular languages are finite.
2. All finite languages are regular.
3. Non deterministic automata can describe more languages than deterministic ones.
4. The star lemma (or pumping lemma) caracterizes regular languages.
5. Rational and regular languages are the same class of languages.

Exercise 2. *Complementation*

Consider the alphabet $\Sigma = \{a, b\}$.

1. Write an automaton recognizing $\mathcal{L}$, the set of words containing '*aba*' as a factor.
2. How do you obtain an automaton recognizing the complement of $\mathcal{L}$?

Consider now the alphabet $\Sigma = \{a, b, c\}$.

3. Adapt your first automaton to write an automaton recognizing $\mathcal{L}'$, the set of words that begin with '*c*' and contain '*aba*' as a factor.
4. How do you obtain an automaton recognizing the complement of $\mathcal{L}'$?

Exercise 3. *Rationality, where are you ?*

Among the following languages, which ones are regular ? In either case justify your answer.

1. $\mathcal{L}_1 = \{a^{2n}, n \geq 0\}$.
2. $\mathcal{L}_2 = \{a^{n^2}, n \geq 0\}$.
3. $\mathcal{L}_3 = \{a^m b^n a^{m+n}, m, n \geq 0\}$.
4. $\mathcal{L}_4 = \{a^p, p \text{ is prime}\}$.
5. $\mathcal{L}_5 = \{w \in \Sigma^*, |w|_a \neq |w|_b\}$.
6. $\mathcal{L}_6 = \{w \in \Sigma^*, |w|_a \not\equiv |w|_b \bmod(2)\}$.
7. $\mathcal{L}_7 =$ the set of words that are palindromes.

Exercise 4. *Rational expressions to english*

For the following rational expression, write down in english the language they describe.

1. $(\varepsilon + \Sigma)(\varepsilon + \Sigma)$.
2. $\left(\Sigma^2\right)^*$.
3. $(b + ab)^*(a + \varepsilon)$.

4. $(ab^*a+b)^*$.

Exercise 5. *English to rational expressions*

Express the following languages with a rational expression (the alphabet is $\Sigma = \{a, b\}$).

1. The set of words such that the length of any sequence of a's is even.
2. The set of words that don't have the same letter consecutively.

Exercise 6. *The Blind Bartender*

A blind bartender plays a game with one of his client. The client put a square plate with four glasses, one standing in each corner, in front of the bartender. Each glass can be either upright or upside down, but the bartender is blind and cannot tell the difference. The bartender's goal is to turn the glasses so they will all be up or all be down. Each round the bartender can turn one, two or three glasses, but between each round the client can rotate the plate by any mutiple of $90°$.

1. Modelize the game with a non deterministic automaton. (Hint : try to reduce the number of configurations to four and the number of actions to three.)
2. Determinize the automaton previously obtained and deduce from it a winning strategy for the bartender.

Exercise 7. *Multiple of three*

We code the integer with their binary representation over the alphabet $\Sigma = \{0, 1\}$ and read them from left to right. Write down an automaton recognizing the multiple of three.

2 Turing Machines

Exercise 8. *Turing Regulation*

1. Let M be a Turing Machine such that any transition moves to the right. Prove that the recognized language is regular.
2. Let L be a regular language. Build a Turing Machine recognizing exactly L.
3. Build a Turing Machine recognizing the language $\{a^n b^n | n \in \mathbb{N}\}$.
4. What can you deduce about Regular Languages versus languages that can be recognized by Turing Machines ?
5. (Bonus) Build a Turing Machine recognizing the language $\{a^n b^n c^n | n \in \mathbb{N}\}$.

Exercise 9. *Some Turing Machines*

Let $\Sigma = \{0, 1\}$ be an alphabet and let x be a word over Σ^*. Build Turing Machines such that :

1. reading x, the machine writes x^{-1} (x written reversely, from the last letter to the first).
2. the machine accepts x if and only if x equals yy^{-1} for some $y \in \Sigma^*$.
3. the machine accepts x if and only if x equals yy for some $y \in \Sigma^*$.