ERRATA FOR MY ARTICLES

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Contents

1. Représentations p-adiques et équations différentielles

Example 2.8.1. Replace $\mathbf{A}^+_{\max}$ by $\mathbf{A}_{\max}$.

Sections 3.3, 5.5. Kedlaya has completely changed his article [34], so that most references to it are now incorrect.

Theorem 4.10. Theorem 4.10 is actually due to Forster, see : O. Forster, Zur Theorie der Steinschen Algebren und Moduln, Math. Zeitschrift, 97, p. 376ff, 1967.

Proposition 2.24. The log map is not defined for $x = 0$. In addition, I only define it on $\tilde{\mathbf{A}}^+$ but later, I use it on $\tilde{\mathbf{A}}^\dagger$ (for example : $\log(\pi_K)$). The (easy) extension to $\tilde{\mathbf{A}}^\dagger$ is done by Colmez in [Col08].

Proof of lemma 5.27. Replace $\mathrm{GL}_d(\mathbf{A}^{\dagger,r}, K)$ by $\mathrm{GL}_d(\mathbf{A}^{\dagger,r}_K)$ and $\mathrm{M}_d(\mathbf{A}^{\dagger,r}, K)$ by $\mathrm{M}_d(\mathbf{A}^{\dagger,r}_K)$.

Matrices. I wrote all matrices "the wrong way". For example, if f and g are two semilinear maps, then in my notation, $\mathrm{Mat}(fg) = f(\mathrm{Mat}(g))\,\mathrm{Mat}(f)$. To recover the usual notation, one needs to transpose everything (this is done in my other articles).

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Proof of proposition 5.15. It is not true that $\iota_n(N_s) = K_n[\![t]\!] \otimes_K \mathrm{D}_{\mathrm{dR}}(V)$. What is true is that the image of ι_n is dense for the t-adic topology. This is what is proved and used in the rest of the proof.

Page 229, line 3. Replace $\tilde{\mathbf{A}}^+$ by $\tilde{\mathbf{A}}$.

The ring $\mathbf{B}_K^\dagger$. I say that the ring $\mathbf{B}_K^\dagger$ is a ring of power series with coefficients in F, but that is not always the case. It is a ring of power series with coefficients in K_0', the maximal unramified extension of F inside K_∞, which may be larger than F. Since it is true that $(\mathbf{B}_K^\dagger)^{\Gamma_K} = F$, this does not affect the results of the paper, and most proofs go through unchanged.

Monodromy. In order to recover the (φ, N)-module $\mathrm{D}_{\mathrm{st}}(\cdot)$, one should take $N(\log(\pi)) = -p/(p-1)$ instead of $N(\log(\pi)) = -1$.

Diagram on page 271. In the diagram at the top of the page, replace ∇_M by the connexion attached to ∂_M.

Lemma 2.7. Replace $k \gg 0$ by $k \gg -\infty$ in $\sum_{k \gg 0} p^k [x_k]$.

Propositions 2.11 and 2.12. These are only true if I is such that $[\tilde{p}]/p-1$ or $[\tilde{p}^{p^n}]/p-1$ belong to $\tilde{\mathbf{A}}_I$. Otherwise, replace $\tilde{\mathbf{A}}_I$ by $\widetilde{\mathbf{B}}_I$.

Corollary 2.20. The condition on r should be that $r \leqslant s, t$ or in other words that $[s;t] \subset [r;+\infty[$.

Diagram on page 280. Technically not a mistake, but in the lower right of the diagram, $\overline{k}$ can be replaced by $\mathcal{O}_{\mathbf{C}_p}/p$.

Page 233. After the proof of lemma 2.5: $\widetilde{\mathbf{B}}_I = \cap_{[r;s] \subset I} \widetilde{\mathbf{B}}_{[r;s]}$. The intersection should be taken over a nested (increasing) family of intervals whose union is I, not all intervals.

2. Bloch and Kato's exponential map: three explicit formulas

Introduction. Not a mistake, but an omission: it may look as though it was a bright idea to use the theory of (φ, Γ)-modules in order to study Perrin-Riou's exponential map, but note that actually, Fontaine defined (φ, Γ)-modules with that purpose in mind. Therefore, this article is really about checking that Fontaine's construction does what it was supposed to do in the first place.

3. Limites de représentations cristallines

Definition III.4.1. The definition of a Wach module is incomplete. One needs to add the condition that $\mathbf{B}_F \otimes_{\mathbf{B}_F^+} \mathrm{N}$ is an étale (φ, Γ)-module over $\mathbf{B}_F$.

4. Construction de (φ,Γ)-modules : représentations p-adiques et B-paires

Lemma 1.1.11. The proof is incomplete (the map $x \mapsto \sum_{i=0}^{h-1} \varphi^i(\omega) \otimes \varphi^i(x)$ does not even have values in $\mathbf{Q}_{p^h} \otimes_{\mathbf{Q}_p} \mathbf{B}_{\mathrm{e}}$). Let $M = (\varphi^{i+j}(\omega))_{0 \leqslant i,j \leqslant h-1}$ and let $N = (n_{ij})_{0 \leqslant i,j \leqslant h-1}$ denote its inverse. Given $x \in (\widetilde{\mathbf{B}}^+_{\mathrm{rig}}[1/t])^{\varphi^h=1}$, let $x_j = \sum_{i=0}^{h-1} \varphi^{i+j}(\omega) \otimes \varphi^i(x)$. We have $x_j \in \mathbf{B}_{\mathrm{e}}$ and $x = \sum_{i=0}^{h-1} n_{0i} x_i$. This shows that the map $\mathbf{Q}_{p^h} \otimes_{\mathbf{Q}_p} \mathbf{B}_{\mathrm{e}} \to (\widetilde{\mathbf{B}}^+_{\mathrm{rig}}[1/t])^{\varphi^h=1}$ is surjective. It is injective by a standard argument since $\mathbf{Q}_{p^h}$ is a field: if $\alpha_1 \otimes x_1 + \cdots + \alpha_r \otimes x_r$ maps to 0, we can assume that r is minimal and that $\alpha_1 = 1$. In this case, by applying φ, we get $\alpha_i \in (\mathbf{Q}_{p^h})^{\varphi=1} = \mathbf{Q}_p$ for all i and hence $\alpha_1 \otimes x_1 + \cdots + \alpha_r \otimes x_r = 0$ in $\mathbf{Q}_{p^h} \otimes_{\mathbf{Q}_p} \mathbf{B}_{\mathrm{e}}$.

Proposition 3.3.5. In the definition of W (statement of proposition 3.3.5), replace $\mathbf{B}^+_{\mathrm{dR}} \otimes_{K_\infty} D_\infty$ by $\mathrm{Fil}^0(\mathbf{B}_{\mathrm{dR}} \otimes_{K_\infty} D_\infty)$.

Proposition 3.3.10. There is an argument missing from the proof, namely the following lemma: let D be an F-vector space with an action of G_F, such that $P(\nabla) = 0$ where $P(X) \wedge P(X+j) = 1$ for all $j \in \mathbf{Z}_{\geqslant 1}$, and let W be a $\mathbf{B}^+_{\mathrm{dR}}$-lattice of $\mathbf{B}_{\mathrm{dR}} \otimes_F D$ that is stable under G_F. If we set $\mathrm{Fil}^i D = D \cap t^i \cdot W$, then $W = \mathrm{Fil}^0(\mathbf{B}_{\mathrm{dR}} \otimes_F D)$.

Page 117. In the first paragraph of page 117, P and Q seem to be exchanged at various points.

5. Equations différentielles p-adiques et (φ, N)-modules filtrés

Theorem I.3.3. In item (2), it is better to require that $\mathbf{B}^{\dagger,pr}_{\mathrm{rig},K} \otimes_{\mathbf{B}^{\dagger,r}_{\mathrm{rig},K}} \mathrm{D}_r$ is the $\mathbf{B}^{\dagger,pr}_{\mathrm{rig},K}$-module generated by $\varphi(\mathrm{D}_r)$. This implies that $\mathrm{Mat}(\varphi) \in \mathrm{GL}_d(\mathbf{B}^{\dagger,pr}_{\mathrm{rig},K})$, which is used later in the proof.

Theorem III.2.3. There is an argument missing from the proof, namely the following lemma, which is now lemma 7.6 of [Ber13]: Let D be an F-vector space, and let W be a $\mathbf{B}^+_{\mathrm{dR}}$-lattice of $\mathbf{B}_{\mathrm{dR}} \otimes_F D$ that is stable under G_F, where G_F acts trivially on D. If we set $\mathrm{Fil}^i D = D \cap t^i \cdot W$, then $W = \mathrm{Fil}^0(\mathbf{B}_{\mathrm{dR}} \otimes_F D)$.

Example IV.2.8. I think that at least of these is fishy (need to check that).

6. Familles de représentations de de Rham et monodromie p-adique

Lemma 2.1.1. For item (2), one needs to assume that M is finitely presented. In this case, the natural map $M \otimes_S \prod S/\mathfrak{m}_x \to \prod M/\mathfrak{m}_x M$ is a bijection (Bourbaki AC, §I.2, exercise 9), and the rest of the proof works.

Section 2.3. It would be better to define a family of representations as a projective S-module, rather than a free S-module.

7. Sur quelques représentations potentiellement cristallines de $\mathrm{GL}_2(\mathbf{Q}_p)$

Item (v) at the beginning of §2.4. There is no map $\varphi^{-m} : \mathbf{B}_{\mathrm{cris}} \to \mathbf{B}_{\mathrm{dR}}$. However whenever this map is used in the paper, it is used on a space on which it is defined.

8. La correspondance de Langlands locale p-adique pour $\mathrm{GL}_2(\mathbf{Q}_p)$

Page 169. In the definition of the parameter spaces, replace the condition "$w(s) \geqslant 1$" by "$w(s) \in \mathbf{Z}_{\geqslant 1}$" (twice).

9. Lifting the field of norms

Proposition 4.2. It is not true in general that $\mathcal{N}(T) = T$. For example if $P(T) = T^2 - a$ and $p = 2$, then $\mathcal{N}(T) = -T - a$ (the equation $\mathcal{N}(T) = T$ does hold if $(-1)^{q-1}P(T)$ is a monic polynomial of degree q and $P(0) = 0$).

What is true is that if $P(T) \in T \cdot \mathcal{O}_E[\![T]\!]$, then $\mathcal{N}(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times$. Let $n(T)$ denote the power series $n(T) = \mathcal{N}(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times$ and let $n^{\circ -1}(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times$ denote its composition inverse. Let $\mathcal{N}'$ be defined by $\mathcal{N}'(f(T)) = n^{\circ -1} \circ \mathcal{N}(f(T))$. We then have $\mathcal{N}'(T) = T$ and the proof of proposition 4.2 works with $\mathcal{N}'$ instead of $\mathcal{N}$: if $k \geqslant 1$, then $\mathcal{N}'(T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E^k \mathbf{A}_K) \subset T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E^{k+1} \mathbf{A}_K$. This implies, by induction on k, that $(T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E \mathbf{A}_K)^{\mathcal{N}'(x)=x} \subset T \cdot \mathcal{O}_E[\![T]\!]^\times$. We have $F_g(T) \in T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E \mathbf{A}_K$ and $\mathcal{N}'(g(T)) = g(T)$ if $g \in \Gamma_K$ and hence $F_g(T) \in (T \cdot \mathcal{O}_E[\![T]\!]^\times + \varpi_E \mathbf{A}_K)^{\mathcal{N}'(x)=x} \subset T \cdot \mathcal{O}_E[\![T]\!]^\times$.

In the statement of prop 4.2, one should therefore assume that $P(T) \in T \cdot \mathcal{O}_E[\![T]\!]$. The only place where this prop is used is lemma 4.5, in which $P(T)$ does belong to $T \cdot \mathcal{O}_E[\![T]\!]$.

10. Multivariable (φ, Γ)-modules and locally analytic vectors

Lemma 5.3. Technically not a mistake, but the condition that I does not contain 0 is not necessary. The lemma (and theorem 5.4 and corollary 5.5) is true if 0 belongs to I.

Lemma 4.3. There are arguments missing from the proof, in particular the fact that there exists $C(I)$ such that $V(x, I) \geqslant V(tx, I) - C(I)$ if $x \in \widetilde{\mathbf{B}}^I$. There should also be a sublemma to the effect that if $y \in t_\pi(\widetilde{\mathbf{B}}_F^I)^{F\text{-la}}$ then $\nabla(y) \in t_\pi(\widetilde{\mathbf{B}}_F^I)^{F\text{-la}}$ as well.

Theorem 4.4. The proof should start by saying that we take $x \in \tilde{\mathbf{A}}^{[r;s]}$. In addition, there is a problem with the proof of item (1). Indeed, u^d/π does not belong to $\tilde{\mathbf{A}}^{[0;s]}$ so it is not true that if $x \in \tilde{\mathbf{A}}^{[r;s]}$, then there exists k_n such that $(u^d/\pi)^{k_n} \cdot x \in \tilde{\mathbf{A}}^{[0;s]} + \pi^n \tilde{\mathbf{A}}^{[r;s]}$.

The element u^d/π belongs to $\tilde{\mathbf{A}}^{[0;s]}$ if $r = s$, so that the proof works for $r = s$. Now take $x \in \tilde{\mathbf{A}}^{[r;s]}$. The proof tells us that $x = \varphi_q^{-m}(f(u))$ where $f(Y)$ converges on the annulus corresponding to $[q^m s; q^m s]$. We can therefore write $f(Y) = f^+(Y) + f^-(Y)$ where $f^+(Y)$ is the positive part and converges on $[0; q^m s]$ and $f^-(Y)$ is the negative part and converges and is bounded on $[q^m s; +\infty[$. The element $x^- = \varphi_q^{-m}(f^-(u))$ belongs to both $\widetilde{\mathbf{B}}^{[r;s]}$ (since $x^- = x - x^+$) and to $\widetilde{\mathbf{B}}^{[s;+\infty[}$, so that it belongs to $\widetilde{\mathbf{B}}^{[r;+\infty[}$.

We now claim that if the power series $f^-(Y)$ converges on the annulus $[q^m s; +\infty[$ and if $f^-(u)$ belongs to $\widetilde{\mathbf{B}}^{[q^m r;+\infty[}$, then $f^-(Y)$ converges on $[q^m r; +\infty[$. In the cyclotomic case, this is proved in lemma II.2.2 of [CC98]. The proof in the LT case is analogous. This implies that $f(Y)$ converges on the annulus corresponding to $[q^m r; q^m s]$.

11. Lubin's conjecture for full p-adic dynamical systems

Corollary 2.7. It is not true that $\mathrm{L}_{\mathcal{F}}(a) = \mathrm{L}_{\mathcal{F}}(b)$ implies that $a = b$, even if $\mathrm{L}_{\mathcal{F}}(a)$ is in $\mathrm{Fil}^1 \mathbf{B}_{\mathrm{dR}}$. We need a and b themselves to be in $\mathrm{Fil}^1 \mathbf{B}_{\mathrm{dR}}$. The additional argument is the one that is in the proof of lemma 4.1 of [Spe18]. Assume that $\tau = \mathrm{Id}$ (otherwise twist everything by τ). Let $x_n = \theta \circ \varphi_q^n(x)$. We have $g(\mathrm{L}_{\mathcal{F}}(x_n)) = \eta(g) \cdot \mathrm{L}_{\mathcal{F}}(x_n)$ for all $n \geqslant 1$ so that $\mathrm{L}_{\mathcal{F}}(x_n) = 0$ for all $n \geqslant 1$. The x_n are zeroes of $\mathrm{L}_{\mathcal{F}}$ that converge to 0 so that $x_n = 0$ for $n \gg 0$. This implies that $\varphi_q^n(x) \in \mathrm{Fil}^1 \mathbf{B}_{\mathrm{dR}}$ for $n \gg 0$ and the proof of corollary 2.7 now works with $\varphi_q^n(x)$ for $n \gg 0$ instead of x.

12. Iwasawa theory and F-analytic Lubin-Tate (φ, Γ)-modules

Theorem 3.5.3. In the second displayed formula, $\exp^*_{F_n, V^*(1-j)}$ should be replaced by $\exp^*_{F_n, V(\chi_\pi^j)^*(1)}$

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