JM **FOCUS ON FLUIDS**

Lagrangian turbulence in the woods

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A recent statistical analysis, proposed by Shnapp (*J. Fluid Mech.*, vol. 913, 2021, R2), of Lagrangian velocity measurements in a wind tunnel in the presence of a canopy (a forest or urban morphology), using three-dimensional particle tracking velocimetry (Shnapp *et al.*, *Sci. Rep.*, vol. 9, issue 1, 2019, pp. 1–13), is a great read. In this strongly anisotropic situation, despite the additional roughness induced by the canopy, it is shown that fluctuations of Lagrangian velocity increments over small time scales display very similar behaviour as those observed in homogeneous and isotropic turbulent flows. This is all the more true when focussing on the non-Gaussian and intermittent nature of these fluctuations. At much larger time scales, of the order and greater than the characteristic turnover time scale of the flow, anisotropies implied by the presence of the canopy are quantified using averages of the fluctuating kinetic energy conditioned upon the direction of Lagrangian velocity with respect to the mean Eulerian flow. Shnapp (2021) evidences that, indeed, the canopy modifies the velocity along the trajectories at large scales, in particular its variance, but leaves unchanged its local regularity, as it is pinpointed by the power-law exponents of the structure functions.

Key words: turbulent flows, turbulent boundary layers, intermittency

1. Introduction

The Lagrangian investigation of laboratory and numerical turbulent flows has been intensively developed over the last thirty years, as reviewed in Yeung (2002), Toschi & Bodenschatz (2009) and Pinton & Sawford (2012), following an intense and vast effort aimed at characterizing with precision the statistical behaviour of the Eulerian velocity field (Frisch 1995). Following velocity along the path of fluid particles is not only important from a fundamental point of view (Monin & Yaglom 1971; Tennekes & Lumley 1972), it is also an appropriate way to describe the mixing and dispersion properties of emitted tracers in geophysical situations (LaCasce 2008).

Fluctuations of velocity along trajectories were initially observed in direct numerical simulations (DNSs) of the Navier–Stokes equations in controlled situations aimed at

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investigating some aspects of homogeneous and isotropic turbulence (Yeung & Pope 1989; Chevillard *et al.* 2003; Biferale *et al.* 2004; Bentkamp, Lalescu & Wilczek 2019), and soon after in von Kármán swirling flows (Voth, Satyanarayan & Bodenschatz 1998; Mordant *et al.* 2001). Despite the artificial nature of the boundary conditions of DNSs, and the strong anisotropy of the experimental realisation of such flows driven by propellers, a remarkable agreement was observed in the statistical properties of the Lagrangian velocity in both situations (Arneodo *et al.* 2008). The Johns Hopkins Turbulence Database (Yu *et al.* 2012) could, for instance, be used to further confirm the picture which follows.

We recall in a few words some of the key ingredients of the phenomenology of homogeneous and isotropic turbulence in the Lagrangian framework, as reviewed by Chevillard *et al.* (2012). Using the notation of Shnapp (2021), the statistical and multiscale nature of a Lagrangian velocity component $v_i(t)$, with $i \in \{1, 2, 3\}$, is well captured by the following probabilistic ansatz:

$$\Delta_\tau v_i = \mathcal{B}\left(\frac{\tau}{T_L}\right)\Delta_{T_L}v_i, \tag{1.1}$$

where the velocity increment $\Delta_\tau v_i(t) = v_i(t + \tau) - v(t)$ is introduced. Equation (1.1) relates an equality in probability law between the random variable $\Delta_\tau v_i$, made up of the instances of the increments along the trajectories at a given time scale τ, and its instances $\Delta_{T_L}v_i$, at the large integral time scale of the flow T_L at which velocity decorrelates, weighted by a random scale-dependent multiplier $\mathcal{B}$. At large scales $\tau \gg T_L$, $\mathcal{B}$ tends to the deterministic value 1, meaning that the increment is statistically equal to $\Delta_{T_L}v_i$, usually taken to be a Gaussian random variable of zero average and of variance $2\langle v_i^2 \rangle$, as dictated by observations. In the inertial range $\tau_\eta \ll \tau \ll T_L$, where τ_η is the Kolmogorov dissipative time scale, $\mathcal{B}$ fluctuates in the same way as $(\tau/T_L)^h$, the randomness being encoded in the exponent h. Dimensional arguments, mostly based on the irrelevance of viscosity at these scales (Tennekes & Lumley 1972), suggest that $\langle h \rangle \approx 1/2$, at any scale τ. Using the language of the multifractal formalism (Frisch 1995), in this statistically averaged sense, we can say that the Lagrangian velocity shares the same local regularity as that of the Brownian motion. Further analyses of experimental and numerical data (Chevillard *et al.* 2003; Arneodo *et al.* 2008) indicate that, indeed, h fluctuates around its mean value, independently of both the Reynolds number and the geometry of the flow, which is known as intermittency. The level of Lagrangian intermittency is observed in the right proportion compared to that measured in the Eulerian framework, consistently with the elegant theory of Borgas (1993).

2. Overview

Much more could be said on the statistical behaviour of the multiplier $\mathcal{B}$ entering in (1.1), in particular on the rich and predictive physics that has been developed to include the differential action of viscosity at small scales $\tau \ll \tau_\eta$, where fluctuations of the velocity increment are similar to those of acceleration. Let us keep in mind that this ansatz is well posed and closed from a probabilistic point of view if we furthermore assume that h and $\Delta_{T_L}v_i$ are statistically independent. It is then possible to derive explicit predictions for the probability density function (p.d.f.) of $\Delta_\tau v_i$ and its moments (i.e. the structure functions), at any scale τ, for a given Reynolds number and a prescribed level of intermittency.

The novelty of the analysis of Shnapp (2021) is to show that this aforementioned Lagrangian phenomenology, initially designed for isotropic turbulent flows, gives a fair account of the fluctuations of velocity along the trajectories obtained in his wind tunnel

(Shnapp *et al.* 2019). Despite the presence of a strong mean flow, and a model canopy laid out on the bottom of the tunnel, Shnapp (2021) evidences that the subset of particles flying just above this rough surface exhibits a statistical behaviour in quantitative agreement with the theoretical predictions for the velocity increment p.d.f.s and moments obtained within this formalism, with furthermore the same level of intermittency. Noticing that the characteristic height of the canopy is of the order of $U_\infty T_L$, where U_∞ is the wind mean velocity, this important observation further illustrates that Lagrangian fluctuations at small scales are universal and decoupled from the large scale flow. Nonetheless, signatures of the anisotropic nature of the canopy are evidenced when comparing the variance and correlation time scales of the different components v_i, which impact $\Delta_{T_L} v_i$, changing weakly the distribution of $\mathcal{B}$, similarly to what was observed by Ouellette *et al.* (2006) and Huck, Machicoane & Volk (2019).

To characterize more precisely the anisotropic nature of the large scale flow from a Lagrangian perspective, Shnapp (2021) decomposes the set of the trajectories according to four quadrants which represent different directions of Lagrangian velocity with respect to the mean Eulerian flow. This original method of classification allows him to analyse the distribution of kinetic energy depending of the amplitude of the streamwise component, and makes some connections with the fluctuating nature of the drag induced by the canopy.

3. Future

The results of Shnapp (2021) remarkably show that Lagrangian tracking experiments are an original and fertile characterization of realistic turbulent flows, such as wind tunnels (Ayyalasomayajula *et al.* 2006; Shnapp *et al.* 2019), jets (Poulain *et al.* 2004; Viggiano *et al.* 2021) and channel flows (Stelzenmuller *et al.* 2017). These newly developed techniques shed new light on Eulerian measurements and simulations of modelled canopies (Bai, Katz & Meneveau 2015; Glick *et al.* 2020), and their consequences for the intermittency phenomenon (Katul *et al.* 2006; Dupont *et al.* 2020).

From a theoretical perspective, the Lagrangian framework naturally calls for the stochastic modelling of the trajectories using random walks, as they were developed for isotropic flows (Sawford 1991; Pope 2002; Viggiano *et al.* 2020). In the spirit of recent propositions made by Innocenti *et al.* (2020) and Shnapp *et al.* (2020), generalizing these approaches to anisotropic situations sounds like a fantastic perspective.

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