Rise-Time Regimes of a Large Sphere in Vibrated Bulk Solids

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We report experiments on the rise time *T* of a single large sphere within a sinusoidally vibrated bed (amplitude *a*) of uniform particles (diameter *d*). At fixed acceleration, we identify three distinct behavioral regimes both from visual observations and from the typical increase of *T* with frequency *f*. We observe two convective regimes separated by a critical frequency and, for low *a* and high *f*, a "nonconvective" regime. In the latter, the bed crystallizes and a size dependent rise is evidenced. We show the relevance of the nondimensional parameter a/d and deduce a scaling law of the form $f \propto d^{-1/2}$. [S0031-9007(97)02305-3]

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Over approximately the last ten years, significant attention has been given to the phenomenon of size segregation in granular mixture. This is motivated in part by the fact that size segregation is often an undesirable outcome of handling and/or processing operations of bulk solids [1]. In a vibrated bed of monodisperse dry granular material, behaviors (e.g., heaping [2,3], compaction [4], convection [5–8], fluidization [9–11], surface waves [12–14], or arching [15]) are highly dependent on the level and form of the external vibration, and are likely to influence the process of size segregation in a polydisperse bed.

In general, a large ball placed at the bottom of a vibrated bed will rise to the surface [16]. Depending on the properties of the system (e.g., ball size), the ball may be trapped on the surface or may reenter the bed [7,17]. If trapping occurs, the primary components of a mixture may eventually separate according to their size. In order to understand why a large ball (or intruder) rises, experiments and simulations have aimed at identifying possible mechanisms. Two main driving processes have been proposed. One is the formation of a bulk convective flow of the bed particles which, at the same time, carry the intruder [7,17–21]. The other is related to local rearrangements under gravity of the bed geometry, as supported by numerical simulations [22–24], and is possibly controlled by collective effects like arching [25] or bridges [26]. Experimentally, a "nonconvective" and size-dependent rising has been observed, but only in a two-dimensional bed [18,25]. In all cases, the time dependence of the phenomenon is not understood.

In this paper we report the influence of the macroscopic behavior of a monodisperse bed on the rise time *T* of a single large sphere. Three rise regimes are identified from the specific relationship between *T* and the frequency *f*. Our results show the distinct features of two convective flows: one where heaping occurs and the other where heaping is not present. For the first time, we observe in a three-dimensional bed a dependence of *T* on the intruder size. This occurs when the bed becomes so compact that it crystallizes. Finally, we emphasize the relevance of the dimensionless amplitude a/d which allows us to predict the rapid increase of *T* with frequency at high accelerations.

The experimental system consists of a fixed acrylic cylinder of inner diameter $D_{\text{cyl}} = 11.43$ cm and a piston mounted onto a Bruel & Kjar shaking head. The piston motion is controlled using an accelerometer with a feedback loop. We emphasize that the bed is excited only through vibrations of the piston. If the walls were moving, the sides of the granular bed during the flight would still be subjected to the shearing motion of the vibrating walls. If the walls are fixed, one can be sure that energy is provided only when the bed is in contact with the piston. Then the shearing effect of the walls is due only to the bed motion. We note also that heaping can be observed when a bed is continuously pushed upward by a piston [27]. The consequence is that, in our experiment, heaping can be observed even when the relative acceleration is smaller than unity. The cylinder is filled with monodisperse acrylic spheres of diameter $d = 3.175$ mm while a single large sphere (diameter $D = 8d$) of the same material is placed on the center of the piston floor. In order to reduce the buildup of static charge, the inside cylinder walls and particles are treated with a household antistatic agent. The initial undisturbed bed depth is $H \cong 16d$, and the bed aspect ratio is $H/D_{\text{cyl}} = 0.44$. A sinusoidal signal is used to vibrate the piston over a range of amplitudes $a/d \leq 2$ and frequencies $5 < f < 75$ Hz, corresponding to relative accelerations Γ between 0.6 and 11 ($\Gamma \equiv a\omega^2/g$, where $\omega \equiv 2\pi f$ and $g = 9.81$ ms^{-1}). The large sphere moves upwards until it emerges from the bed attaining its highest altitude above the piston. Its rise time *T* to reach the surface is carefully measured with a stopwatch. We point out that the sinusoidal signal used is a constant current drive. A constant voltage drive is usually used because the mechanical resonance is damped and that improves the sound quality of a loudspeaker, for instance. However, vibration of a granular bed is another issue. First, there is no need to ensure a flat response in frequency since the power supply is adjusted to obtain the desired level of vibration. Second, when the bed collides with the piston, the sinusoidal motion is perturbed regard-

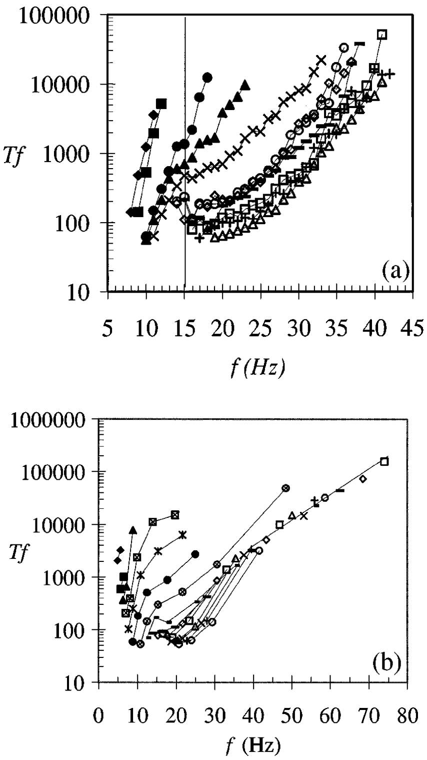

FIG. 1. (a) Dimensionless rise time Tf as a function of frequency f, for a range of relative acceleration Γ: (◆) = 1.5, (■) = 2, (●) = 2.5, (▲) = 3, (X) = 3.5, (○) = 4.5, (◇) = 5, (▬) = 6, (□) = 7, (+) = 8, (△) = 9. The vertical line delimits the first and second regime. Curves connect data of same Γ: (b) other data with (◆) = 0.6, (■) = 0.8, (▲) = 1, (⊠) = 1.25, (∗) = 1.5, (●) = 2, (⊗) = 3, (■) = 4, (▬) = 5, (◇) = 6, (□) = 7, (Δ) = 8, (X) = 9, (+) = 10, (○) = 11. For clarity, symbols are not connected in the third regime; only a single line shows the linear trend.

less of the type of signal used. In fact, we observed that the behavior of a granular bed changes when a sinusoidal vibration of given acceleration and frequency is applied by using either a constant voltage or a constant current drive. It seems that the way the piston reacts to the collision is at the origin of this difference. With a constant voltage drive, the system tends to resist the collision by increasing the current (and thus the applied force), which is controlled mainly by electrical and magnetic parameters. With a constant current drive, a constant sinusoidal force is applied to the piston and the collision can be described by a purely mechanical equation which is unaffected by a change in voltage during the collision (i.e., no electrical-mechanical coupling) [28]. Hence the choice of excitation is important. The constant current drive used in our experiments has the advantage of providing a better defined system in the sense that the applied force is always known, and it appears that it gives the closest agreement with previous studies [8,16].

Figure 1 shows the log of dimensionless rise time Tf (number of vibration cycles) plotted against f, where each curve represents a fixed value of Γ. Because of a restriction in the maximum peak to peak amplitude of the vibration apparatus, we were not able to obtain high accelerations at low frequencies. We note that the choice of Tf instead of T does not affect the qualitative features discussed. Mainly, three regimes can be described. First, we observe that heaping exists only below a critical frequency $f \cong 15$ Hz. When heaping is present, Tf scales exponentially with f, as can be seen from the nearly straight lines in Fig. 1(a). In this first regime, a strong internal convective flow, coupled with surface avalanches, carriers the intruder upwards.

The second behavioral regime appears for frequencies beyond 15 Hz and smaller than approximately 40 Hz, where Tf shows a more complex variation with f. Close to 15 Hz, Tf varies slowly with f, but increases rapidly at higher frequency. In this second regime, the bed surface no longer exhibits heaping. However, we think that convection still provides a driving mechanism. This is justified partially by our observations of the bulk motion, but also by the similarities between the scaling law we deduced and the one suggested by the recent granular convection experiments of Knight *et al.* [9]. They tapped a cylindrical container filled with uniform glass beads and counted the number of taps required for several tracer particles to reach the bed surface. Their results imply that $Tf = \tau(e^{H/\xi} - 1)$ with $\tau \propto e^{f/f_0}$ and $\xi \propto a \propto \Gamma/f^2$ (true for $f \geq 15$ Hz). Our data for Tf scale well with a law of the form $Tf \propto (\Gamma - \Gamma_c)^{-\beta}[e^{(f/f_{\rm cr})^2} - 1]$ where $\Gamma_c \cong 1$, $f_{\rm cr} \cong 15$ Hz, and $\beta \cong 2$. The fact that the scalings are different may be due to the fixed wall condition, to the low filling height used (for which Knight *et al.* observed significant deviations from their proposed scaling), to the lower aspect ratio $H/D_{\rm cyl}$ and/or the mode of excitation. Despite the differences between our experiments and those of Ref. [8], the scalings both contain a contribution growing like e^{f^2}, which is the essential term describing the quadratic shape of the curves appearing in Fig. 1(a) for $f > 15$ Hz. Consequently, we identify the second regime as a manifestation of the convective flow observed in [8]. We note that, at fixed frequency, there is a variation with Γ given by the term $(\Gamma - \Gamma_c)^{-\beta}$, qualitatively consistent with early observations [16]. The information given by $f_{\rm cr}$ is also important because its value corresponds well to the border between the first and second regimes.

In the third regime, the bed crystallizes into a hexagonal structure as can be seen from the top surface shown in Fig. 2. This regime is clearly observed when the amplitude is much smaller than the diameter of the bed particles, here $a/d \leq 0.25$. The corresponding rise time split appears in Fig. 1(b) for f between about 40 and 75 Hz and $\Gamma \geq 4$ [Fig. 1(b) also shows roughly the previous regimes]. The spread with Γ is minimal and the points almost lie on a straight line yielding exponential behavior. Thus when the frequency is fixed, the amplitude of vibration does not significantly affect the time response of the system. In fact, we will show later that a/d characterizes the evolution of

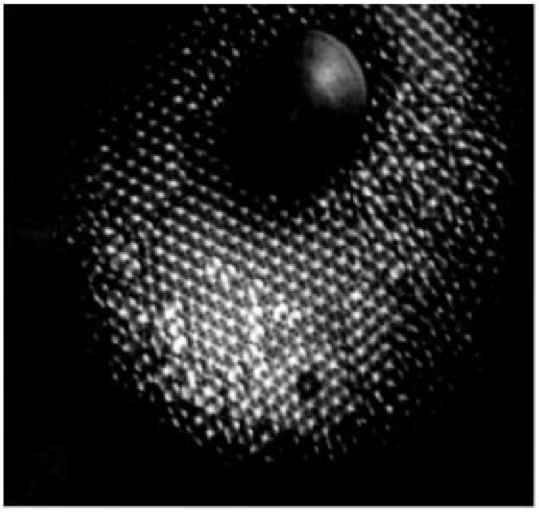

FIG. 2. View of the top surface when the bed crystallizes. Note the regular packing typical of a hexagonal structure.

the bed behavior from the second regime, where convection is observed, to the third regime where noncollective particle motions allow structural reorganizations. A clear distinction from the convective regimes appears when one looks at the motions of the particles around the intruder near the surface. Indeed, the particles are falling into the voids created below the intruder, while they would rise in the case of a convective flow. In addition, the mean velocity of the large ball depends on the size ratio $\Phi = D/d$, as shown in Fig. 3 for $f = 50$ Hz and $a/d = 0.1$. The larger the ball, the faster it rises to the surface. *A priori*, this result is in contradiction with the occurrence of a convective flow which drags all the particles at nearly the same velocity, whatever their size, density, or shape is. Previously, effects of size ratio had been seen in a two-dimensional bed [19,20], but the extension of this result to three dimensions was until now not straightforward. In a two-dimensional (2D) system where a regular packing is the natural state readily sustainable at low accelerations, size dependent [18] as well as convective behaviors have been observed [18,19] , while in a three-dimensional system (3D), only convection has been detected [7,8]. However, in our 3D experiments, a strongly ordered structure was created by shaking at small amplitudes and high accelerations and consequently, we have been able to discover

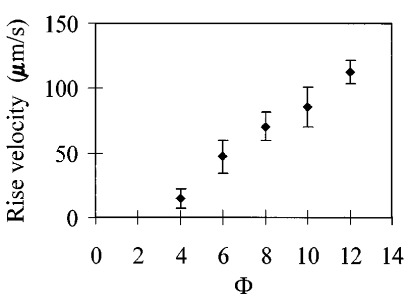

FIG. 3. Average rise velocity (μms^{-1}) as a function size ratio Φ, when $f = 50$ Hz and $a/d = 0.1$.

a size ratio dependence of T coupled with an apparently "nonconvective" regime. Far from being in a quasistatic state, the particles fixed in position by the "crystal" still have a significant rotational motion. Thus they can feel the repeated action of their neighbors and may eventually detect the presence of the intruder.

In order to examine the effect of vibration amplitude, the same data as in Fig. 1(a) are shown in Fig. 4 replotted against a/d for fixed values of Γ. There is a general decay of Tf with a/d, and for Γ greater than approximately 4, the data tend to collapse onto a single curve. Only the data for the first and second regimes are shown, but the same decreasing trend has been observed when the third regime is included. The curve exhibits the important feature that Tf increases quickly as a/d is reduced. Although a similar variation has been observed by Ahmad *et al.* [16], they did not notice the important role played by the amplitude. For $\Gamma \geq 4$, we will discuss the idea that the parameter controlling this variation is a/d. Ahmad *et al.* placed a large sphere in a vibrated bed of sand having a mean diameter $d_A = 0.5$ mm (the subscript refers to Ahmad) contained within a cylinder of 20.32 cm in diameter. The bed aspect ratio was $H/D_{\mathrm{cyl}} = 0.625$ ($H/D_{\mathrm{cyl}} = 0.44$ in our experiment). We performed an analysis of their data by selecting the a/d_A values at which there is an increase of about an order of magnitude in T, from where T varies slowly with f (see Fig. 4 in [16]). This gives us a measure of the change in the bed behavior. For a range of Γ between 5 and 10, fairly consistent values were obtained having a mean of $a/d_A \cong 0.37 \pm 0.08$. By following the same procedure with our data, we find that $a/d \cong 0.4 \pm 0.1$. Since we obtain comparable values of a/d, we suspect that the value of the amplitude alone is not enough to describe the response of the system but that it is a/d which correctly characterizes the rapid growth of T towards the small amplitude. The corresponding frequencies in the two experiments are completely different. We can show that they depend on the diameter of the bed particles. For this purpose we assume that the experiments of Ah-

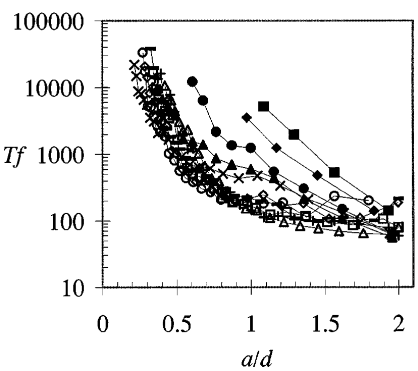

FIG. 4. Dimensionless rise time Tf as a function of dimensionless amplitude a/d, for a range of relative acceleration Γ: (◆) = 1.5, (■) = 2, (●) = 2.5, (▲) = 3, (X) = 3.5, (○) = 4.5, (◇) = 5, (▬) = 6, (□) = 7, (+) = 8, (Δ) = 9.

mad and ours are equivalent if the values of a/d and of Γ are the same in both experiments. Consequently, there is a relationship between the frequencies and ratio of bed particle diameters given by $(f/f_A) = \sqrt{d_A/d}$. Numerically, this gives $f_A \cong 2.5f$, which agrees reasonably well with the actual scaling between the two systems. This last result enforces the idea that the amplitude must be scaled by the diameter of the bed particles. The validity of the previous analysis relies on the hypothesis that disparities in particle properties or in geometry can be neglected as well as the fixed wall condition and the excitation type. At last, we note that if, in general, Tf decreases with Γ at fixed a/d, it is not the case when $a/d < 1$ and $\Gamma > 4$, where Tf increases with Γ at fixed a/d.

This Letter presents experimental results indicating the existence of three regimes in which a single large sphere can rise to the surface of a vibrated bed of uniform particles. The first regime, where convection is associated with heaping, corresponds to a variation of Tf like e^f at fixed Γ. Heaping, which is mostly due to shearing at the walls [3], exists as long as the shearing frequency is smaller than a critical frequency, and values of Γ control the amount of heaping, not its existence. The second regime, closely related to recent experiments of granular convection [8], corresponds to a variation of Tf like e^{f^2} at fixed Γ. Although the two convective regimes display different dynamic features, it cannot be concluded that the origin of convection is different. It is the type of vibration used (i.e., sinusoidal current of constant amplitude) which allows us to differentiate the two convective regimes. The interaction between the bed and the piston during the collision is the key to understanding how energy is transmitted into the granular media and how this affects the behavior. In contrast, we believe that the presence of fixed walls simply enhances the possibilities to observe heaping. The third regime can be seen as a 3D equivalent of the 2D size-dependent regime observed experimentally by Duran *et al.* [18] and Cooke *et al.* [19]. The third regime is nonconvective in the sense that the rise velocity depends on the size ratio. The particles are confined in a very stable crystal-like structure where it is likely that the rise phenomenon is controlled by structural defects due to the presence of the large sphere. Then, the more perturbed the structure, i.e., the larger the sphere, the faster is the rise. At last, the dimensionless amplitude a/d appears to be an important scaling parameter. The rapid increase of rise time when a/d becomes small can be understood as a finite size effect due to the discrete nature of the bed. Further investigation is required to better understand the transition parameters, and also the meaning of Γ. One key may be to look at the correlations between the bed collision times and the accelerations. However, it is also important to take in account the mechanical response of the vibration system during the collision with the bed.

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