Cours ENSL:
Big Data – Streaming, Sketching, Compression
Today: Plagiarism Detection

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Last time: Sketching – Streaming

Application Context

- Internet of Things (IoT) compress locally, send summaries
- Datacenters: dedicated to linear algorithms, inefficient for other classes
- In both contexts, there is a strong need to have data driven algorithms (where placement is imposed by data) and whose complexity is linear

Positionning

- w.r.t. traditional courses on algorithms
  - Exact algorithms for polynomial problems
  - Approximation algorithms for NP-Complete problems
  - Potentially exponential algorithms for difficult problems (going through an ILP for example)
- Here, we will consider extreme contexts
  - not enough space to transmit input data (sketching) or
  - not enough space to store the data stream (streaming)
  - not enough time to use an algorithm other than a linear complexity one (plagiarism)
- Compared to the more "classical" context of algorithms:
  - we aim at solving simple problems and
  - we are looking for approximate solutions only because we have very strong time or space constraints.

Compression associated to a specific function f

- More formally, given f,
- we want to compress the data X but still be able to compute $\simeq f(X)$.
- Sketching: we are looking for C_f and g such that
  - the storage space $C_f(X)$ is small (compression)
  - from $f(X)$, we can recover $f(X)$, ie $g(C_f(X)) \simeq f(X)$
- Streaming: additional difficulty, the update is performed on the fly.
  - we cannot compute $C_f(X \bigcup \{y\})$ from $X \bigcup \{y\}$
  - since we cannot store $X \bigcup \{y\}$
  - so we need another function h such that . $h(C_f(X), \{y\}) = C_f(X \bigcup \{y\})$
- and one last difficulty:
- very often, it is impossible to do in deterministic and exact / deterministic and approximate
- but only with a randomized and approximation algorithm.
- How to write this ?
  - Given α and ϵ, we are looking for an estimator Z such that
  - $Pr(|Z - f(X)| \geq \epsilon f(X)) \leq \alpha$.

Crash Course in probabilities

- Z random variable with positive values
- $E(Z)$ is the expectation of Z
- definitions and properties ?
  - $E(Z) = \int \lambda P(Z = \lambda) d\lambda$ or $E(Z) = \sum_j jP(Z = j)$
  - $E(Z) = \int P(Z \geq \lambda) d\lambda$ or $E(Z) = \sum_j P(Z \geq j)$
  - $E(aX + bY) = aE(X) + bE(Y)$
- To measure the distance from Z to $E(Z)$, we use the variance $V(Z)$
  - Definition?
  - $V(Z) = E((Z - E(Z))^2) = E(Z^2) - E(Z)^2$
- How to measure the difference between Z to $E(Z)$?
  1. Markov: $Pr(Z \geq \lambda) \leq E(Z)/\lambda$
  2. Chebyshev: $Pr(|Z - E(Z)| \geq \lambda E(Z)) \leq \frac{V(Z)}{\lambda^2 E(Z)^2}$
  3. Chernoff: If $Z_1, \ldots, Z_n$ are Independent Bernouilli rv with $p_i \in [0.1]$ and $Z = \sum Z_i$, then
     $Pr(|Z - E(Z)| \geq \lambda E(Z)) \leq 2 \exp(\frac{-\lambda^2 E(Z)}{3})$.

Morris Algorithm: Counting the number of events

- Step 1: Find an estimator Z
  - Z must be small (of order of $\log \log n$)
  - we need to define an additional function g
  - such that $E(g(Z)) = n$
- Morris algorithm
  - $Z \to 0$
  - At each event, $Z \to Z + 1$ with probability $1/2^Z$
  - When queried, return $f(Z) = 2^Z - 1$
- Proof scheme
  - $E(2^Z) = n + 1$ and $V(2^Z) \leq n^2/2$ and apply Chebychev to obtain $Pr(|f(X_n) = \tilde{n} - n| \geq \epsilon n) \leq \frac{1}{2\epsilon^2}$
  - Do the same experiment plenty of times to obtain $Pr(|f(X_n) = \tilde{n} - n| \geq \epsilon n) \leq \alpha$
  - To obtain a lower constant, use above scheme to get $Pr(|f(X_n) = \tilde{n} - n| \geq \epsilon n) \leq 1/3$ and then use the median of several experiments to conclude

2nd example: how to count the number of unique visitors

Context

- We assume that visitors are identified by their address ($i_k \in [1, n]$)
- We observe a flow of m visits $i_1, \dots, i_m$ with $i_k \in [1, n]$
- How many different visitors ?
- Deterministic and trivial algorithms:
  - solution in n:n bit array
  - solution in $m \log n$: keep the whole stream!
- We have proved (when $m = \Theta(n)$) that
  - we cannot do better with exact and deterministic algorithms
  - we cannot do better with approximated and deterministic algorithms
- With only $O(\log^k n)$ (for some k), it is possible by using both randomization and approximation.

Idealized algorithm (Flajolet & Martin)

We start with an idealized algorithm (one that cannot be implemented in practice).

- Let us choose a random h function from $[1, n]$ to $[0, 1]$
- Why idealized?
  - Problem 1: to store such a random function, you must define the images for in each of the n points... at least $\Omega(n)$ bits
  - Problem 2: and in addition we would have to store real values!
  - Let us assume for now that storing such a function costs $\Theta(1)$
- How to keep track of the number of unique visitors?
- We keep $Z \longrightarrow \min_{i \in \text{stream}} h(i)$. Intuition?
  - If you see the same visitor k times, it won't change Z
  - If we see t different visitors, then the values taken by h split $[0, 1]$ in $t + 1$ intervals... and all should have the same size in expectation... and this size is $\frac{1}{t+1}$ including the first !
- so we should return $\frac{1}{Z} - 1$!

Non Idealized algorithm – Hash functions

- To make it practical, we will consider a large (but not too large) family of functions $\mathcal{H}$ from $[1, p] \rightarrow [1, p]$
- Notion of k-wise independence
    - $\forall i_1, \dots, i_k, \forall j_1, \dots, j_k$, $i_k \neq i_l$, and if we pick a random h function in $\mathcal{H}$, then
    - $P(h(i_1) = j_1$ and $h(i_k) = j_k)$ $1/p^k$
    - a larger value for k provides a "better" family
- Example: the set of polynomials $\mathcal{H}^k_{\text{poly}}$ of degree k in F_p
    - evaluation cost? for degree k, k mult & and adds
    - independence? how many polynomials such that $(h(i_1) = j_1$ and $h(i_k) = j_k$
    - exactly one, Lagrange polynomial: $P = \sum_{r=1}^{k} \frac{\prod_{l \neq r}(X - i_l)}{\prod_{l \neq r}(i_r - i_l)} \times j_r$
    - choice? picking a function at random in $\mathcal{H}^k_{\text{poly}} \rightarrow$ picking $k+1$ coefficients.
    - and thus the family $\mathcal{H}^k_{\text{poly}}$ is k-independent
- can be easily generalized to different sizes $F_p \longrightarrow F_q$

Finding Similar Itemsets

General Idea

- 2 type of difficulties related to
  - the number of objects: N objects $\longrightarrow N^2$ comparisons
  - the objects themselves : large texts,...
- Applications
  - pages with a lot of text in common (mirror sites, approximate mirror)
  - plagiarism (today)
  - group news that deal with the same event
  - Amazon, Netflix: users with the same taste
  - dual: Amazon, Netflix: products with the same fans
- we will concentrate on texts, the first step only is application specific
  - Order of magnitude: 10^6 documents, size a few MB not huge (a few TB)
  - Distributed over a datacenter: large number of nodes $10^3 - 10^6$ nodes
- Two goals:
  - avoid moving data between the nodes (small and shared bandwidth)
  - avoid performing 10^{12} comparisons: both for time and data movements

Techniques

- 3 steps
  1. Shingling : conversion of a large text into a (large) set
  2. Min-hashing: assign to each text a (small) similarity-preserving signature
  3. Locality Sensitive Hashing: detect suspect pairs by collision detection
- randomized approximation algorithm $\longrightarrow$ errors: false positive and false negative
- what is crucial in our context ? Complexity: we want to deal with linear complexity algorithms only !
- Remark: we assume that the output is (at most) of linear size (otherwise, we have no chance !)

Shingling

- $k-$shingle
  - sequence of k successive characters in the text
  - $\{abcab\}$ et $k = 2$?
- Observation (admitted) close texts $\longrightarrow$ a lot of shingles in common
- A text: represented by the set of shingles it contains
- How many shingles ? with $k = 10$
- The data structure should enable to perform comparisons easily...
  - 30^{10} shingles $= 2^{49}$
  - size if stored as a vector of bits 70 TB
  - what happens in practice? Solution? shingles $\longrightarrow$ tokens
  - first use of hashing functions: adapt the size, control collisions

Min-hashing

- Each text is associated to a set of items
- We need to define a similarity between sets.
- Jaccard Similarity: $Sim(C_1, C_2) = \frac{C_1 \cap C_2}{C_1 \cup C_2}$
- $d(C_1, C_2) = 1 - Sim(C_1, C_2)$ is a distance (proof later)
  - one vector per document
  - one row per token token
- How to compute the similarity between two documents ?
- Problems:
  - we do not want to deal with N^2 pairs
  - we cannot centralize all N pairs at a single node

Minhashing

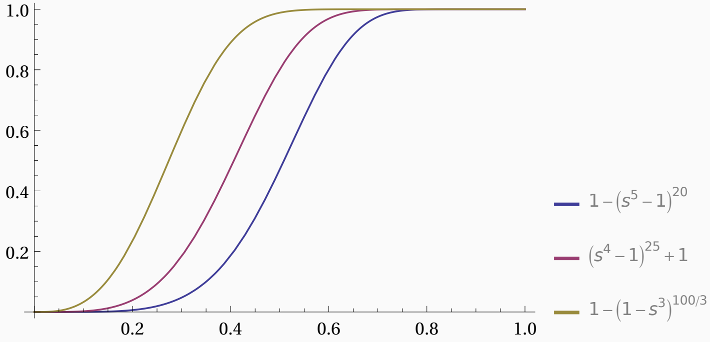
- Let us suppose that (C_1, C_2) are stored at the same place
- Goal: build a small similarity preserving signature for each document.
- General Idea: build a random game whose expected value (to win) is $Sim(C_1, C_2)$
- Do we really need to have (C_1, C_2) at the same place to play the game ?
- Do we really need permutations ?
- How many hash functions do we need in order to obtain a good precision ?

- So far: we have a very compact summary of each document (250×4B integers$= 1$kB)
- Last step: given a suspicion threshold $s \in [0, 1]$, return all pairs (C_1, C_2) such that $Sim(C_1, C_2) > s$
- Without doing all comparisons!
- Order of magnitude:
  - 10^6 documents $\longrightarrow 1GO$, Ok en mémoire
  - 10^{12} comparisons with 10^{-6}s per comparison 12 days)-;
- Goal: go from quadratic to linear complexity
- using hash functions again and collision detection
- now, we want close vectors to collide, and distant vectors not to collide

locally sensitive hashing

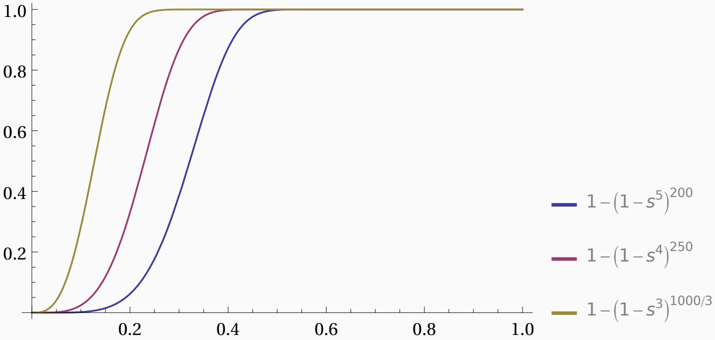
- split the summary (250 integers) into b blocks of size r ($rb = 250$)
- let h_k be the hash function associated to the k-th block
- collision: (almost) only if both vectors coincide on this block. Solution: use a large number of buckets (with respect to 10^6) $\longrightarrow 10^9$ is Ok in practice, very few false positive.
- a pair (C_1, C_2) is suspicious if $\exists k, h_k(B_k^1) = h_k(B_k^2)$ where B_k^i is the $k-$th block of C_i
- what happens ?
  - if r is too small ? too many false positive
  - if r is too big ? we will miss similar itemsets and get false negative
- Given r (and thus b) and $s = Sim(C_1, C_2)$, what is the probability that a collision occurs ?

with $N = 100$ and $r = 3, 4, 5$



false positive ? false negative ?

with $N = 1000$ and $r = 3, 4, 5$



Practical implementation ?

- distributed documents.
- keep everything local (until the computation of signatures)
- keep everything local (compute the hashing of each block) 1 document + 1 block $\longrightarrow$ 4B !
- gather all information related to one block number to the same node (4B + 1B for the index) $\longrightarrow$ 5MB
- detect all suspicious pairs (very few and send them to a specific node)...
- very few communications !

Locality Sensitive Hashing and Nearest Neighbors Search

($k-$) nearest neighbors

- Metric space with distance, set P of points
- preprocessing allowed on P
- Query: given a point, find its (k) closest neighbor(s)
  - example for spam classification: start with a huge annotated emails
  - one word = one item
  - return the k closest emails, majority vote to determine if it is spam or not
- Approach # 1: no preprocessing, just look through all possible items
  - space $O(dn)$
  - query $O(dn)$
- Approach # 2: if d=1
  - space $O(n)$ just keep the boundaries
  - query $O(\log n)$ just a basic binary search
- Approach # 3: if d=2
  - Voronoï diagrams: space $O(n)$ and computing cost $O(n \log n)$
  - query time easy (locate the cell)
  - As dimension increase, the description increases exponentially with d
- all exact (known) approaches in high dimension either have
  - exponential space $O(n^d)$
  - or exponential query time !
  - (same for $kd-$trees)
  - in very. large dimension, the naive algorithm is not that bad !

- Given a set of P points, construct a data structure such that
  - on query q, we return p in P such that
  - $d(p, q) \leq c \min_{p' \in P} \ d(p', q)$
- (r_1, r_2)PLEB problem: point location in equal balls
  - given a set P of points and r_1, r_2
  - construct a data structure to answer as follows
  - If $\exists p \in P$ st $d(p, q) \leq r_1$, return YES and any $p' \in P$ s.t. $d(p', q) \leq r_2$
  - If there is no $p \in P$ st $d(p, q) \leq r_2$, return NO
  - elif don't care what algorithm returns

Locality Sensitive Hashing (Indyk, Motwani)

- Usually, we want hashing functions to "shuffle" items as much as possible
  - When writing $P(h(i_1) = j_1 \;\&\; h(i_2) = j_1) = 1/p^2$, we say that the distance between images should not depend on the distance between initial points
  -
- Here, we want to detect "collisions"
  - we want close points to have a high probability to collide
  - we want distant points to have a low probability to collide
  - just as in the context of plagiarism.
- General idea
  - hash items into many different buckets (with different functions)
  - declare that there is a collision if two items fall into of the buckets
- Formal definition $\mathcal{H}$ a family of hash function $U \longrightarrow S$
- (where U is the set of points, S the set of buckets)
- is said to be (r_1, r_2, p_1, p_2) locality sensitive if
  - If $d(p, q) \leq r_1$, then $P_{h \in \mathcal{H}}(h(p) == h(q)) \geq p_1$ and
  - If $d(p, q) \geq r_2$, then $P_{h \in \mathcal{H}}(h(p) == h(q)) \leq p_2$
- of course $r_1 < r_2$, $p_1 > p_2$

Example

- Let $H^d = \{0,1\}^d$ equipped with Hamming distance (number of different coordinates)
- Let $\mathcal{H} = \{h_i, \forall i, \text{ where } h_i(b_1, \ldots, b_d) = b_i\}$
- $\mathcal{H}$ is $(r, cr, 1 - r/d, 1 - cr/d)$ locality sensitive
  - if p, q are at distance at most r, they have at least $(d - r)$ coordinates in common and thus a probability at least $1 - r/d$ to be hashed similarly,
  - if p, q are at distance at least cr, they have at most $(d - cr)$ coordinates in common and thus a probability at most $1 - cr/d$ to be hashed similarly.

Theorem
Suppose $\exists(r_1, r_2, p_1, p_2)$*-LSH family, then there is an algorithm for* (r_1, r_2)*-PLEB with answer queries with constant probability (it might be wrong), and that uses space* $O(dn + n^{1+\rho})$ *and query time* $O(n^\rho)$ *(evaluation of hash functions), where* $\rho = \frac{\log(1/p_1)}{\log(1/p_2)}$ *(complexity decreases when* ρ *decreases, ie when* $p_2 << p_1$*).*

Sketch of the proof — Algorithm

- let (k, l) be parameters (t.b.d. later), let G be a family of hash functions from U to S^k (new buckets), $g(p) = (h_{g_1}(p), \dots, h_{g_k}(p))$ each h_{g_i} being randomly chosen in H.
- Preprocessing:
  - (1) choose $g_1, \dots, g_l$ (other parameter) independently from G
  - (2) for each $p \in P$, store $g_1(p), \dots, g_l(p)$
- On query
  - (1) search the points of P in $g_1(q), \dots, g_l(q)$, but stop after the first $2l$ points in (the unlikely) case there are more than $2l$.
  - (2) If there is one point p such that $d(p, q) \leq r_2$ return it and return YES, otherwise return NO

Proof (continued)

- Intuition (1): if q and p are "close", then one the g_i will send them into the same k-bin.
- Intuition (2): it is unlikely that they are $2l$ distant (useless) points in the set (that would prevent to find the useful point)
- (2) There are at most $2l-1$ points st $d(p,q) > r_2$ and $\exists j,\ g_j(p) = g_j(q)$ with constant probability
  - Let $k = \log_{1/p_2} n$, what is the expected number of points st (2) holds ?
  - If $d(p,q) > r_2$ then for each h, the probability of collision is at most p_2
  - so the expected number of times p is written in any g_j is p_2^k, and the expected number of times it is written for a given g is lp_2^k
  - and the number of "bad points" written for g is therefore at most $nlp_2^k = l$
  - Markov says $P(X > \lambda) < E(X)/\lambda$ so $P(\text{more than } 2l \text{ bad points}) \leq 1/2$
- (1) If $p \in P$ with $d(p,q) \leq r_1$, then $\exists j,\ g_j(p) = g_j(q)$ with constant probability
  - If $d(p,q) \leq r_1$ then for each h, the prob of collision for h is at least p_1
  - the probability of collisions in one bucket is p_1^k
  - the probability of a collision in at least one of the l buckets is $1-(1-p_1^k)^l$ $= 1-(1-n^{\rho})^l$
  - choice of l ? if we set $l = n^{\rho}$ then the probability is at least $1 - 1/e \simeq 0.63$.
- to increase the probability, use the classical and tricks
- Check space and time complexities

Conclusion on sketching/streaming/compression

- Goal: data flow X and a function to evaluate f
- streaming: maintain a summary $C_f(X)$ enough to compute $f(X) =\simeq g(C_f(X))$
    - Solution: Use approximation randomized algorithms
    - $\forall \epsilon, \delta,\ Pr(\text{relative error} \geq \epsilon) \leq \delta$
    - enough (and often necessary) to change space complexities (from $\log n \rightarrow \log\log n$, $n \rightarrow \log n$, from n^d to $n^\rho d$)
    - at the price of sometimes large constants
    - but constants are pessimistic
    - and very small ϵ and α are not always required (plagiarism)
- General Idea:
    - Do less communications (same a lot of energy, time)
    - But more local computations (cheap)
    - crucial for IoT and datacenters
- Method:
    - Find an estimator Z tel que $E(Z) =$ what we want to estimate
    - go for $+$ and $++$ versions to control the probability
    - hash functions are a very powerful and versatile tool:
        - to shuffle potentially correlated entries (Unique Visitors)
        - to adapt the size of sets (Plagiarism)
        - to create short summaries (Min-Hashing)
        - to detect close items (Locality Sensitive Hashing)

A few resources on the web

- Algorithms for Big Data (CS 229r), Jelani Nelson (Harvard, now MIT)
- Mining Massive Data Sets (CS 246) Jure Leskovec, Anand Rajaraman, Jeffrey D. Ullman (Stanford).
- Algorithms for Big Data (CS 598) Chandra Chekuri (Urbana Champaign)
- Data Streams Algorithms (CS711) Andrew McGregor (UMass Amherst).
- Dealing with Massive Data (COMS 6998) Sergei Vassilvitskii (Columbia).