Task-graph scheduling to minimize memory

Loris Marchal

Joint work with Henri Casanova, Mathias Jacquelin,
Thomas Lambert, Yves Robert, Oliver Sinnen & Frédéric Vivien.

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Outline

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Motivation and previous work

Parallel tree processing

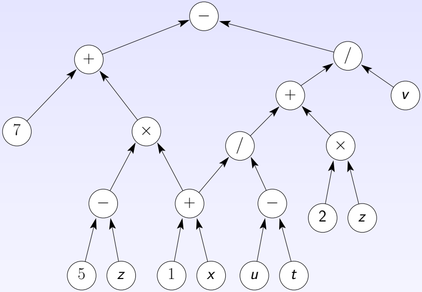
Series-Parallel graphs

Summary and Perspectives

Related Work: Register allocation

How to efficiently compute the following arithmetic expression with the minimum number of registers ?

$$7 + (1 + x)(5 - z) - ((1 + x)/(u - t) + 2z)/v$$



Pebble-game rules:

- Inputs can be pebbled anytime
- If all ancestors are pebbled, a node can be pebbled
- A pebble may be removed anytime

Objective: pebble all outputs using minimum number of pebbles

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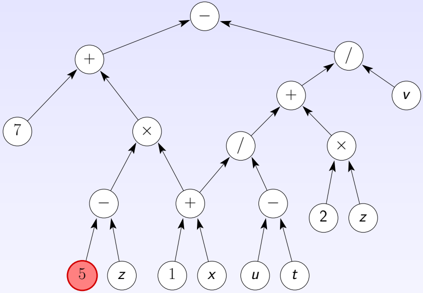

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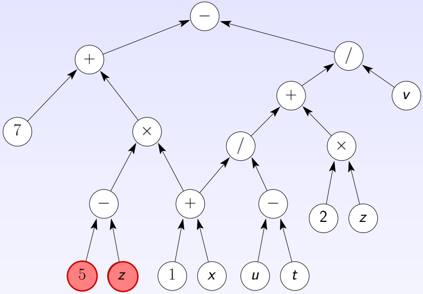

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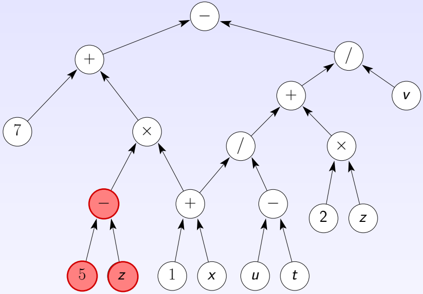

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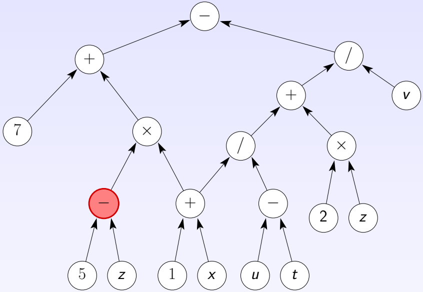

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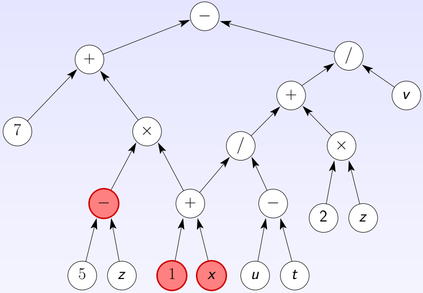

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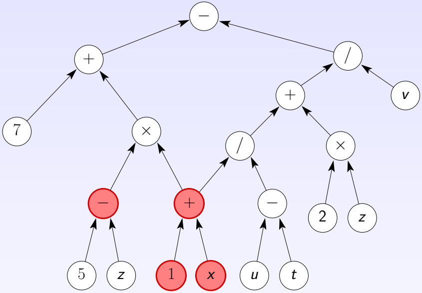

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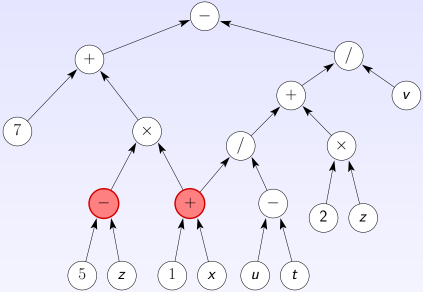

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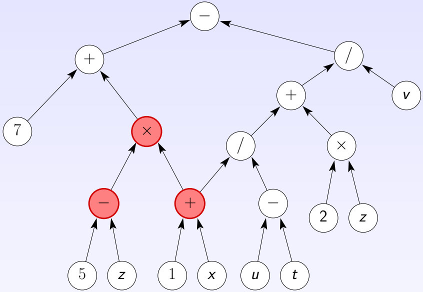

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Complexity results

General problem on DAGs:

- P-Space complete [Gilbert, Lengauer & Tarjan, 1980]
- Without re-computation: NP-complete [Sethi, 1973]

Problem on trees:

- Polynomial algorithm [Sethi & Ullman, 1970]

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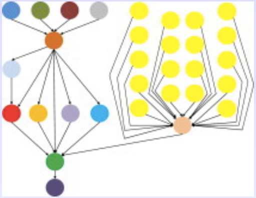
New motivation: scientific computing

- Workflows with large data files
- Bad evolution of performance for computation vs. communication: $1/\text{Flops} \ll 1/\text{bandwidth} \ll \text{latency}$



- Gap between processing power and communication cost increasing exponentially

	annual improvements
Flops rate	59%
mem. bandwidth	26%
mem. lantency	5%

- Avoid communications
- Restrict to in-core memory (out-of-core is expensive)

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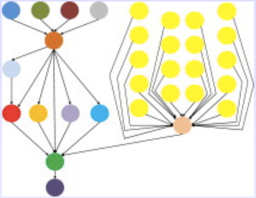

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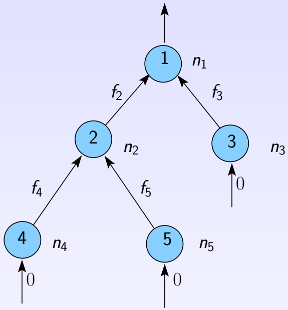
Existing algorithms on trees

- Context: multifrontal sparse factorization
- Assembly tree: the DAG of the application is a tree
- Large tree with large input files

Two existing algorithms:

- Best post-order traversal [J. Liu, 1986]
- Best traversal [J. Liu, 1987]

Introduction: tree-shaped workflows



- In-tree of n nodes
- Output file of size f_i
- Execution file of size n_i
- Input files of leaf nodes have null size
- Memory required for node i:

$$MemReq(i) = \sum_{j \in Children(i)} f_j + n_i + f_i$$

NB: top-down schedule = mirror of bottom-up schedule

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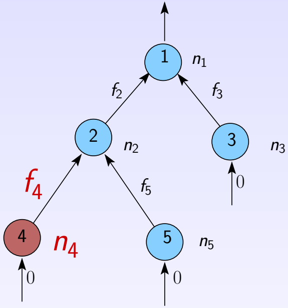

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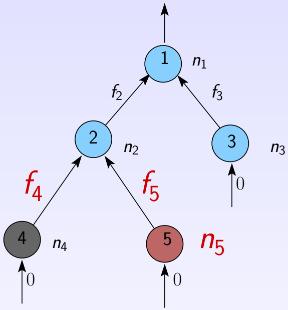

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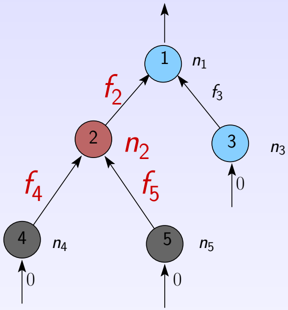

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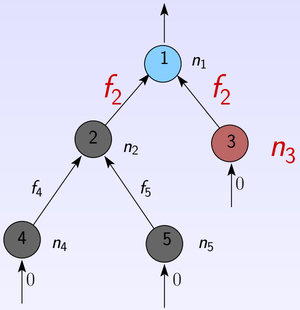

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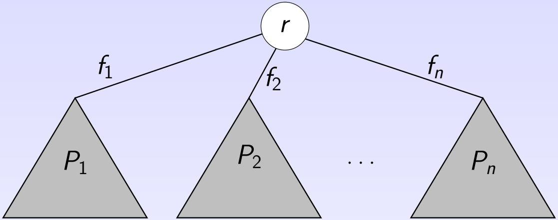
Liu's best post-order traversal for trees



- For each subtree T_i: peak memory P_i, residual memory f_i
- For a given processing order $1, \dots, n$, the peak memory is:

$$\max\{P_1,\ f_1 + P_2,\ f_1 + f_2 + P_3,\ \dots, \sum_{i<n} f_i + P_n,\ \sum f_i + n_r + f_r\}$$

- Optimal order:
- Postorder traversals are dominant when:

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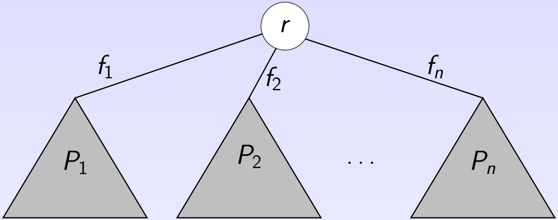

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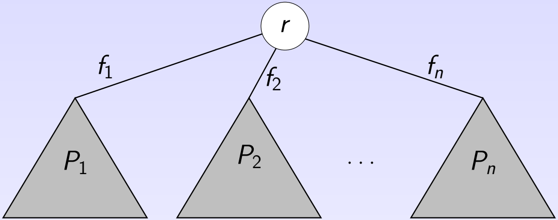

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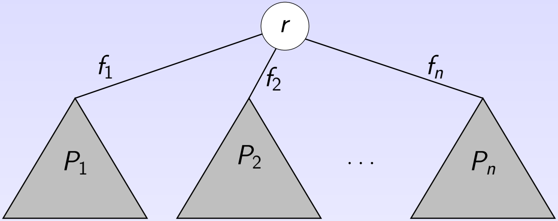

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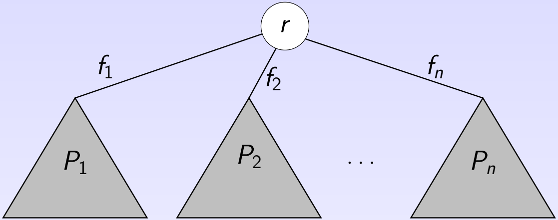

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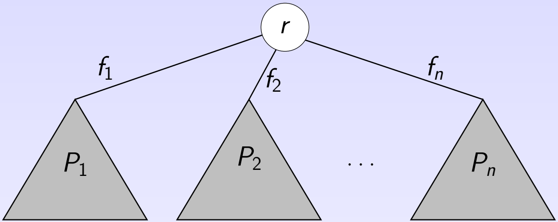

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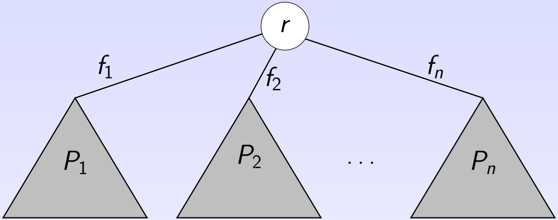

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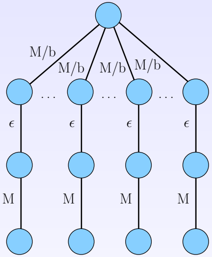
Postorder traversals are arbitrarily bad in the general case

There is no constant k such that the best postorder traversal is a k-approximation.



- Minimum peak memory: $M_{\min} = M + \epsilon + (b-1)\epsilon$
- Minimum postorder peak memory traversal: $M_{\min} = M + \epsilon + (b-1)M/b$

	actual assembly trees	random trees
Non optimal traversals	4.2%	61%
Maximum increase compared to optimal	18%	22%
Average increased compared to optimal	**1%**	12%

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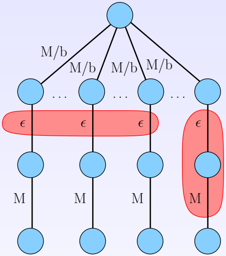

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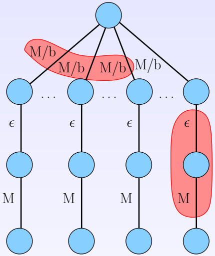

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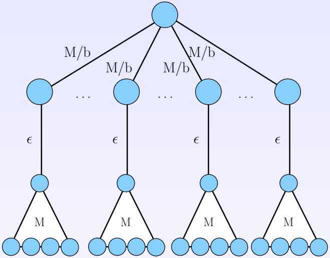

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Post-Order is not optimal...but almost!

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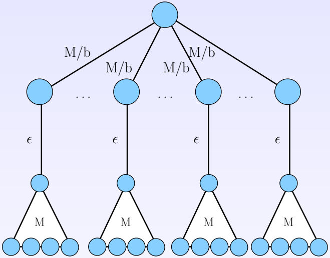

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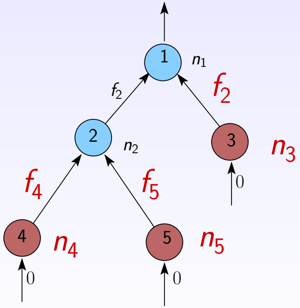
Parallel tree processing

Series-Parallel graphs

Summary and Perspectives

Parallel tree processing

- p identical processors
- Node i has execution times p_i
- Parallel processing of nodes $\Rightarrow$ larger memory
- Trade-off time vs. memory



NP-completeness in the pebble game model

Background:

- Makespan minimization NP-complete for trees ($P|trees|C_{\max}$)
- Polynomial when unit-weight tasks ($P|p_i = 1, trees|C_{\max}$)
- Pebble game polynomial on trees

Pebble game model:

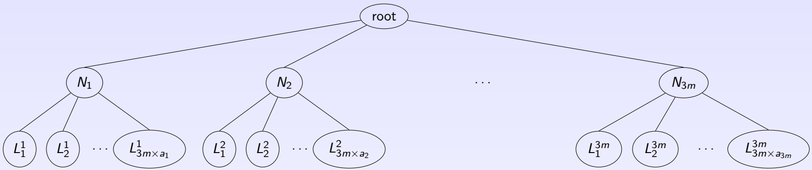
- unit execution time: $p_i = 1$
- unit memory costs: $n_i = 0, f_i = 1$
  (pebble edges, equivalent to pebble game for trees)

Theorem

Deciding whether a tree can be scheduled using at most B pebbles in at most C steps is NP-complete.

NP-completeness – proof

Reduction from 3-Partition:



Schedule the tree using:

- $p = 3mB$ processors,
- at most $B = 3m \times B + 3m$ pebbles,
- at most $C = 2m + 1$ steps.

Joint minimization of both objectives

No zenith approximation:

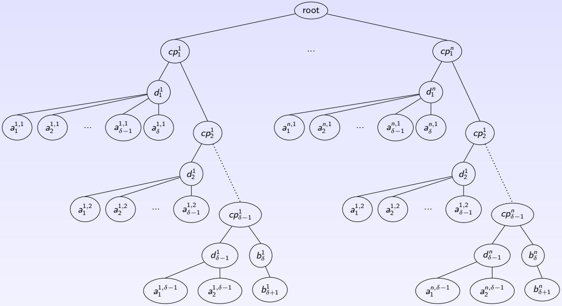
Theorem

There is no algorithm that is both an α-approximation for makespan minimization and a β-approximation for memory peak minimization when scheduling tree-shaped task graphs.

(proof sketch on next slide)

No zenith approx. – proof

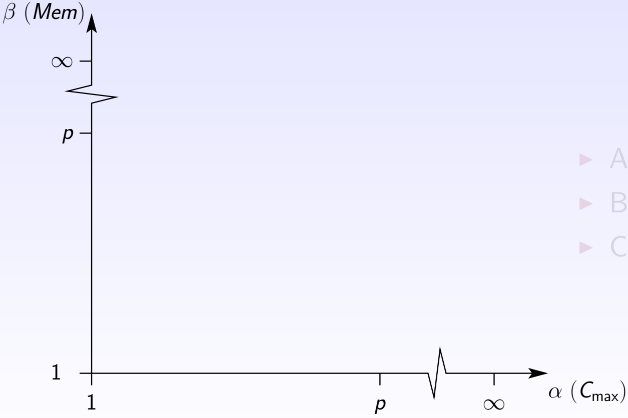
- n identical subtrees, largest in-degree is δ
- $M_{seq} = \delta + n$; $C^*_{max} \geq \delta + 2$ (critical path = height + 1)



- To achieve $\alpha C^*_{max} = \alpha(\delta + 2)$ each cp_k^i node needs to finish at $\alpha(\delta + 2) - 1$
- Calculate number of edges in each subtree, each edge present during a least two steps
- Calculate average memory with $\alpha(\delta + 2) - 2$ steps $\Rightarrow$ lower bound lb
- By setting $\delta = n^2$, we show that lb on memory is greater than 2β for any β we choose $\rightarrow$ contradiction

Approximability overview with fixed p

Inexistence of solutions which are α-approximation for the makespan and β-approximation for the memory, with **fixed number of processors**.



- A: $(\alpha, \frac{\sqrt{p+1}}{2\alpha} + \frac{1}{\alpha^2})$
- B: $(1, p-1)$
- C: $(\alpha, 1)$

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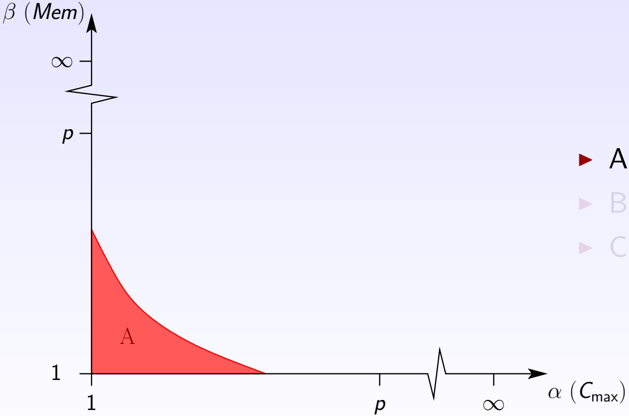

- A: $(\alpha, \frac{\sqrt{p+1}}{2\alpha} + \frac{1}{\alpha^2})$
- B: $(1, p-1)$
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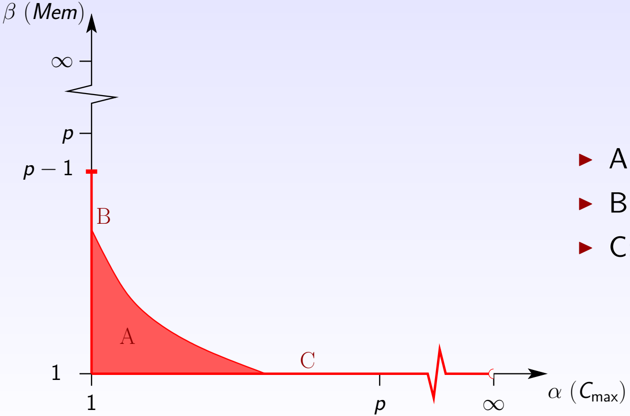

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Heuristics for weighted trees – 1/2

List-scheduling heuristics:

- Put ready nodes in a queue (sorted with some criterion)
- Schedule them whenever a processor is ready

Leaf nodes sorted using best sequential postorder

Two list-scheduling heuristics:

- Deepest-First (longest critical path, makespan oriented)
- Inner-First (memory oriented, sort of parallel postorder)

Performance:

- $(2 - 1/p)$-approximation for makespan
- Unbounded ratio for memory

Heuristics for weighted trees 2/2

Another memory-oriented heuristic:

- Split tree into subtrees
- Process p subtrees in parallel
- Process remaining nodes sequentially

$$C_{\max} = \max_{p \text{ largest subtrees } T_i} p(T_i) \quad + \sum_{\text{remaining nodes } j} p_j$$

Optimal subtree splitting (for makespan):

- Start with a single subtree (the tree)
- Split largest subtree until it is a single leaf node
- Store solution at each step
- Take the solution with minimal makespan

Memory guarantee:

- p-approximation algorithm

Optimization:

- Simple load-balancing of all subtrees to the processors

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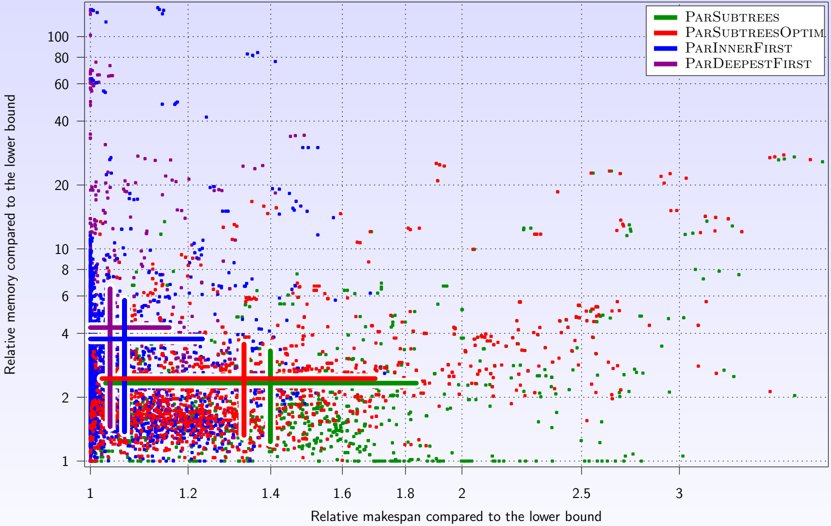
Experimental testbed

- 76 assembly trees of a set of sparse matrices from University of Florida Sparse Collection
- Metis and AMD ordering
- 1, 2, 4, or 16 relaxed amalgamation per node
- 608 trees with:

number of nodes:	2,000 to 1,000,000
depth:	12 to 70,000
maximum degree:	2 to 175,000

Results

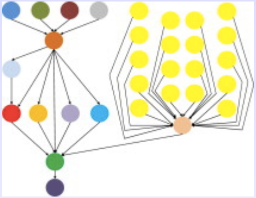


- Memory lower bound: best sequential postorder
- Makespan lower bound: $\max\left\{\frac{W}{p}, W_{\text{critical path}}\right\}$

Outline

Series-Parallel graphs: Motivation

- Not all scientific workflows are trees
- But most workflows exhibit some regularity
- Large class of workflows: Series-Parallel graphs



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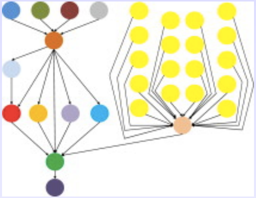

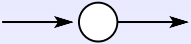

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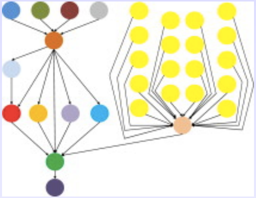

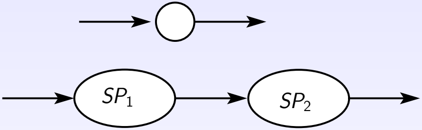

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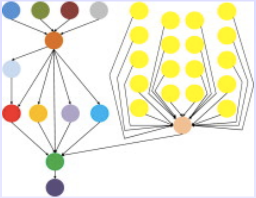

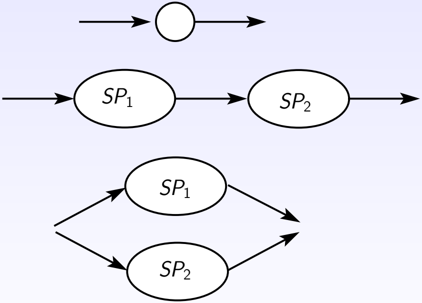

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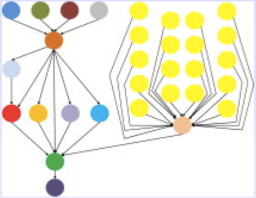

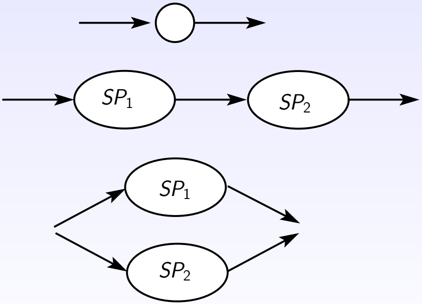

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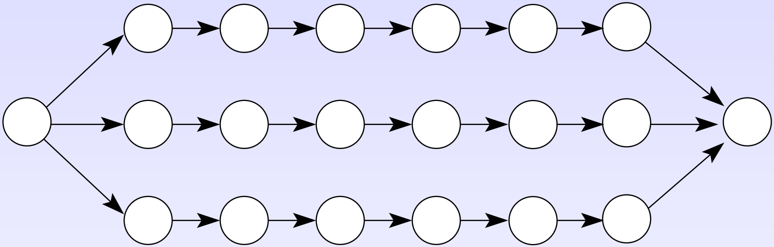
First step: fork-join graphs



Select edges with minimal weight on each branch: $e_1, \ldots, e_B$

Theorem

There exists a schedule with minimal memory which synchronises at $e_1, \ldots, e_B$.

Algorithm:

1. Apply optimal algorithm for out-trees on the left part
2. Apply optimal algorithm for in-trees on the right part

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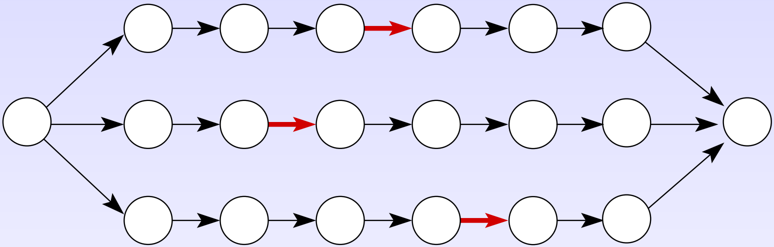

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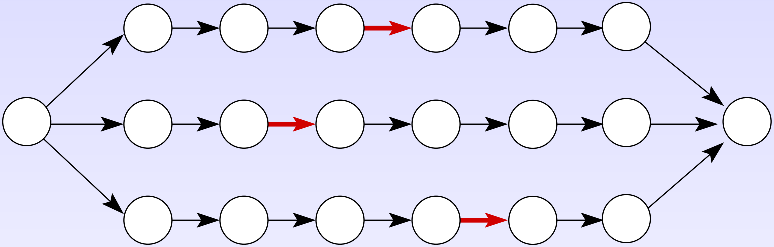

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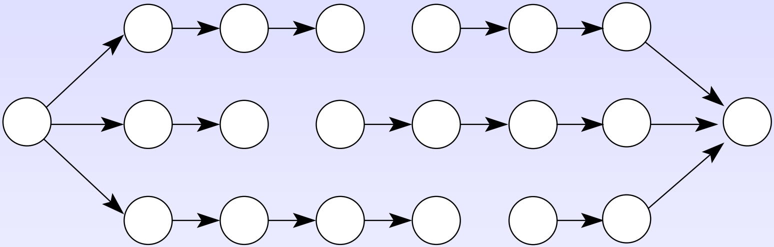

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General Series-Parallel graphs: work in progress

Recursive algorithm:

- Apply fork-join algorithm starting with innermost parallel composition
- Replace parallel composition with sequential schedule

Good candidate for optimal algorithm:

- Always optimal in brute-force simulations
- Sketch of proof, adapted from Liu

Outline

Summary and Perspectives

- Comprehensive study of tree-shaped task graphs (postorder, optimal sequential, complexity and heuristics for parallel processing)
- Adaptation to Series-Parallel graphs

Future work:

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- Extend results to other class of regular graphs ($2D$ grids, etc.)
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Thank you !